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Optical Measurement of Radiometric Aperture Areas

Ulf Griesmann

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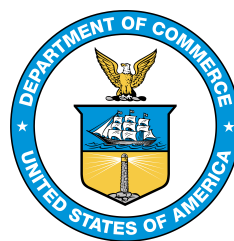
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Sensor Science Division

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Abstract

The National Institute of Standards and Technology (NIST) calibrates the areas of radiometric and photometric apertures using a coordinate measuring microscope, which provides direct traceability of aperture areas to the meter, the length unit of the International System of Units (SI). This measurement system combines a two-dimensional motion stage with laser interferometers for position feedback, and a high-magnification optical transmission microscope. For area measurements, apertures are mounted on the motion stage in the focal plane of an illuminator. The microscope is then used to locate points on the edge of the apertures in the coordinate system of the motion stage. The aperture area is calculated from a set of aperture edge positions that have been measured along the full length of an aperture's edge. This publication describes the measurement instrument, provides a summary of the general measurement procedure, and documents the software algorithms that are applied in the measurements. The traceability of aperture area measurements to the SI, and the components of measurement uncertainty that contribute to the total measurement uncertainty stated in the calibration reports are explained.

Keywords

aperture area; laser interferometry; optical microscopy; coordinate measuring microscope; radiometric apertures.

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1. Introduction

Apertures with precisely known areas are used in radiometry and photometry to define the optical phase space into which radiant flux is emitted or received. A simple application is the measurement of irradiance, the radiant flux received at a surface per unit area (or its photometric analog, illuminance), which requires an aperture with known area to be placed in front of an optical radiation sensor.

Several different methods have been developed for aperture area measurements that are traceable to the length unit of the International System of Units (SI). For apertures without a knife edge, a coordinate measuring machine (CMM) can be used to locate the aperture edge, and the area is calculated from the edge locations [1]. (Areas of knife edge apertures have also been measured with a CMM using a specially designed fiber probe [2].) In a second category of measurement methods, the area of a radiometric aperture is calculated from the fraction of light that is transmitted by an over-filled aperture [3, 4]. The beam illuminating the aperture is synthesized from a Gaussian beam that is rastered over the aperture so as to emulate a light field with nearly uniform flux. A third group of measurement systems are coordinate measuring systems that rely on an optical probe, e.g. involving a laser beam [5] or a microscope [6, 7], to find the location of an aperture edge. As in the case of the tactile CMM, the aperture area is calculated from the edge points using the methods described in the following Sec. 2.

Beginning in the 1990s the National Institute of Standards and Technology (NIST) has worked on improved methods for the measurement of radiometric aperture areas to meet the need for aperture area measurements with combined relative standard uncertainties of about 10^{-4} . As part of this effort, a custom optical coordinate measuring machine was constructed at NIST for absolute aperture area measurements that are traceable to the SI meter [8–10]. This aperture area measurement (AAM) system consists of a planar air bearing stage with laser interferometer position feedback on both motion axes, that is used to precisely locate the edge of an aperture in the focal plane of a high-magnification optical microscope. A set of edge points along the full length of the aperture edge is determined using the motion stage and the calibrated microscope, and the aperture area is calculated from the edge points. The AAM instrument has participated in an international comparison of aperture area measurements [11], and it has been used to provide a calibration service for aperture area since the early 2000s.

During a recent renovation of the AAM instrument, several instrument components were updated or replaced. The operating software for the AAM instrument and the software for data analysis was also replaced to take advantage of now widely available algorithms for image processing. This special publication describes the renovated AAM instrument, the traceability of aperture area measurements to the SI, and it documents key software algorithms used during aperture area measurements and

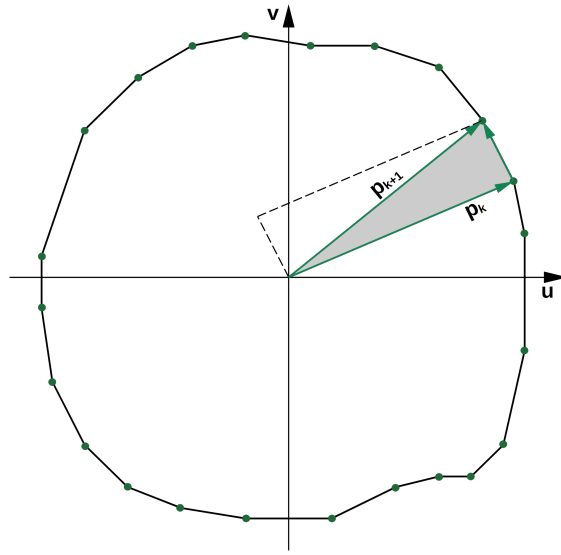


Fig. 1: Calculation of aperture areas from a set of aperture edge points, which are ordered to create a polygon with positive orientation. The exterior product $\mathbf{p}_k \wedge (\mathbf{p}_{k+1} - \mathbf{p}_k)$ can be visualized by the parallelogram indicated with the dashed lines.

in the measurement data analysis. Much of the following discussion centers on circular apertures because those are by far the most common type of radiometric aperture. The AAM instrument is, however, not limited to the measurement of circular apertures and ways to extend the measurements to other aperture shapes are indicated where appropriate.

2. Area Measurement Concept

A radiometric aperture is understood here as a precisely machined opening in a thin sheet made of an opaque material, such that the edge of the aperture can be described by a simple closed curve in a plane. The AAM instrument measures the position of the aperture edge in the plane by locating a set of discrete points $\mathbf{p}_k$ on the edge of the aperture as indicated in Fig. 1 (k is an index enumerating the edge points). The exterior product [12, 13] of vectors $\mathbf{p}_k$ and $\mathbf{p}_{k+1} - \mathbf{p}_k$ is a 2-vector (or bivector) with a magnitude (oriented area) twice that of the triangle shown in gray in Fig. 1. It follows that the area of each triangle is given by

$$\frac{1}{2} \mathbf{p}_k \wedge (\mathbf{p}_{k+1} - \mathbf{p}_k), \quad (1)$$

where $\wedge$ denotes the exterior product. Summing the areas of the triangles corresponding to all the edge points $\mathbf{p}_k$ results in the area A enclosed by the polygon:

$$\begin{aligned} A &= \frac{1}{2} \sum_{k=0}^{N-1} \mathbf{p}_k \wedge (\mathbf{p}_{k+1} - \mathbf{p}_k) \\ &= \frac{1}{2} \sum_{k=0}^{N-1} \mathbf{p}_k \wedge \mathbf{p}_{k+1} . \end{aligned} \quad (2)$$

In Eq. 2 it is assumed that $\mathbf{p}_N \equiv \mathbf{p}_0$. When edge points $\mathbf{p}_k$ are described by pairs of coordinates (u_k, v_k) in a coordinate system with an orthonormal basis, $\mathbf{p}_k = u_k \mathbf{e}_u + v_k \mathbf{e}_v$, Eq. 2 becomes the formula known as Gauss’s area formula [14] or the surveyor’s area formula [15]

$$A = \frac{1}{2} \sum_{k=0}^{N-1} u_k v_{k+1} - v_k u_{k+1} . \quad (3)$$

For the common case of a circular aperture with radius R , and N equally spaced edge points that have been measured along the aperture edge, the area formula in Eq. 2 underestimates the area of the circular aperture. The difference between the area of the circle and the area of the inscribed polygon with N vertices, or area defect δA , is given by:

$$\delta A = \pi R^2 \left[1 - \frac{N}{2\pi} \sin\left(\frac{2\pi}{N}\right) \right] . \quad (4)$$

The area defect δA in Eq. 4 becomes zero as $N \rightarrow \infty$. Eq. 4 can be used to estimate the minimum number of edge position measurements along a circular aperture edge that are required for an aperture area measurement to not exceed a given area defect. For example, at least 257 equally distributed aperture edge position measurements are required to achieve a relative area defect smaller than 10^{-4} when the aperture area is calculated with Eq. 3 from the edge point measurements.

In addition to the surveyor’s formula Eq. 3, which applies to apertures with arbitrary shape, areas of apertures designed to have circular, elliptical, or rectangular shapes can be estimated by numerically fitting an aperture edge model to the set of measured edge points $\mathbf{p}_k$. The aperture area is then calculated from the parameters of the edge model, for example from the aperture radius in the case of circular apertures. Following a fit of an aperture model, the residuum of the fit must be examined to ensure that the aperture edge is adequately described by the model. An extensive scientific literature exists on algorithms for the numerical fitting of circles and ellipses to points in a plane (see e.g. references [16–19]). Rectangular or polygonal aperture edges are modeled as sequences of best-fit line segments [20] (unless otherwise noted, “best-fit” always refers to a least-squares fit).

3. Motion Stages

At the heart of the AAM instrument is the planar motion stage with two orthogonal axes shown in Fig. 2(a) and (b). It is constructed on a precision-ground flat granite base, which provides the horizontal guide surface for a system of vacuum-preloaded air bearing stages. The arrangement of the air bearing stages is illustrated in a simplified diagram Fig. 3 to help explain the arrangement of the motion stages shown in Fig. 2(a). A steel bar, visible in Fig. 2(a) at the back of the motion stage, is mounted to the granite base and serves as the air bearing guideway for the X-axis of the stage system. This steel bar has a precision-ground vertical guide surface parallel to the X-Z plane, and facing the observer in Fig. 2(a), which serves as the guide surface for the X-axis of the motion stage. The motion stage consists of an aluminum bar that moves along the guide surface on the X-axis guideway using vacuum-preloaded air bearings, and a second, orthogonal bar that is the guideway for the Y-axis. Both guideways form the L-shaped structure seen in Figs. 2 and 3. The L-shaped guideway structure is supported against the granite base by three air bearings at the ends and at the corner of the L-shaped stage. The square platform at the center of Figs. 2(a) and (b) is used to move parts under test in the field of view of a microscope during measurements. This measurement platform is guided along the Y-axis guideway with a vacuum-preloaded air bearing and it is supported on the granite base by a set of vacuum-preloaded air bearings. The air gap of the bearing at the base of the measurement platform varies approximately linearly between 6 μm and 10 μm for air supply pressures between 2000 hPa (30 psi) and 4000 hPa (60 psi). The stage uses two linear motors for motion in the X- and Y-directions. The nominal workspace of the stage is 100 mm $\times$ 100 mm.

A third axis of motion in the direction orthogonal to the X-Y plane is provided by a linear stage that is mounted to a granite arch above the air bearing stages as can be seen in Fig. 2(a). This Z-axis stage is actuated by a precision lead screw. It has a range of motion of approximately 40 mm and a position repeatability of the Z-axis is about 0.5 μm . The repeatability is limited primarily by the large Abbe offset of 150 mm in the Z-axis stage between the lead screw of the stage and the glass scale encoder that is used for position feedback. The microscope used by the AAM system is installed on the Z-axis stage for positioning and focusing.

Both axes of the X-Y motion stage are controlled using helium-neon laser interferometers as position encoders. The two orthogonally polarized modes emitted by a helium-neon laser at vacuum wavelengths near 632.992 nm [21] are each used in two plane mirror interferometers to provide position feedback for one of the axes of the X-Y air-bearing motion stage. The mirrors of the laser interferometers can be seen in the image of the motion stage in Fig. 2(b). The laser beams of the two interferometers intersect at the optical axis of the microscope so that Abbe offset errors are eliminated for the X-Y motion stage. Distances measured with the laser interferometers depend on the refractive index of air in the interferometer paths. The laser interferometers

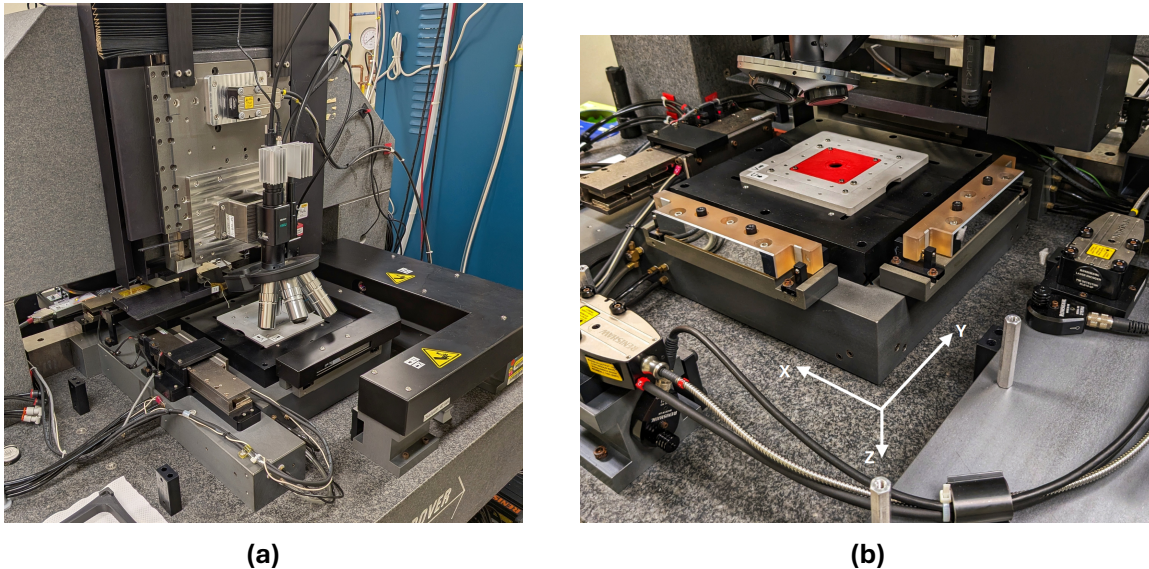


Fig. 2: (a) Overview of the AAM instrument with laser interferometer controlled planar X-Y air bearing stages and the vertical Z-stage that positions the microscope. (b) X-Y air bearing motion stage with the protective covers over the interferometer mirrors and the laser interferometers removed.

have compensators with sensors for air pressure and air temperature in the laser path, which continuously correct the laser wavelengths in air using a mathematical model for the refractive index of air [22, 23] and transmit a corrected position feedback signal to the motion controllers. (The effects of the air's water or carbon dioxide content on the refractive index is negligible for the resolution of the laser interferometers in the AAM instrument.) An additional set of pressure and temperature sensors, which are periodically re-calibrated, is used to validate the built-in sensors. The laser interferometers control the position of the X-Y stage with a resolution of 10 nm.

3.1. Stage coordinate systems and transforms

The air bearing guideways of the X-Y stage, the linear Z-axis stage, and the motion control logic used by the motion controllers for the stages define a right-handed Cartesian coordinate system, the *machine* coordinate system. For an observer standing in front of the AAM instrument and facing the instrument, the X-axis points from the right to the left, the Y-axis from the front to the back, and the Z-axis from top to bottom as indicated in Figs. 2(b) and 3(a). The origins of the X-Y-Z machine coordinates are close to the negative limits of the X,Y, and Z motion stages.

When working with the AAM instrument the machine coordinate system is inconvenient because it is difficult for a user to intuit the stage positions from the machine coordinate values. A second coordinate system, the *aperture coordinate system*, is,

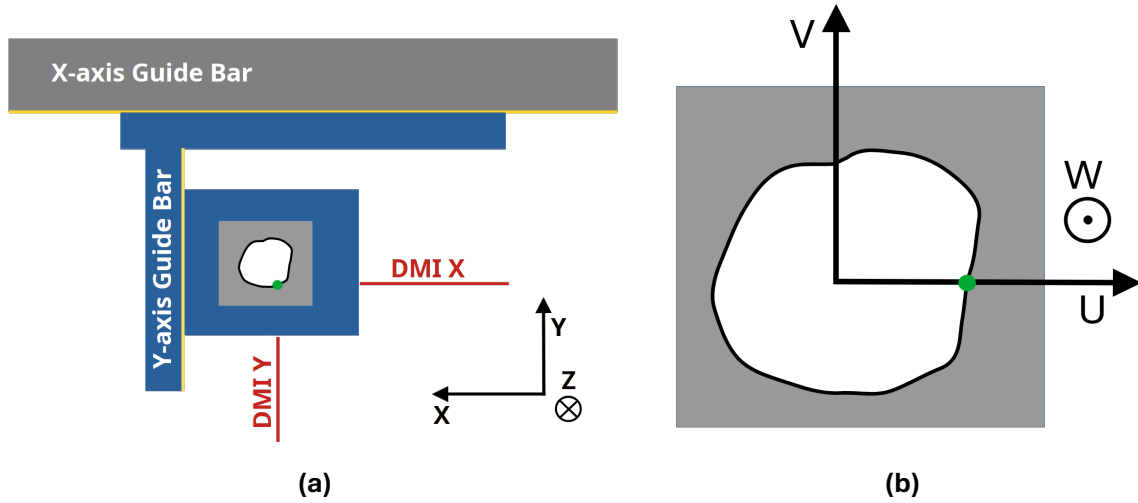


Fig. 3: Coordinate systems defined for the AAM instrument. (a) X-Y-Z are machine coordinates used by the motion control system with displacement measuring interferometers (DMI) as position encoders. (b) The U-V coordinates parametrize the aperture plane. They are defined with their origin at the aperture center and describe points *on the aperture or part under test*. The W coordinate is the distance of the microscope's focal plane from the aperture plane. The optical axis of the microscope is indicated by the green dot.

therefore, introduced to make it easier to describe or make aperture area measurements. The aperture coordinate center has its origin at the center of an aperture, or test part, as shown in Fig. 3(b). A point (u, v) in the aperture coordinate system is the point in the aperture plane that is at the center of the field of view of the microscope. For example, in the case of a circular aperture with radius R , aperture U-V coordinates $(R, 0)$ mean that the edge of the aperture is at the center of the microscope's field of view. Apertures are usually mounted at the center of the motion stage shown in Figs. 2, so that the origin of the U-V aperture coordinate system coincides with the motion stage center. For an observer standing in front of the AAM instrument, the U-axis points from the left to the right, the V-axis from the front to the back. The vertical W axis is defined as the distance of the microscope's focal plane from the aperture plane. The w coordinate is zero when the microscope is focused at the nominal aperture plane, $w > 0$ means that the microscope is focused above the aperture, and $w < 0$ when the microscope is focused below the aperture plane. The X-Y-Z machine coordinates and U-V-W aperture plane coordinate systems are illustrated in Figs. 3. The following linear transform converts machine coordinates (x, y, z) reported by the mo-

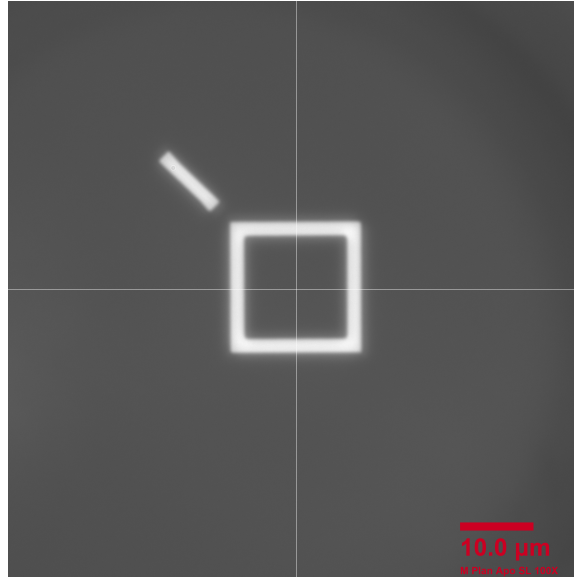


Fig. 4: Fiducial mark on the NIST SRM 5001 grid standard imaged with 100× magnification in reflectance mode. The cross-hair lines indicate the center of the microscope’s field of view.

tion controllers into aperture plane coordinates (u, v, w) :

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}. \quad (5)$$

The veracity of Eq. 5 is best illustrated with an example. A user has mounted an aperture with radius R at the center of the motion stage and wants to bring the edge of the aperture into the field of view of the microscope. This is accomplished by moving the aperture in the U-V coordinate system from the origin $(0,0)$ to the point $(u,v) = (R,0)$, which requires a *positive* displacement R in the machine coordinate axis X. Similarly, a *negative* displacement in the Y-axis is required to move the point $(0,R)$ in the U-V coordinate system into the field of view of the microscope. x_0 and y_0 in Eq. 5 are usually chosen so that the origin of the U and V coordinates, i.e. $(u,v) = (0,0)$, is at the motion stage center, and z_0 is set so that the microscope is nominally focused at the aperture plane when $w = 0$. The inverse transform from aperture plane coordinates to machine coordinates is readily obtained from Eq. 5.

3.2. Stage errors

Distance measurements of the X and Y motion axes are individually traceable to the SI meter through their displacement measuring laser interferometers. This does not, however, hold for coordinates in the X-Y plane reported by the X-Y motion stage, because the X-Y stage positions are affected by a residual non-orthogonality (skewness)

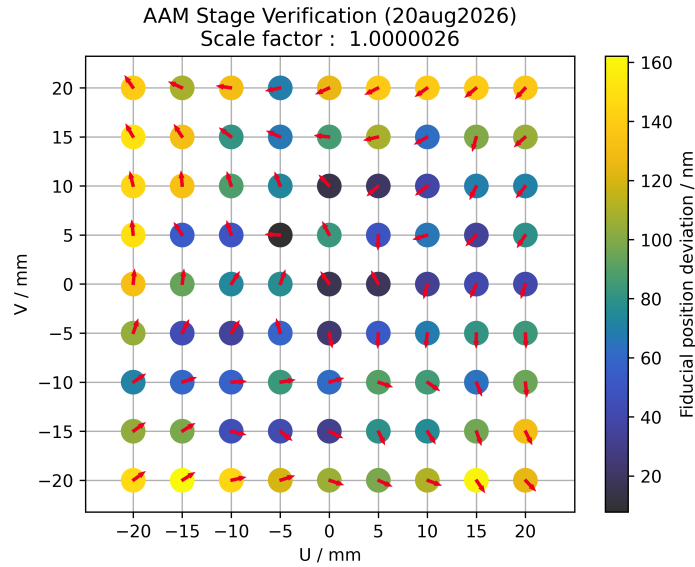


Fig. 5: Position error field of the AAM instrument at the X-Y stage center. Shown are the deviations of the measured fiducial positions from the calibrated fiducial positions on SRM 5001. The magnitude of the deviations is encoded in the color scale and arrows indicate the directions of the deviations.

of the motion axes and the flatness errors of the laser interferometer mirrors. These stage errors are characterized using NIST Standard Reference Material (SRM) 5001 [24, 25], a two-dimensional grid photomask with fiducial marks on a square grid with 5 mm spacing. SRM 5001 has a group of different fiducials at each grid point. The fiducial marks used to measure the errors of the X-Y stage are square frames with an overall width of 18 μm and a frame line width of 2 μm as shown in Fig. 4. The frame centers in the grid have certified positions with position uncertainties of around 10 nm. The fiducial position uncertainties on SRM 5001 are traceable to the SI meter through the NIST length scale interferometer [26]. For a measurement of the X-Y stage error, the positions of the grid fiducials are measured in the coordinate system of the X-Y motion stage. Each of the grid fiducials is imaged with the microscope (see Sec. 4) with a high-magnification objective, the position of the frame fiducial in the image is calculated using a correlation method described in Sec. 6.5, the frame fiducial is centered in the field of view, and the position is recorded. The deviations of the frame fiducial positions from the nominal 5 mm $\times$ 5 mm grid are calculated by aligning the measured fiducial center positions to their nominal positions in a least-squares sense using a numerical procedure known as a Procrustes analysis [27].

The Procrustes analysis estimates the rotation, translation, and scaling that must be applied to the measured fiducial positions to optimally align them with the known calibrated positions of the fiducials on SRM 5001. The residual differences between the aligned measured fiducial positions and the calibrated positions are shown in Fig. 5 for

a 40 mm × 40 mm sub-area of the fiducial grid on an SRM 5001 with 81 fiducial marks. Rotation and translation are determined solely by the placement of the fiducial grid in the coordinate system of the motion stage and the choice of the coordinate origin. The scale factor is of much greater significance, because it results from differences in the traceability paths to the SI meter between the laser interferometers of the AAM instrument and the SRM 5001. In the measurement shown in Fig. 5 the deviation of the scale factor from one, or scale error, amounts to about 3 nm per mm of stage motion, which remains well below the stage position uncertainty discussed in Sec. 8.

Fig. 5 further indicates a skewness error in the motion stage that can be traced to an approximately 3.5 μrad ($2 \times 10^{-4}^\circ$) deviation from orthogonality of the two motion axes. Finally, the noise floor in Fig. 5 of around 50 nm is set by the uncertainties of the calibrated fiducial positions of the SRM 5001, and the uncertainty of the position measurements with the AAM instrument that are discussed in Sec. 8 and listed in the uncertainty budget Table 1. For apertures with diameters less than about 10 mm the position error caused by the stage axes skewness error is not a significant contributor to the uncertainty budget.

SRM 5001 is manufactured on a quartz glass substrate, which is among a group of materials with good long-term stability [28]. This property makes SRM 5001 useful for the validation of the AAM instrument. Validation measurements using the SRM 5001 grid standard are performed periodically in addition to measurements of check apertures.

4. Microscope and Illuminators

In NIST's AAM instrument, a high-magnification digital optical bright-field microscope is used to detect aperture edges with high spatial resolution. Figure 6 shows the AAM microscope and the camera (the camera heat sink is removed), and an aperture that is installed on the motion stage for an aperture area measurement. The microscope is mounted on the vertical motion stage for positioning and focusing. The mounting bracket for the microscope is made from a titanium alloy with a coefficient of thermal expansion of approximately 1/3 of that of aluminum alloys to improve its thermo-mechanical stability. Aperture edges are generally imaged with the microscope objective that has the highest numerical aperture (NA) while providing sufficient working range. This is usually one of several available objectives with 100× magnification. At the time this publication is being written, images are acquired with a CMOS camera that has a sensor pixel size 2.74 μm × 2.74 μm and a sensor size of 2840 × 2840 pixels. For a microscope objective with 100× magnification this translates into a field of view of approximately 78 μm × 78 μm with an effective pixel size in the image of 27 nm.

The microscope has two modes of illumination. A reflected light (epi-illumination) mode that illuminates an object through the microscope objective, and a transmit-

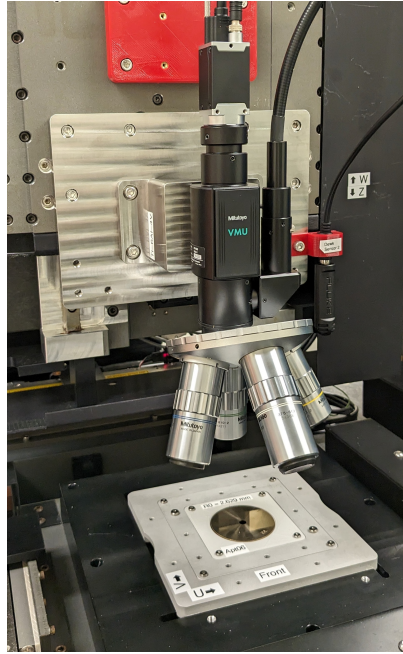


Fig. 6: AAM microscope with camera and an aperture installed for measurement.

ted light mode (trans-illumination) in which an object is illuminated by a condenser that is installed below the aperture stage shown in Fig. 2 in an opening that is machined through the granite base of the AAM instrument. Aperture area measurements are always made in transmitted light mode. An aperture edge is illuminated through a long working distance Köhler illuminator below the aperture by light from a high-power light emitting diode (LED) with a center wavelength near 525 nm. Spatially coherent imaging conditions are adequately realized when the microscope objective has a NA at least three times that of the illuminator [29–31]. Our aperture area measurements are always made with high-magnification objectives that have numerical apertures ranging from 0.55 to 0.9. Consequently, the numerical aperture of the illuminator is maintained in a range from about 0.15 to 0.2 to maintain coherent illumination conditions.

4.1. Image coordinate systems and transforms

The location of each image pixel in the field of view of the microscope must be mapped into the stage coordinate system described in Sec. 3.1. The mapping of pixel coordinates to stage coordinates is illustrated in Fig. 7. Images are received from the camera as square matrices of brightness values $\mathbf{B}$. The first element (pixel) of a brightness matrix B_{00} is the image origin and it corresponds to the upper left corner of the image. The first index of image pixel B_{ik} is the row index i , the second index the column index k . A stage position reading is the position of the optical axis in the stage coordinate system

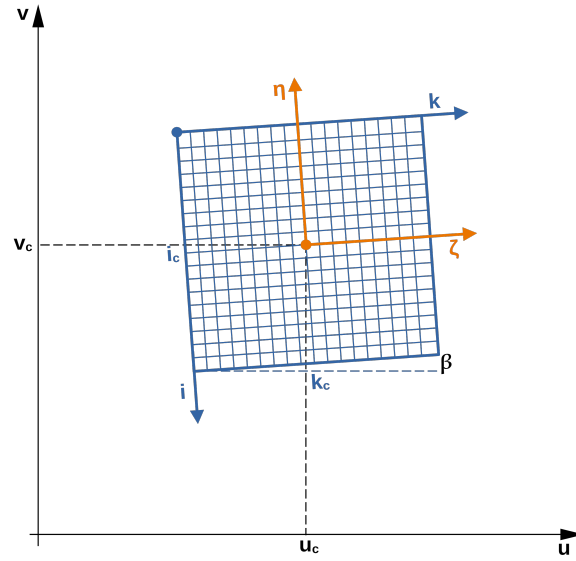


Fig. 7: Image and stage coordinate systems. (i, k) are sensor pixel coordinates, where i is the row index and k the column index, and (ζ, η) are pixel coordinates relative to the optical axis at the image center. (u, v) are the stage coordinates defined in Sec. 3.1. β is the mis-alignment angle between image sensor coordinates and motion stage coordinates.

(see Sec. 3.1). Since the imaging sensor is centered on the optical axis of the microscope, it is convenient to transform the image pixel coordinates (i, k) into coordinates (ζ, η) in a coordinate system that has its origin at the image sensor center as shown in Fig. 7. This is accomplished with the following coordinate transformation:

$$\begin{aligned}\zeta &= k - k_c \\ \eta &= (n_i - 1 - i) - i_c,\end{aligned}\tag{6}$$

where n_i is the number of rows in the image sensor array and (i_c, k_c) are the pixel coordinates of the image sensor center.

Image sensor coordinates (ζ, η) are converted to motion stage coordinates (u, v) by first multiplying the sensor coordinates with a scale factor σ , which is the same in ζ and η directions, to convert pixel units to length units, and then rotate the scaled coordinates to account for any angular misalignment β between the image sensor coordinates and the stage coordinates:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \sigma \cdot \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \zeta \\ \eta \end{pmatrix} + \begin{pmatrix} u_c \\ v_c \end{pmatrix}.\tag{7}$$

In Eq. 7 (u_c, v_c) are the stage coordinates of the image center. As will be discussed in the following Sec. 4.2, the linear transformation in Eq. 7 was found to be adequate. Non-linear effects, e.g. due to the distortion of microscope objectives are small and they are treated as uncertainties.

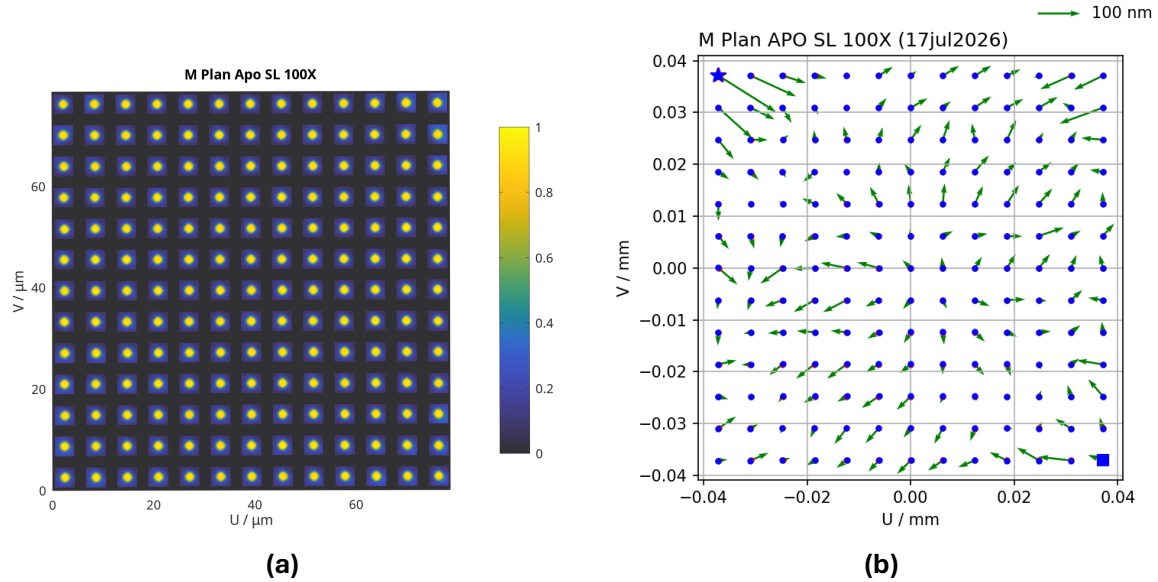


Fig. 8: (a) Image montage of a 5 μm diameter circular fiducial that is moved on a 13 × 13 square grid in the field of view of the microscope with a 100× magnification objective. (b) Image to stage mapping function error for the calibrated microscope objective.

4.2. Microscope calibration (virtual calibration grid)

For each objective the two parameters scale factor σ and alignment angle β in the image coordinate mapping function Eq. 7 must be calibrated in a way that permits SI-traceable distance measurements with the microscope. The AAM instrument uses a calibration method that takes advantage of the SI-traceable motion stage (see Sec. 3). A 3.125 mm thick chrome-on-glass photomask with several circular chrome coated areas with diameters 2 μm, 5 μm, and 10 μm is installed on the AAM motion stage. These circular fiducials are imaged using the reflection mode of the microscope. The microscope's reflection mode is chosen for this calibration because the thin photomask can then be supported on its entire back side instead of only the edges, and aberrations caused by the illumination of the fiducials through the photomask substrate are avoided. The choice of illumination has otherwise no bearing on the microscope calibration. Objectives with 100× magnification are best calibrated with the 2 μm fiducial. The fiducial is then moved through the field of view of the microscope objective on a grid pattern. At each grid position an image is acquired and the corresponding motion stage position is recorded. From the fiducial images the position of the fiducial center is estimated (see Sec. 6.5), resulting in a grid of fiducial positions in pixel coordinates with a corresponding grid of stage coordinates, the virtual calibration grid. An example is shown in Fig. 8a, which is a montage of 13 × 13 fiducial images on a square grid in the field of view of a 100× magnification microscope objective.

The problem of finding parameters σ and β of the linear coordinate mapping function Eq. 7 such that the grid of transformed image coordinates has the smallest distance from the measured stage coordinates is an optimization problem also known as the generalized Procrustes problem, which, in various forms, is encountered in many fields of science and engineering. Numerical algorithms for the solution of Procrustes problems have been developed since the 1970's [27, 32], and modern implementations are readily available [33]. Figure 8b shows the residual differences between the transformed image coordinates using the optimized transformation in Eq. 7 and the stage coordinates for one of the microscope objectives with 100 $\times$ magnification. Apart from the image corners, the maximum mapping error – the deviation of the linear model in Eq. 7 from the measured fiducial positions in the field of view of the microscope – is approximately 1.5 pixels or 42 nm. The approximately rotation symmetric shape of the mapping error suggests that it is primarily the result of residual optical distortion in the microscope objective. This residual error is treated as a position uncertainty in the analysis of aperture area measurements. The optimal mapping function parameters that were obtained in a microscope objective calibration are used for all subsequent measurements with the microscope objective. The microscope objective calibration is repeated whenever a change is made to the mounting of the camera on the microscope or the microscope objective. The calibration is also repeated at regular intervals.

4.3. Location of the physical aperture edge

The light from the microscope condenser that illuminates an aperture is diffracted at the aperture edge. Diffraction leads to a characteristic brightness¹ distribution in the image of an aperture edge as shown in Fig. 9a. In the image, near the aperture edge is a bright region resulting from constructive interference. In case of aperture edge images observed with the AAM instrument the transition from the bright (open) area to the dark (blocked) area occurs within a band that is about 100 pixels wide. The brightness distribution near the aperture edge depends on the degree of partial coherence in the imaging system, and on aberrations such as defocus and spherical aberrations that may be present in the imaging system.

The imaging of an opaque straight edge is a prototypical optical problem that has been investigated many times in experiment and theory due to its significance in many optical applications, including microscopy (see e.g. [29] and references therein). The degree of partial coherence in the imaging system of a microscope can be parameterized by the ratio S of the objective's numerical aperture NA_o to the numerical aperture

¹The term “brightness” is used here to refer to the magnitude of the pixel values in a camera image. It is assumed to be proportional to the radiant power received by the image sensor.

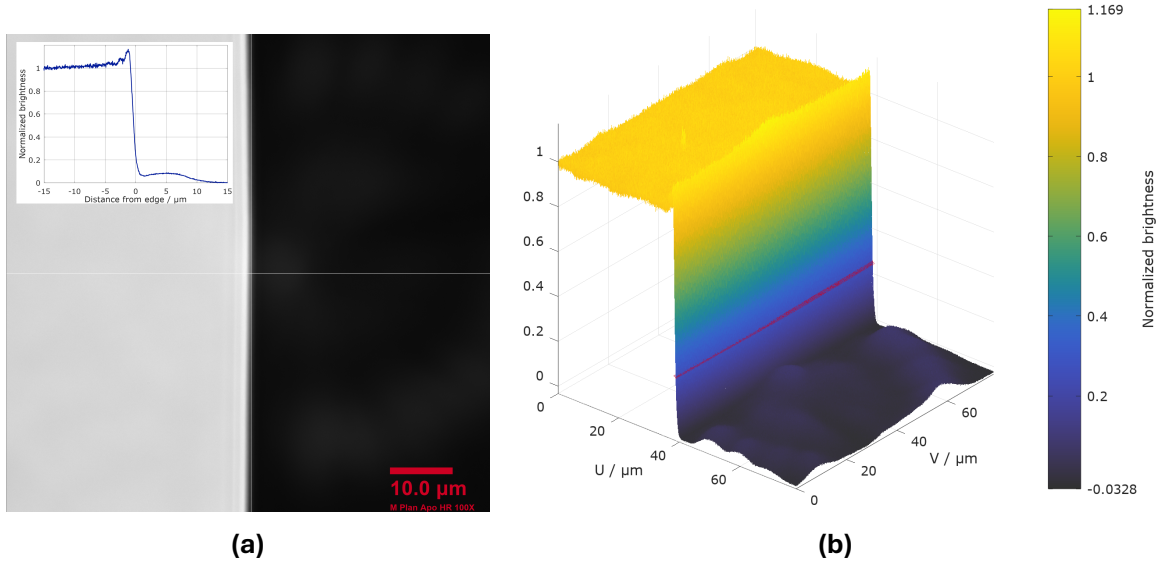


Fig. 9: (a) In-focus transmitted light image of the edge of an aperture with 5 mm diameter at 100× magnification. The inset shows a horizontal section of the brightness distribution near the image center. (b) Normalized brightness with contour indicating the physical edge at a brightness value of 0.25.

NA_i of the illumination system (condenser)

$$S = \frac{NA_o}{NA_i}. \quad (8)$$

Sometimes the coherence ratio is defined as the inverse of Eq. 8 [29]. The degree of partial coherence determines which intensity value in an edge image such as Fig. 9a corresponds to the true location of the physical aperture edge [29]. For fully incoherent illumination $S \ll 1$ the brightness at the edge in the normalized aperture edge image is 1/2. For full illumination $S = 1$ and the intensity at the edge in normalized coordinates is 1/3 [34]. Finally, for coherent illumination $S \gg 1$ and the brightness at the true edge is 1/4. Which illumination condition to realize in a measurement setup is a matter of choice and depends largely on practical considerations. Most microscopy applications use full illumination to achieve the highest possible resolution. In the AAM instrument, full illumination is difficult to realize. The condenser in the illumination system must have a sufficiently large working distance to accommodate apertures with different sizes and form factors. This restricts the numerical aperture of the condenser and it is impossible to match the numerical aperture of high-magnification objectives. It is primarily for this reason that the microscope in the AAM is configured as a coherent imaging system. In practice, the numerical aperture of the illuminator is set to a value between 0.15 and 0.2, which ensures that $S \geq 3$ for the objectives that are used for aperture area measurements. In this case the intensity at the edge in normalized aperture edge images is 0.25 [29, 30, 35]. In Fig. 9b a contour line at nor-

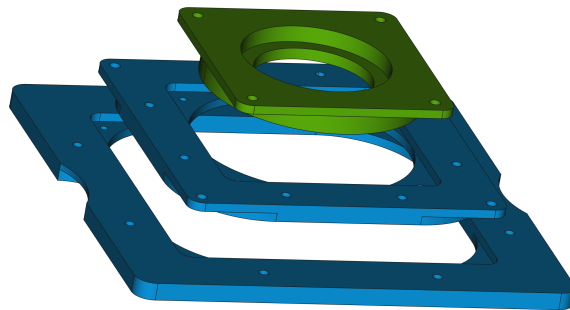


Fig. 10: Exploded view of the universal aperture mount. Aluminum frames for different size apertures are shown in blue and a 3D-printed aperture adapter is shown in green.

malized brightness 0.25 indicates the true edge location for the aperture edge image Fig. 9a.

5. Measurement Procedure for Circular Apertures

The typical workflow of an aperture area calibration consists of the following steps. First, the aperture is mounted in the AAM instrument and left to come to thermal equilibrium with the laboratory environment for at least one hour. The aperture is then located in the coordinate system of the motion stage, and the aperture plane is determined. Images of the aperture edge are taken along the full edge of the aperture and the images are stored for subsequent analysis. During the analysis of a set of aperture edge images each image is normalized, and the brightness contour corresponding to the physical aperture edge is calculated in image coordinates. The image coordinates of the edge contours are then transformed into motion stage coordinates, and the aperture area is calculated. Finally, the measurement uncertainty is evaluated.

Apertures must be mounted on the XY motion stage of the AAM instrument such that the aperture is coincident with the focal plane of the microscope illuminator. At the center of the AAM motion stage is a three-point mount for a plate with the dimensions of a 6 inch standard photomask (152 mm $\times$ 152 mm $\times$ 6.25 mm). The top surface of the plate is at the focal plane of the microscope illuminator.

Apertures come in a wide variety of shapes and outside dimensions with aperture diameters typically ranging from a few mm to about 50 mm. A degree of flexibility is therefore required to mount apertures in the AAM instrument. The adapter shown in Fig. 10 is used as a “universal aperture mount”. It consists of two nested aluminum frames and a custom-designed aperture adapter that is 3D-printed from ABS (acrylonitrile butadiene styrene) plastic. These customized adapters can be designed quickly from templates using parametric solid modeling software [36] and they can be fabricated within an hour. The inner aluminum frame can be removed to accommodate larger apertures. 3D-printed plastic adapters were found to be sufficiently stable for

the duration of aperture area measurements, which typically only take several minutes. When compared to aluminum adapters, no detrimental effect on the measurement results was observed. The illuminator has a working distance of 11.6 mm, which restricts the space available below the aperture plane (illuminator focal plane). Above the aperture plane the space is restricted by the working distance of the microscope objective. In some cases, e.g. when an aperture is in a recess, and depending on the orientation in which the aperture is measured, an objective with a larger working distance may be required. Several high-magnification objectives with working distances ranging from 1.3 mm to 13 mm are available. Measurements are generally made with the objective that has the highest NA for the required working distance.

When an aperture is mounted on the motion stage, it is important to wait until it is in thermal equilibrium with the surrounding stage and air. At least two hours are allowed to pass after the installation of an aperture before a measurement is made. This thermal “soaking” period can be skipped for the laboratory check standards, which are kept in the AAM laboratory at all times and do not require direct handling because they are already installed in individual aperture mounts.

At the beginning of an aperture area measurement, the center of the aperture is positioned at the origin of the U-V stage coordinate system within approximately ± 1 mm. The first step in an aperture area measurement is to precisely locate the aperture in the U-V coordinate system of the motion stage. For circular apertures this is done by seeking and locating the aperture edge in several, usually six or eight, directions beginning at the stage origin. (See Sec. 6.3 for a summary of the edge finding algorithm.) From this initial set of edge locations preliminary values for the center and radius of the aperture can be estimated to within about 10 μm by finding the least-squares best-fit circle. In addition, the position of the aperture on the optical axis W, and the angular orientation of the aperture plane in the U-V-W coordinate system must be determined. At each of the initial set of edge positions the aperture edge is brought into focus using either the transmission mode or the epi-illumination mode of the microscope. The focusing is either done manually by observing the microscope image or an auto-focusing algorithm, described in Sec. 6.4, can be used. The W-coordinate at the best focus is recorded for all edge locations and the aperture plane is estimated by calculating the best-fit plane [20], either in a least-squares sense or by minimizing the absolute distance of the measured edge points from the best-fit plane.

When the aperture position and orientation in the U-V-W coordinate system have been measured, a set of aperture edge images is taken along the aperture edge. While the aperture edge images are acquired, the preceding measurement of the aperture plane is used to keep the aperture edge in focus along the whole aperture edge. Sampling an aperture edge with adequate edge point density may require numerous edge images, perhaps up to a few hundred. For example, the edge of an aperture with 5 mm diameter is imaged typically at 180 locations with equal angular spacing.

5.1. Aperture materials and coefficients of thermal expansion

In calibration reports, aperture areas are reported at 20 °C [37]. While the temperature in the AAM laboratory is maintained close to the reference temperature of 20 °C, it does vary between 19.5 °C and 20.5 °C. Measured aperture areas must thus be corrected for the thermal expansion of the aperture using the linear coefficient of thermal expansion (CTE), α , of the aperture material. The change in the aperture area, ΔA , of an aperture with area A due to a change in temperature T is calculated using the following equation:

$$\Delta A = 2\alpha A \Delta T + \alpha^2 A (\Delta T)^2, \quad (9)$$

where $\Delta T = T - 20^\circ\text{C}$ is the deviation of the temperature from 20 °C. For small temperature deviations within a few °C of 20 °C only the linear term in Eq. 9 needs to be considered in the calculation of the area correction. Similarly, when a calibrated aperture is *used* at a temperature different from the 20 °C reference temperature, the aperture area must be adjusted accordingly using Eq. 9.

Radiometric apertures are made from many different materials including pure metals, metal alloys, and ceramics. Only for some pure metals that are used as aperture materials have measurements of the CTE been published in the scientific literature. The CTE of pure silicon is known with low uncertainty [38]. The doping of silicon with either phosphorus or boron has only a small effect on the CTE [39], which is negligible in the context of aperture area measurements. Measurements of the CTE including uncertainties have also been reported for copper [40], and nickel [41]. For many other aperture materials, especially alloys, the uncertainties of available CTE data, e.g. from material data sheets, are often not known because the CTE depends on the composition of the material, the amount of impurities present, and on the processes that were used to produce the material. In this case it is customary to assume a relative uncertainty in the CTE of 10%.

5.2. Aperture area tilt correction

When an aperture is installed at the center of the motion stage for an aperture area measurement, as shown in Fig. 6, the aperture plane can, generally, not be brought into perfectly parallel alignment with the U-V motion plane. The residual tilt of the aperture plane with respect to the U-V motion plane is described by two tilt angles α_u and α_v , which quantify the tilt in U and V direction. The aperture tilt is measured at the beginning of each aperture area measurement by measuring the W coordinates of the best focus position for a set of points with coordinates (u, v) along the aperture edge. A best-fit plane is estimated for the in-focus edge points in the U-V-W coordinate system, which yields the two tilt angles α_u and α_v . Tilt angles between 0.01° and 0.1° are typical. An aperture tilt angle exceeding 0.1° can in most cases be traced to inadequate aperture mounting, which can be corrected.

Aperture tilt reduces the measured aperture area caused by a foreshortening effect of the aperture dimensions. For example, a circular aperture with area A and a normal vector that has an angle γ relative to the w axis of the U-V-W coordinate system will project an ellipse with an area $\tilde{A}$ on the U-V plane:

$$\tilde{A} = A \cos \gamma = A \sqrt{\cos^2 \alpha_u + \cos^2 \alpha_v - 1}. \quad (10)$$

For the small tilt angles encountered in aperture area measurements the measured aperture areas $\tilde{A}$ can be corrected for the effect of aperture tilt using the following formula:

$$A = \frac{\tilde{A}}{\sqrt{1 - (\alpha_u^2 + \alpha_v^2)}}. \quad (11)$$

For the small tilt angles typically encountered in aperture area measurements when the apertures are mounted correctly, the tilt correction does not lead to significant changes in the measured aperture area.

6. Algorithms, Data, and Software

This section describes several algorithms and procedures that were developed specifically for aperture area measurements with the AAM instrument, and it documents the data format used for the archiving of image data and for the storage of other measurement data.

6.1. Image normalization and level finding

Aperture edge images – a typical image is shown in Fig. 9a – are normalized such that the dark opaque area is at zero, and the brightness in the open aperture area is equal to one. The normalization is complicated by the camera dark noise and stray light, which cause a typical pixel value floor of about 150. A histogram is calculated from all pixel values in an aperture edge image as shown in Fig. 11. The histogram has two prominent peaks corresponding to the dark and bright areas of aperture images. The mean value of the pixel values in the maximum bin at low pixel brightness is taken as the dark level B_d of the brightness distribution in the image. The mean of the pixel values in the maximum bin in the second peak at high pixel values is the bright level B_b within the open aperture. The dark level is subtracted from the image and then the image is scaled so that the mean image value in the open aperture is equal to one:

$$b(\zeta, \eta) = \frac{B(\zeta, \eta) - B_d}{B_b - B_d}, \quad (12)$$

where $b(\zeta, \eta)$ stands for the normalized pixel brightness and ζ and η are the pixel coordinates introduced in Fig. 7. The normalization algorithm in Eq. 12 has proven to be

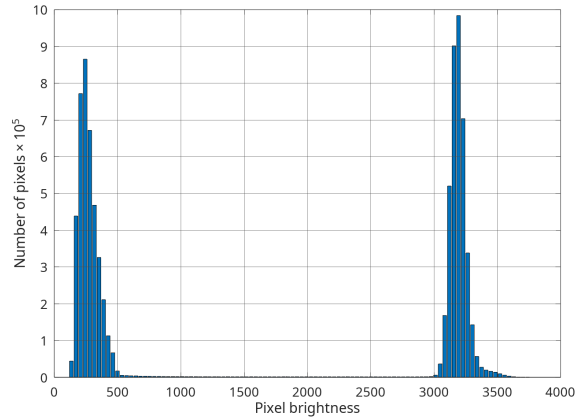


Fig. 11: Pixel brightness histogram with 100 bins for the aperture edge image shown in Fig. 9a. The tail of pixels with brightness higher than the peak near 3200 in the histogram stems from the brightness overshoot due to diffraction of light at the aperture edge.

robust. It fails in very few cases, when the aperture edge quality is exceptionally poor or the aperture edge is too thick and it is not possible to unambiguously focus the microscope on the aperture edge. In those cases some edge images had to be excluded from the aperture area calculation.

As described in Sec. 4.3, for coherent illumination the image brightness in the normalized image has a value of 0.25 at the position of the physical aperture edge. The 0.25 iso-line in normalized aperture edge images is calculated with sub-pixel accuracy using an implementation of the marching squares algorithm for iso-line computation that is available as a package for the Python programming language [42, 43].

6.2. Image gradients

All aperture edge images, such as the one shown in Fig. 9, are characterized by a narrow transition region between the open aperture and an opaque region where light transmission is blocked. This suggests that the image gradient is a useful observable for operations that require knowledge of the location of the aperture edge in the field of view of the microscope. For example, the image gradient can be used in the focusing of the microscope or to locate the aperture edge in the field of view of the microscope.

Image gradients are calculated using a Farid-Simoncelli derivative operator [44] with 5×5 kernel size. This derivative algorithm proved to be robust even when some noise is present in aperture edge images. Several aperture image metrics used by the software for the AAM instrument are based on the gradient magnitude $g(\zeta, \eta)$ of normalized aperture edge images:

$$g(\zeta, \eta) = \|\nabla b(\zeta, \eta)\|. \quad (13)$$

∇ is a discretized gradient operator acting on aperture edge images.

Gradient magnitude images are used to define several image metrics that are useful when dealing with aperture edge images. For example, the gradient magnitude can be used as a focusing criterion in microscopy [45]. The focus of an aperture edge image can be determined by evaluating the slope of the transition from the dark to the bright region in the image. This leads to the definition of a *thresholded mean gradient magnitude*

$$\bar{g} = \frac{1}{n} \sum_{\zeta, \eta} g(\zeta, \eta) \quad \text{For } \zeta, \eta \text{ such that } b(\zeta, \eta) \in [L, H], \quad (14)$$

where L and H are lower and upper image brightness thresholds. The sum includes all sensor pixels, n in total, for which normalized pixel brightness values $b(\zeta, \eta) > L$ and $b(\zeta, \eta) < H$. The lower and upper threshold can be chosen so that only pixels in the vicinity of the true aperture edge are included in the calculation of the mean gradient magnitude. The approximate position and orientation of an aperture edge in an aperture edge image can be found using the centroid of the gradient magnitude. Analogous to Eq. 14, a *thresholded gradient magnitude centroid* can be defined:

$$(\zeta_c, \eta_c) = \frac{1}{\bar{g}} \left(\sum_{\zeta, \eta} \zeta \cdot g(\zeta, \eta), \sum_{\zeta, \eta} \eta \cdot g(\zeta, \eta) \right) \quad \text{For } \zeta, \eta \text{ such that } b(\zeta, \eta) \in [L, H]. \quad (15)$$

The gradient centroid defined by Eq. 15 is used by the software for the AAM instrument to position aperture edges at the center of the field of view of the microscope.

6.3. Edge finding

Efficient measurement of aperture areas requires an algorithm for the automated finding of aperture edges such as the one shown in Fig. 9 in the U-V coordinate system of the motion stage. Initially the aperture center is positioned at the optical axis of the microscope. The whole field of view is then illuminated. The average brightness is calculated and will be used to normalize all subsequent images. The search for the aperture edge uses the bi-directional radial search algorithm with decreasing step size shown as Algorithm 1. Beginning at the aperture center, a step is made in the desired radial direction with a large step size of 1 mm, or larger for apertures with large diameter. An image is taken at the new position, the pixel values in the image are averaged and the average value is normalized to the average brightness at the aperture center. This process is repeated, while moving radially in the same outward direction until the normalized average image brightness drops below 0.5. The search direction is then reversed and the step size is halved. The search continues until the normalized average brightness is close to 0.5 within a pre-defined tolerance of a few percent. Once the algorithm has terminated, the aperture edge is in the field of view of the microscope, but the aperture edge will not, in general, be positioned at the center of the field of view.

Algorithm 1: Radial aperture edge finding algorithm in direction ϕ .

```

1 (zeta,eta) ← (0,0) // Stage origin
2 rad ← rad_init // Initial radial increment
3 dir ← 1 // Initial radial direction

4 move_stage_to(zeta,eta)
5 img ← img_init ← get_camera_image()

6 while abs(mean(img/img_init) - 0.5) > tol do
7     zeta ← dir · rad · cos  $\phi$ 
8     eta ← dir · rad · sin  $\phi$ 
9     move_stage_to(zeta, eta)
10    img ← get_camera_image()

11    if dir > 0 and mean(img/img_init) < 0.5 - tol or
12    dir < 0 and mean(img/img_init) > 0.5 + tol then
13        rad ← rad/2
14        dir ← -dir
15    end
16 end

```

An additional refinement step is needed to center the aperture edge. An aperture edge image is taken and normalized as described in Sec. 6.1. The centroid of the gradient magnitude is then calculated as described in Sec. 6.2 and Eq. 15, and the gradient centroid is positioned at the center of the field of view. Figure 9 shows an aperture edge that has been centered using the gradient centroid. The algorithm described in this section was found to be robust and sufficiently fast. Typically, for an aperture with 10 mm diameter it takes a few seconds to find the aperture edge.

6.4. Focusing

The focusing of aperture edge images is complicated by the coherent imaging configuration of the microscope (see Sec. 4.3). Unlike in incoherent imaging where a defocus results merely in a broadening of the edge transition region, coherent imaging is characterized by the emergence of interference phenomena near the optimal focus, for example the emergence of a bright fringe at the aperture edge when the image moves into focus as can be seen in Fig. 9a. The interference effects complicate the use of conventional focusing metrics such as the image variance [45], because the focusing metric is no longer a smooth function of the amount of defocus with a single peak at the best focus. Focusing is additionally complicated by the small depth of field (DOF) of high magnification microscope objectives. The DOF ranges from 0.3 μm to 0.9 μm for the 100 $\times$ magnification objectives used in the AAM, and can be much smaller than the thickness of many radiometric apertures, which may have edge thicknesses up to

20 μm or more. In this case the focus metric can show a broad, flat maximum without clear indication of an optimal focus position.

Focusing in the AAM instrument is generally based on the thresholded gradient magnitude described in Sec. 6.2. It is assumed that the focusing of the microscope is optimal when the mean gradient magnitude is at a maximum in the transition region between dark and bright areas in aperture edge images. The auto-focus function measures the mean gradient magnitude for several positions of the vertical W-axis near the focus. A parabola is then fitted to the mean gradient data, the W-coordinate of the peak is calculated, and the vertical stage is positioned at that coordinate. In practice, it can be preferable to focus the stage interactively using both the visual camera feedback and the edge image gradient information to bring the microscope into optimal focus along the aperture edge.

6.5. Fiducial finding

The calibration of the motion stage error with the NIST SRM 6001 two-dimensional grid standard described in Sec. 3.2, and the virtual calibration grid for microscope objective calibrations described in Sec. 4.2 rely on estimating the (ζ, η) coordinates of a fiducial mark in the field of view of the microscope. In the case of the stage calibration, square box fiducials are used, like the one shown in Fig. 4, and circular fiducials are used for the microscope objective calibration. In both cases a correlation method is used to determine the fiducial position. The image of the fiducial obtained with the microscope is convolved with a binary mask that has the nominal shape of the fiducials using a suitable correlation function from the scipy library [46]. The correlation of the nominal fiducial shape and the measured fiducial has a maximum at the fiducial position. A quadratic two-dimensional polynomial is then fitted to the correlation function in the vicinity of its maximum, and the position of the peak in the image coordinates is calculated from the coefficients of the best-fit polynomial.

6.6. Data formats

Aperture edge images are stored in the FITS (Flexible Image Transfer System) image format [47, 48]. The FITS image format is an established scientific image format that was originally developed for the storage of astronomical images. The FITS format is fully documented, has broad software support, and is in the Public Domain, which makes the format suitable for the archiving of image data that are acquired during aperture area calibrations. All other data files produced by software for the AAM instrument use the XML format [49], a data serialization format that is human readable, fully documented, and is supported by numerous software libraries. All image data and other files produced in the context of aperture area calibrations are stored with checksums so that the integrity of measurement data can be confirmed.

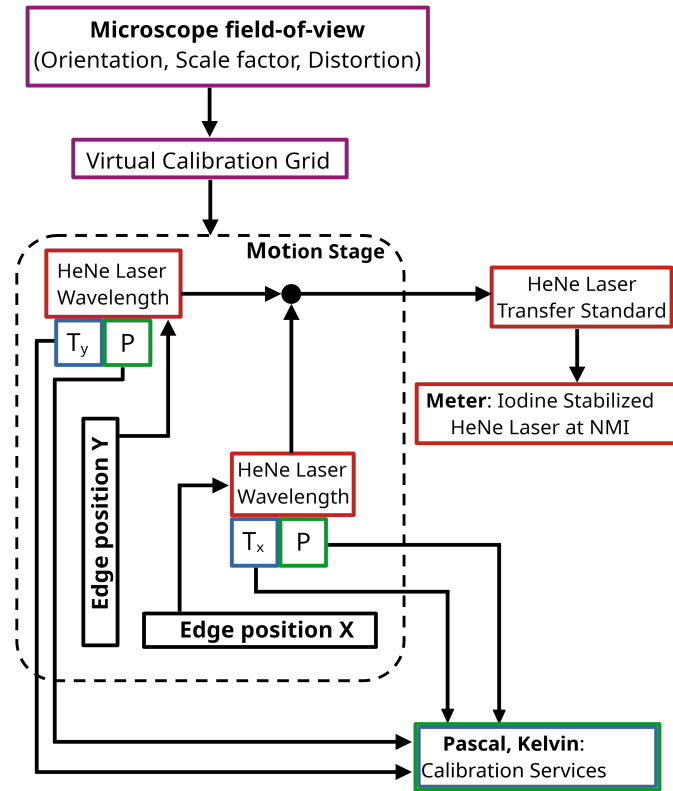


Fig. 12: Traceability of aperture edge position measurements to realizations of the SI units for length (m), pressure (Pa), and temperature ($^{\circ}\text{C}$, K). T and P denote the air temperature and pressure measurements needed to calculate the laser wavelengths in air.

7. Traceability

Aperture area measurements are traceable to the length unit meter of the International System of Units (SI) through the laser interferometers that serve as position encoders for the X and Y motion axes [21], with the exception of stage error caused by the residual non-orthogonality of the axes. The stage error, whether it is used to correct the position data reported by the motion controllers or treated as an uncertainty, is traceable to the meter through NIST SRM 5001 [24] as described in Sec. 3.2. The traceability of aperture edge position measurements, and thus aperture area measurements, is illustrated in Fig. 12. Distance measurements made with the microscope are traceable to the SI meter through the calibrated motion stage using the virtual calibration target described in Sec. 4.1.

An additional set of pressure and temperature sensors (shown with green frames in Fig. 12) is used as transfer standards to validate the sensors built into the control electronics of the laser interferometers. The sensors for temperature and pressure are

calibrated periodically through a calibration service. Even though the wavelengths of helium-neon lasers are linked to an atomic transition in neon, the laser requires periodic calibration (typically every three to five years), because the laser wavelengths gradually change as the laser tube ages, primarily due to the loss of helium from the laser tube by diffusion. The laser wavelengths are calibrated, using a transfer standard, against the wavelength of the light emitted by an iodine-stabilized helium-neon laser at a National Metrology Institute (NMI).

8. Uncertainty

For all measurements, the quoted expanded uncertainty U is the combined standard uncertainty u_c multiplied with a coverage factor k of 2. The aperture area of each customer-provided radiometric aperture is measured at least three times to estimate the measurement repeatability. The measurement repeatability, and the uncertainties enumerated in Sec. 8 are combined in quadrature to obtain the combined standard uncertainty u_c of the aperture area [50, 51]. In the estimation of the combined standard uncertainty the following contributors to the measurement uncertainty are considered:

1. **Laser interferometers:** Both displacement-measuring laser interferometers have the same position uncertainty because they are constructed identically, except for a minute difference in interferometer wavelength. The following uncertainty components contribute to the position measurement uncertainty of the laser interferometers [52]:
 - (a) For the helium-neon laser used in the AAM instrument, the relative short term stability of the laser vacuum wavelength is 2×10^{-9} within an hour, which is typical for frequency-stabilized helium-neon lasers. The measurement time for apertures never exceeds an hour, and, therefore, the position measurement uncertainty due to the short term stability limit is negligible when compared to other contributors to the combined uncertainty.
 - (b) The relative laser wavelength uncertainty over a period of three years, the recommended calibration interval for the helium-neon laser, is specified by the laser system manufacturer as 1×10^{-7} . For a displacement of 10 mm this corresponds to a position uncertainty of 2 nm, which is also small compared to other contributors to the combined uncertainty, and remains below the laser interferometer resolution of 10 nm for all but the largest aperture dimensions.
 - (c) The laser interferometers have an inherent position uncertainty due to electronic noise and non-linearity of 3 nm according to the manufacturer specifications, which is less than the interferometer resolution of 10 nm.

- (d) The uncertainties of the temperature and pressure sensors that are used to determine the refractive index of air result in an uncertainty in the laser interferometer wavelengths. The wavelength in air, λ_{air} , with refractive index n is reduced relative to the vacuum wavelengths λ_{vac} according to the following equation

$$\lambda_{\text{air}} = \frac{\lambda_{\text{vac}}}{n} . \quad (16)$$

The refractive index of air primarily depends on air temperature, air pressure, and, to a lesser degree, the water and carbon dioxide content of the air. For the purpose of estimating the effect of the uncertainties in air pressure and air temperature measurements on the air wavelength a simplified “shop floor” formula for the refractive index of air that is valid near 633 nm can be used:

$$n = 1 + \frac{7.86 \cdot 10^{-4} p}{273 + T} , \quad (17)$$

where p is the air pressure in kPa and T the air temperature in °C [53]. The humidity of air is ignored in the correction of the laser wavelengths for the refractive index of air by the AAM laser interferometers. The temperature sensors used by the wavelength compensation electronics of the laser system have an uncertainty of 0.2 °C, and the pressure sensor has an uncertainty of 2 hPa as specified by the manufacturer. It follows that the associated uncertainty in displacement measurements is 1.1 nm per mm of travel, or 11 nm for an aperture with 10 mm diameter where the diameter is the largest distance traveled during the measurement.

- (e) The laser interferometers in the AAM instrument have a deadpath, the optical path difference between interferometer test and reference arms at the zero position, of 40 mm. The uncertainty u_d in displacement measurements caused by the deadpath d can be calculated as follows:

$$u_d = d \frac{\lambda - \lambda_0}{\lambda_0} , \quad (18)$$

where λ_0 is the air wavelength at the time the zero position of the laser interferometer was established, and λ is the air wavelength when a distance measurement is made at a later time. The deadpath uncertainty can be estimated using Eqs. 16 and 17. Since the air pressure in the laboratory changes much more slowly than the air temperature, the uncertainty due to the deadpath can be estimated from the temperature change during a measurement alone. For a temperature change of 1 °C the position uncertainty resulting from the interferometer deadpath amounts to 37 nm.

- (f) The distance from the center of the motion stage to the interferometer mirrors is approximately 100 mm (see Fig. 2). This unsensed length is not ac-

counted for by the distance measurements made with the laser interferometer [52], and it changes when the temperature changes during a measurement. The temperature change during the few minutes it takes to complete a measurement is typically within a band of 0.02 °C. In the absence of thermal inertia, this temperature change would result in a change in the relevant stage dimension of 15 nm for aluminum, the stage material. This value can be taken as an upper limit for the uncertainty because the typical measurement time of a few minutes is insufficient for the stage to reach thermal equilibrium.

2. **XY stage errors:** As discussed in Sec. 3.2, the motion stage of the AAM instrument has a position error that is primarily caused by a small squareness error of the two motion axes of 3.5 μrad ($2 \times 10^{-4}^\circ$). The resulting stage error field is shown in Fig. 5. The component of the stage error that is caused by the squareness error of the motion stage has a magnitude of approximately 2.4 nm mm^{-1} . For example, a displacement of 5 mm accrues a position error of 12 nm resulting from the squareness error of the motion stage, which is close to the resolution of the laser interferometers. This small error is, therefore, treated as a position uncertainty of the X and Y motion axes as mentioned in Sec. 3.2. In addition, for circular apertures, the squareness error of the XY stage has only a minor effect on the measured aperture area.
3. **Image pixel coordinate to stage coordinate mapping:** The mapping that transforms image pixel coordinates into motion stage coordinates in Eq. 7 is established with the calibration procedure described in Sec. 4.1. The mapping function has an uncertainty of approximately one pixel for microscope objectives with 50 $\times$ and 1.5 pixels for objectives with 100 $\times$ magnification. For objectives with 50 $\times$ magnification this corresponds to a position uncertainty of 56 nm, and for 100 $\times$ magnification objectives to 42 nm.
4. **Position readout and camera synchronization:** The position readout of the motion controllers for the X and Y motion axes must be synchronized with the imaging of the aperture edge. Any timing errors result in a position uncertainty. The uncertainty was estimated from repeated stage position readings with simultaneous imaging of a fiducial mark with the microscope. The correlation (mean distance) of fiducial mark position and stage position is taken as a measure of the position error due to synchronization. For the currently used synchronization mechanism in software, the position uncertainty was estimated as 44 nm.
5. **Non-uniform illumination:** Aperture areas are measured in a transmitted light configuration. Even though the Köhler illuminator of the microscope is designed to illuminate an aperture edge uniformly, the illumination is not constant over the field of view caused by optical imperfections in the illuminator, and as the

result of stray light that is scattered at lens surfaces and illuminator apertures. In addition, photon noise and electronic camera noise add a non-uniform image component at high spatial frequencies. The result is that aperture edge images often deviate from the ideal over a range of length scales. For example, noise and artifacts resulting from stray light are clearly evident in the aperture edge image in Fig. 9b.

Non-uniform illumination causes an uncertainty in the aperture edge position because the position of the iso-brightness line at 0.25 in normalized edge images (see Fig. 9b) is affected by the variable illumination in the field of view of the microscope. The illumination non-uniformity is estimated from a bright field image that is taken at the center of the aperture at the beginning of each aperture area measurement. The range of the brightness values in this image is taken as an estimate of the illumination non-uniformity with an assumed box distribution. The resulting position uncertainty is calculated from the brightness variance in the bright field image and the gradient magnitude of the brightness in the image of an aperture edge (typically around 60 per pixel for objectives with 100× magnification).

6. **Structural stability:** The AAM instrument consists of large motion stages built on a granite structure with an overall mass of nearly 1000 kg. This large machine structure is susceptible to thermal influences, heating by motors and equipment, and to mechanical creep in the vertical motion stage. The stability of the structural, and metrology, loop was evaluated by observing the lateral drift in the microscope's field of view of a circular fiducial mark on a photomask while the photomask is held in place by the motion control system. The observed drift distance as a function of time for several observation runs is shown in Fig. 13a. Closer inspection shows that the drift predominantly occurs in the V direction, which suggests that the drift may be caused by creep and relaxation effects in the vertical motion stage after the stage has been moved to focus the microscope. Another significant contributor to the observed drift is the microscope camera which is shown in Fig. 6. Even with an attached heat sink the camera temperature is about 10 °C higher than the surrounding air and it raises the temperature of both the microscope and the structure that holds the microscope on the vertical motion stage, leading to changes in the pointing of the microscope. The dependence of the camera temperature on the image data rate is an additional cause for instability because the data rate can vary over the course of a measurement.

We found that the magnitude of the lateral drift $\delta(t)$ as a function of time t is well described by a simple exponential model:

$$\delta(t) = \kappa \left(1 - e^{-\frac{t}{\tau}} \right), \quad (19)$$

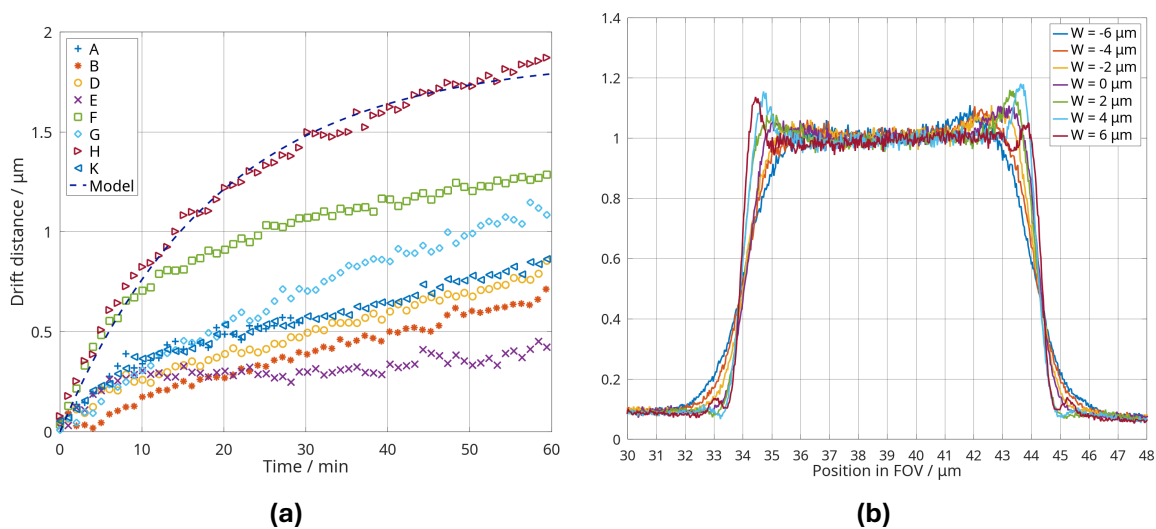


Fig. 13: (a) Drift distance of a circular reflective fiducial in the field of view (FOV) of the microscope at 100× magnification as a function of time. (b) Horizontal section through images of feature B5 on NIST SRM 475 [54] as function of defocus.

where κ is the drift magnitude, and τ is a time constant. The dashed line in Fig. 13 shows a typical worst-case drift model used to estimate the uncertainty stemming from the changes in the structural loop with $\kappa = 1.872 \mu\text{m}$ and $\tau = 19.16 \text{ min}$.

7. **Defocus:** During an aperture area measurement the aperture edge must remain within the depth of focus of the microscope as images of the aperture edge are acquired along the edge. There are two reasons why the aperture edge may move out of focus. The first is the limit imposed by the structural stability of the AAM system discussed before. After the microscope is focused onto an object, over time not only the drift shown in Fig. 13a is observed, but also a gradual defocusing of the object. We can estimate the focus drift by measuring the movement in W direction that is necessary for re-focusing after a period of time. The focus drift was estimated to be $(-55 \pm 27) \text{ nm min}^{-1}$, and it is treated as a measurement uncertainty component. How the focus drift is converted into an edge position uncertainty is described in the following paragraph.

Another reason for imperfect focusing during an aperture area measurement is that, in general, apertures cannot be mounted perfectly co-planar with the XY motion stage. When the aperture is tilted with respect to the motion plane, the aperture edge may move in and out of focus during a measurement unless the focus is adjusted continuously during the measurement. The angular orientation (tilt) of the aperture in the U-V-W coordinate system is measured prior to every aperture measurement by focusing the microscope on the aperture at several (usually six) locations along the aperture edge. The best-fit focal plane is

estimated from the W coordinates of the optimal focus positions and this plane is used to maintain the microscope in focus along the aperture edge during an aperture area measurement. An uncertainty in the focusing arises because of the stage drift described before, and because of imperfections in the vertical motion stage used to focus the microscope. The effect of this uncertainty on the aperture edge position is estimated using a reflective chromium line with a nominal width of $10\text{ }\mu\text{m}$ (feature B5) on the optical line width standard NIST SRM 475 [54, 55]. A horizontal section through an image of the chromium line for varying amounts of defocus is shown in Fig. 13b. The section for $W = 0$ is in focus. As the amount of defocus increases, the image of the line broadens. We use the line broadening as an estimate of the aperture edge position uncertainty caused by defocus. Since the broadening of the chromium line is calculated from the position of two edges, half of the broadening is used as an estimate of the aperture edge position uncertainty. Using the data shown in Fig. 13b, the edge position uncertainty was estimated as $43.8\text{ nm per }\mu\text{m}$ of defocus for microscope objectives with $100\times$ magnification. The defocus uncertainty is estimated by subtracting the best-fit plane from the positions of the microscope during the measurement and calculating the standard deviation of the W coordinate. This is then converted into an edge position uncertainty using the sensitivity factor estimated with SRM 475.

Finally, defocus during an aperture area measurement also occurs when an aperture is not plane because the microscope remains focused on a best-fit plane during the measurement as described in Sec. 5. The planarity error of the aperture is estimated from the set of edge locations that are determined at the beginning of an aperture area measurement. The deviations of the measured edge locations from the best-fit plane are taken as an estimate of the planarity error. The associated defocus uncertainty, estimated as the standard deviation of the edge W coordinate deviations from the best-fit plane, is converted into an edge position uncertainty as described in the previous section, and added, in quadrature, to the other defocus uncertainty contributors. In practice, high-quality apertures have small planarity errors and their contribution to the edge position uncertainty is minor.

8. **Aperture thermal expansion:** Changes in temperature during an aperture area measurement cause an uncertainty in the aperture area. The air temperature near the aperture is monitored during measurements. In general, the temperature variability during measurements is less than $0.1\text{ }^{\circ}\text{C}$. The uncertainty in the aperture area resulting from the temperature fluctuations during an area measurement is estimated using Eq. 9 with the assumption that the aperture temperature follows the air temperature, neglecting thermal inertia.

A typical uncertainty budget for the measurement of a knife-edge aperture with a nominal diameter of 5 mm is presented in Sec. 8.2 below.

8.1. Uncertainty calculation for aperture areas

The uncertainty contributors listed in Sec. 8 fall into two groups that are treated differently. In the first group are uncertainties that are correlated for the aperture edge points derived from a single aperture edge image. For example, the position uncertainty of the laser interferometers applies to a whole aperture edge image, and, therefore, this uncertainty applies to all edge points in the image in the same way. The uncertainty components in this group are the stage position uncertainty, the uncertainty due to the stage error, the image coordinate to stage coordinate mapping uncertainty, the position uncertainty due to camera and stage position readout synchronization error, and the uncertainties due to focus drift and focus variation. The second group contains the uncertainties that apply to each edge point individually regardless of the image from which they originate. An example is the uncertainty caused by the illumination non-uniformity. The uncertainty in aperture area, and best-fit radius, resulting from edge point coordinate uncertainty is estimated using statistical sampling (Monte-Carlo) [56] or bootstrap [57] methods. For example, from 180 edge images a set of at least 360 000 edge points is typically obtained. Normally distributed random numbers with zero mean and with a standard deviation equal to the combined standard uncertainty in the first (correlated) group of uncertainty contributors are generated for each of the aperture edge images and the edge point coordinates for the image are collectively shifted by a random vector. Then, normally distributed random numbers with a standard deviation equal to the combined standard uncertainty of the contributors in the second group is calculated and added to each of the edge point coordinates. Finally, the aperture area is calculated from the modified edge points. This process is repeated many times to build up a distribution of aperture area values. The standard deviation of this distribution is an estimate of the aperture area uncertainty.

Uncertainties caused by the structural instability and temperature variation are treated differently. We estimate the uncertainty due to structural instability using the model of the drift process in Eq. 19. The drift is added to the nominal edge positions on a circle and the effect on the aperture radius is estimated from the radius of the best-fit circle. The uncertainties due to drift processes, and the area uncertainty caused by temperature variation are added, in quadrature, to the uncertainty from the bootstrap analysis. Uncertainty values for all contributors to the measurement uncertainty are tabulated in the calibration reports.

8.2. A typical uncertainty budget

An uncertainty budget for a measurement of the aperture of one of the check apertures (Apt06) in the AAM laboratory is shown in Table 1. This aperture is a circular knife-edge aperture with a diameter of 5.26 mm. During the measurement the aperture was imaged at 280 locations with a 100× magnification microscope objective having a field of view of 80 μm $\times$ 80 μm . The edge images were acquired over a period of 214 s.

Contributors to edge position uncertainty	Uncertainty
Laser interferometer (combined)	$3.99 \times 10^{-5} \text{ mm}$
XY-stage skewness error	$6.31 \times 10^{-6} \text{ mm}$
Camera and position synchronization	$4.40 \times 10^{-5} \text{ mm}$
Focus drift	$8.59 \times 10^{-6} \text{ mm}$
Focus variation	$9.93 \times 10^{-6} \text{ mm}$
Image to stage coordinate mapping	$3.00 \times 10^{-5} \text{ mm}$
Non-uniform illumination	$5.34 \times 10^{-5} \text{ mm}$
Contributors to area uncertainty	
Area uncertainty from position uncertainty	$6.98 \times 10^{-5} \text{ mm}^2$
Structural loop stability	$4.84 \times 10^{-4} \text{ mm}^2$
Aperture temperature variance	$8.99 \times 10^{-5} \text{ mm}^2$
Combined standard uncertainty of aperture area:	$4.97 \times 10^{-4} \text{ mm}^2$
Expanded uncertainty of aperture area (k=2):	$9.94 \times 10^{-4} \text{ mm}^2$

Table 1: Uncertainty budget for the area measurement of a knife-edge aperture (Apt06) that is used as a check aperture for the AAM instrument. The aperture has a design diameter of 5 mm. The measured diameter is 5.258 mm and the aperture area is 21.7138 mm². The first three lines in this table are position uncertainties inherent to the AAM instrument regardless of the application to aperture area measurements.

The upper section of Table 1 lists the contributors to the position uncertainty of aperture edge points. The resulting uncertainty of the aperture area is calculated using the statistical methods described in Sec. 8.1, and listed in the top line of the lower section of Table 1. In the current configuration of the AAM instrument, the two largest contributors to the measurement uncertainty are the illumination non-uniformity present in the microscope condenser, and the limits of the stability of the AAM metrology loop, as is the case in the uncertainty budget shown in Table 1.

9. Example Calibration Report

An example of a calibration report for radiometric aperture area is available at the following link: <https://doi.org/10.6028/NIST.SP.250-102sup1>

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