



**NIST Internal Report  
NIST IR 8590**

# **On the Possibility of Extracting Reference Charpy Specimens from 4340 Steel Round Bars with $D \geq 1$ in.**

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September 2025



U.S. Department of Commerce  
*Howard Lutnick, Secretary*

National Institute of Standards and Technology  
*Craig Burkhardt, Acting Under Secretary of Commerce for Standards and Technology and Acting NIST Director*

NIST IR 8590  
September 2025

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**Publication History**

Approved by the NIST Editorial Review Board on 2025-09-15

**How to Cite this NIST Technical Series Publication**

E. Lucon (2025) On the Possibility of Extracting Reference Charpy Specimens from 4340 Steel Round Bars with  $D \geq 1$  in. (National Institute of Standards and Technology, Boulder, CO), NIST Internal Report (IR) NIST IR 8590.

<https://doi.org/10.6028/NIST.IR.8590>

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**Public Comment Period**

September 17, 2025 – September 16, 2026

**Submit Comments**

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## **Abstract**

We have investigated the possibility of extracting low- and high-energy reference Charpy specimens, to be certified for the indirect verification of impact machines in accordance with ASTM E23, from round bars of 4340 steel with larger diameters than the ones currently used, *i.e.*,  $D \geq 1''$  (25.4 mm). Larger round bars would allow extracting specimens from adjacent locations within the bar cross-section, rather than only in the center. The influence of sampling location on Charpy absorbed energy was found to be statistically significant in most investigated scenarios. However, the Charpy results obtained in every possible scenario all satisfy the NIST acceptability criterion based on sample size, and therefore indicate that the use of larger bars and side-by-side sampling is a viable option, when the currently preferred diameter (5/8") is unavailable. Between 1" and 1 1/4" diameter bars, the former show slightly lower data scatter than the latter, but also correspond to higher estimated cost and material waste per specimen.

## **Keywords**

4340 steel; ASTM E23; material waste; reference Charpy specimens; round bars; sample size; sampling location.

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## 1. Introduction

Since the inception of the Charpy Machine Verification Program by the US Army in Watertown Arsenal, Massachusetts, in the late 1960s [1], the AISI 4340 alloy steel was used to produce reference Charpy specimens of low- and high-energy level. The 4340 steel is a heat-treatable low-alloy steel containing chromium, nickel, and molybdenum, and possesses high toughness and strength in the heat-treated condition. Very different levels of Charpy absorbed energy,  $KV$  (impact toughness), can be achieved by varying the tempering temperature, as shown in Fig. 1 [2];  $KV \approx 20$  J is measured when tempering specimens between  $300\text{ }^{\circ}\text{C} - 400\text{ }^{\circ}\text{C}$ , while absorbed energy increases to  $\approx 100$  J when increasing the temperature to  $600\text{ }^{\circ}\text{C}$ .

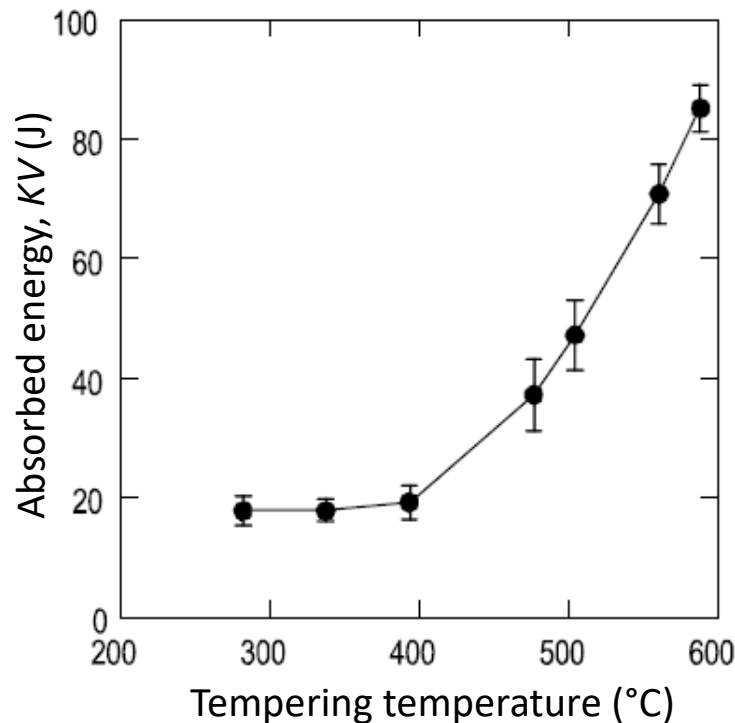


Fig. 1. Charpy absorbed energy for 4340 steel as a function of tempering temperature [2].

Until fairly recently, the steel was procured in the form of  $12.7\text{ mm }^{+3.8\text{ mm}}_{-0.0\text{ mm}}$  ( $0.5''^{+0.15''}_{-0''}$ ) square bars, delivered in the as-rolled condition and in lengths between 2 m and 4 m. The steel was tailor-made according to NIST specifications (originally produced by the US Army) that were relatively detailed and stringent.

A few years ago, the NIST specifications were revised and substantially simplified in order to enable procurement and utilization of readily available 4340 steel (“off-the-shelf”), as opposed to tailor-made. The possibility of using of “off-the-shelf” 4340 steel was recently validated through a specific investigation [3]. Another decision, taken in the spirit of facilitating the procurement of steel suitable for the production of NIST reference Charpy specimens, was switching from square cross-section bars to round bars, which are expected to be more easily

obtainable. The ideal diameter,  $D$ , for the 4340 round bars for obtaining 10 mm × 10 mm Charpy specimens and minimizing waste was 5/8" (15.875 mm). The first procurement of 5/8" diameter round bars of 4340 steel was completed in 2024.

To compare 0.5" square bars and 5/8" round bars in terms of the amount of waste generated after machining Charpy specimens, we used the following assumptions:

- The bar length is taken as 3 m (6 ft).
- The dimensions of a Charpy "blank" (oversized unnotched bar, which is heat treated and eventually machined down to the standard Charpy specimen dimensions), are taken as length = 60 mm, thickness and width = 11 mm, including cutting thickness. The corresponding volume of a Charpy blank is therefore 7,260 mm<sup>3</sup>.
- The number of blanks that can be extracted from a 3 m long bar is 3000/60 = 50.

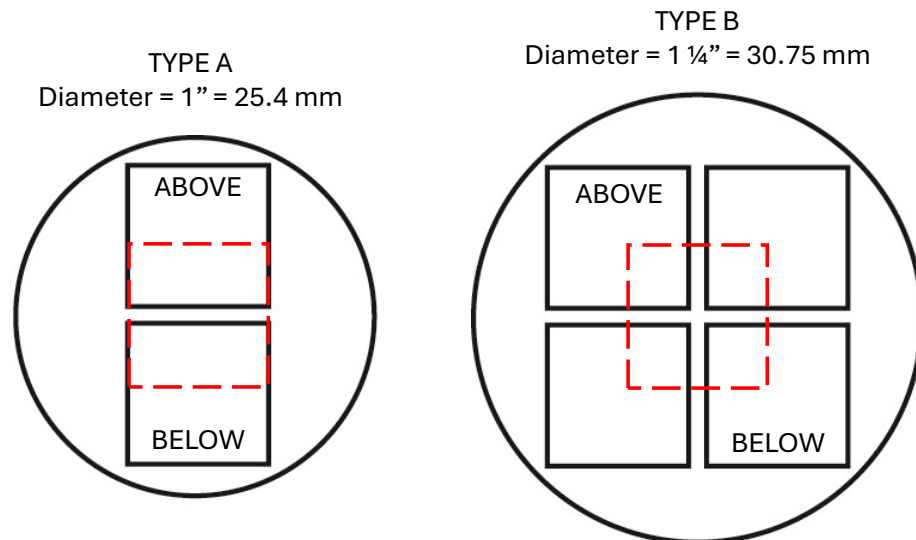
The volume of a 12.7 mm square bar of 3 m length is 483,870 mm<sup>3</sup>, while the volume of a 15.875 mm round bar of 3 m length is 593,798 mm<sup>3</sup>, *i.e.*, 22.7 % larger. The waste resulting from extracting 50 Charpy blanks from a single bar is therefore 120,870 mm<sup>3</sup> per square bar and 230,798 mm<sup>3</sup> per round bar, or 2,417.4 mm<sup>3</sup>/blank and 4,616 mm<sup>3</sup>/blank respectively. The increase of steel waste in going from square to round bars is therefore as high as 91 %.

A possible solution to this sharp increase in waste would be to extract multiple Charpy blanks in side-by-side location from the cross section of larger diameter round bars, namely with  $D \geq 25.4$  mm (1"). However, we must ensure that the blank/specimen position within the cross section of larger diameter bars does not affect in any way the impact toughness of the steel, thus contributing to an increase in variability, and consequently a decrease in the quality of Charpy reference specimens.

The aim of the investigation presented in this report is to compare the absorbed energy measured from 4340 Charpy reference specimens extracted from side-by-side locations within the cross section of round bars with  $D \geq 25.4$  mm, with respect to the values obtained from specimens extracted from the bar center (used as benchmark in this study). Specifically, we considered round bars with  $D = 25.4$  mm (1") and  $D = 31.75$  mm (1 1/4").

## 2. Material and Experimental

As shown in Fig. 2, Type A round bars (diameter = 25.4 mm) allow extracting two side-by-side Charpy blanks, while type B round bars (diameter = 31.75 mm) allow extracting four side-by-side Charpy blanks in a square pattern.



**Fig. 2. Schematic of the extraction of Charpy specimens from round bars with  $D=1''$  (left) and  $D=1\frac{1}{4}''$  (right). The red dashed squares represent the center position (benchmark).**

Using the same assumptions and calculation process previously mentioned:

- The number of Charpy blanks that can be extracted from a 3 m long bar is 100 for Type A and 200 for Type B.
- The bar volume is 1,520,122 mm<sup>3</sup> for Type A and 2,375,191 mm<sup>3</sup> for Type B.
- The corresponding amount of waste is:
  - For Type A: 794,100 mm<sup>3</sup>, or 7,941 mm<sup>3</sup>/blank.
  - For Type B: 923,100 mm<sup>3</sup>, or 4,616 mm<sup>3</sup>/blank.

The use of Type A bars would therefore entail a 72 % increase of the relative steel waste, whereas using Type B bars would not change the relative steel waste with respect to 5/8" diameter round bars.

Assuming a steel price of \$25/lb (\$55.12/kg) and an approximate steel density of  $7.85 \times 10^{-6}$  kg/mm<sup>3</sup>, the cost per Charpy blank using the assumptions above is \$5.14 for bars with diameters 5/8" and 1 1/4", and \$6.58 for 1" diameter bars. Hence, 4340 steel round bars with diameter 5/8" and 1 1/4" are fully equivalent in terms of cost and waste amount, while 1" diameter bars are more expensive and generate more waste.

For the investigation detailed herein, we extracted 30 Charpy specimens from the center of Type A and Type B bars, to used as "benchmark" absorbed energy data (red squares in Fig. 2). Additionally, we also extracted 30 + 30 = 60 Charpy specimens from Type A bars in the two side-by-side positions (identified as "ABOVE" and "BELOW") shown in the left side of Fig. 2, and 30 +

30 = 60 Charpy specimens from Type B bars in two diagonally opposite positions (“ABOVE” and “BELOW”), as indicated on the right side of Fig. 2. For each type, all the bars used came from the same 4340 steel heat.

All 180 tests were performed at  $21\text{ }^{\circ}\text{C} \pm 1\text{ }^{\circ}\text{C}$  using the three ASTM E23 reference Charpy machines (SI, TO, and TK) owned and maintained by NIST in Boulder, Colorado (10 tests per machine/condition for each bar type). Before Charpy testing, Rockwell C hardness (HRC) was measured on 10 specimens per condition (total = 60).

The specimens tested had been heat treated by one of the NIST Charpy Vendors so as to obtain low absorbed energy levels ( $KV = 17\text{ J}$  to  $20\text{ J}$ ).

### 3. Test results

#### 3.1. Hardness measurements

HRC measurements obtained from CENTER, ABOVE, and BELOW specimens for Type A and Type B round bars (10 specimens per condition) are reported in Table 1 for Type A and Table 2 for Type B (SD = standard deviation). Each HRC value is the average of two individual measurements, taken on the ends of the sample on the sample surface opposite to the notch (rear surface).

**Table 1. Rockwell C hardness measurements on Charpy specimens extracted from Type A bars ( $D = 1''$ ).**

Position in the bar					
CENTER		ABOVE		BELOW	
#	HRC	#	HRC	#	HRC
16	49.10	1	49.25	41	49.05
5	48.75	11	48.45	47	49.00
7	49.20	13	49.25	33	48.70
20	49.35	28	49.35	39	49.50
30	48.85	23	48.95	37	49.20
18	49.25	27	48.80	60	49.30
4	49.05	29	49.35	43	48.80
9	49.05	9	48.95	50	49.15
21	48.95	21	49.40	53	49.25
27	49.30	16	48.95	55	48.60
<b>Mean</b>	<b>49.09</b>	<b>Mean</b>	<b>49.07</b>	<b>Mean</b>	<b>49.06</b>
<b>SD</b>	<b>0.19</b>	<b>SD</b>	<b>0.30</b>	<b>SD</b>	<b>0.28</b>

**Table 2. Rockwell C hardness measurements on Charpy specimens extracted from Type B bars ( $D = 1 \frac{1}{4}''$ ).**

Position in the bar					
CENTER		ABOVE		BELOW	
#	HRC	#	HRC	#	HRC
7	49.45	1	49.40	34	48.70
16	49.50	10	49.55	43	49.05
25	49.55	19	49.50	52	48.85
5	49.30	28	49.65	61	49.65
14	49.50	5	48.60	32	48.65
23	49.45	14	49.30	41	49.60
3	49.45	23	49.60	50	49.45
12	49.50	9	48.55	39	49.50
21	49.50	18	48.70	48	49.00
30	49.65	27	48.75	57	48.75
<b>Mean</b>	<b>49.49</b>	<b>Mean</b>	<b>49.16</b>	<b>Mean</b>	<b>49.12</b>
<b>SD</b>	<b>0.09</b>	<b>SD</b>	<b>0.45</b>	<b>SD</b>	<b>0.39</b>

For both type bars, the smallest standard deviation corresponds to the specimens extracted from the center of the bars. The effect is more evident for Type B.

The overall values of mean hardness and standard deviation are 49.07 and 0.26 for Type A bars, 49.26 and 0.38 for Type B bars.

To assess the statistical significance of the mean HRC differences between CENTER, ABOVE, and BELOW specimens for both types of bar, we ran single factor (or one-way) Analysis of Variance (ANOVA) tests [4], using a typical significance level  $\alpha = 0.05$ . The resulting  $p$ -value, if larger than  $\alpha$ , indicates that the null hypothesis (all groups are equal) cannot be rejected.

For both Type A and Type B bars, the hardness values measured in the three locations within the cross section do not show statistically significant differences. However, the  $p$ -value calculated for Type A (0.9686) is much higher than for Type B (0.0533), indicating that hardness is more homogeneous for the smaller diameter bars.

### 3.2. Charpy test results

Table 3 and Table 4 provide absorbed energy values obtained from Type A and Type B bars, respectively, with mean values and standard deviations for each location.

**Table 3. Charpy absorbed energy measurements from Type A bars ( $D = 1''$ ).**

Machine	Sampling location					
	CENTER		ABOVE		BELOW	
	#	KV (J)	#	KV (J)	#	KV (J)
SI	1	18.93	31	19.19	61	18.75
	2	19.57	32	20.49	62	20.04
	3	19.11	33	19.62	63	19.82
	4	19.97	34	18.33	64	17.89
	5	19.97	35	18.40	65	19.61
	6	20.26	36	18.47	66	19.22
	7	20.63	37	18.47	67	19.00
	8	19.82	38	19.34	68	20.52
	9	21.41	39	18.19	69	19.15
	10	20.54	40	19.41	70	20.84
TO	11	18.91	41	19.34	71	19.94
	12	19.08	42	19.51	72	19.86
	13	18.99	43	18.30	73	18.65
	14	19.17	44	19.60	74	19.00
	15	18.99	45	19.86	75	18.48
	16	20.03	46	19.51	76	17.70
	17	18.65	47	19.08	77	19.08
	18	18.99	48	20.29	78	18.73
	19	19.08	49	19.42	79	18.48
	20	19.25	50	19.94	80	17.35
TK	21	17.95	51	17.76	81	16.86
	22	18.26	52	17.56	82	17.36
	23	17.66	53	17.36	83	15.96
	24	17.56	54	18.06	84	17.76
	25	17.36	55	16.56	85	16.86
	26	17.05	56	15.96	86	16.56
	27	16.76	57	16.86	87	17.26
	28	17.26	58	18.06	88	16.66
	29	16.56	59	18.06	89	17.16
	30	16.96	60	17.76	90	16.06
	<b>Mean</b>	<b>18.82</b>	<b>Mean</b>	<b>18.63</b>	<b>Mean</b>	<b>18.35</b>
	<b>SD</b>	<b>1.26</b>	<b>SD</b>	<b>1.11</b>	<b>SD</b>	<b>1.35</b>

**Table 4. Charpy absorbed energy measurements from Type B bars ( $D = 1 \frac{1}{4}$ " ). NOTE: for three tests, absorbed energy was not recorded (N/A) due to a malfunction of the acquisition system.**

Machine	Sampling location					
	CENTER		ABOVE		BELOW	
	#	KV (J)	#	KV (J)	#	KV (J)
SI	1	18.86	31	18.23	61	18.25
	2	19.29	32	18.44	62	18.33
	3	19.50	33	18.58	63	18.68
	4	18.57	34	18.15	64	18.11
	5	18.28	35	17.30	65	18.97
	6	18.43	36	18.44	66	17.90
	7	18.71	37	18.15	67	18.18
	8	18.86	38	N/A	68	17.61
	9	19.94	39	N/A	69	18.97
	10	19.00	40	N/A	70	18.61
TO	11	19.41	41	18.96	71	18.55
	12	19.84	42	18.44	72	18.30
	13	18.89	43	18.19	73	17.61
	14	18.81	44	15.95	74	18.91
	15	18.80	45	19.14	75	18.38
	16	19.15	46	18.79	76	18.30
	17	19.23	47	18.70	77	18.38
	18	18.80	48	19.13	78	18.13
	19	18.46	49	19.04	79	18.64
	20	19.41	50	18.88	80	18.56
TK	21	17.32	51	17.30	81	17.32
	22	15.72	52	17.80	82	17.72
	23	18.12	53	16.90	83	16.92
	24	18.22	54	17.60	84	16.92
	25	18.12	55	18.00	85	17.32
	26	18.02	56	16.90	86	17.72
	27	17.22	57	16.50	87	16.62
	28	17.02	58	16.70	88	17.22
	29	16.42	59	17.80	89	17.32
	30	17.52	60	17.30	90	16.92
	Mean	18.46	Mean	17.97	Mean	17.98
	SD	0.98	SD	0.87	SD	0.67

Once more, one-way ANOVA was used to statistically assess differences between mean values of absorbed energy calculated from different sampling locations for the two types of bars. The corresponding  $p$ -values are shown in Table 5.

**Table 5.  $p$ -values obtained from ANOVA analyses of Charpy test results.**

Bars	Machine	$p$ -value	Bars	Machine	$p$ -value
Type A ( $D = 1$ " )	SI	0.0241	Type B ( $D = 1 \frac{1}{4}$ " )	SI	0.0042
	TO	0.0102		TO	0.0034
	TK	0.1075		TK	0.4658
	ALL	0.3423		ALL	0.0346

In most cases, differences were found statistically significant ( $p < 0.05$ ), although the variability is lower for the smaller bars.

### 3.2.1. Calculation of sample sizes

The sample size,  $n_{SS}$ , is a statistical parameter used by the NIST Charpy Machine Verification Program [2] to verify the acceptability of a lot of reference Charpy specimens, based on 75 tests performed on the three ASTM E23 reference machines (25 tests per machine):

$$n_{SS} = \left( \frac{3s_p}{E} \right)^2 \quad (1)$$

In Eq (1),  $E$  is the larger between 1.4 J and 5 % of the mean absorbed energy, and  $s_p$  is the pooled standard deviation, given by:

$$s_p = \sqrt{\frac{s_1^2 + s_2^2 + s_3^2}{3}} \quad (2)$$

with  $s_1$ ,  $s_2$ , and  $s_3$  = standard deviations of the three reference machines.

The lot of reference specimens is deemed acceptable, and can therefore be certified, if  $n_{SS} \leq 5.0$ . The implication is that the average absorbed energy obtained by a NIST customer after testing a set of 5 certified reference specimens is statistically comparable with the certified absorbed energy established by NIST. The sample size can also be interpreted as a quality index of the variability of the reference lot – if it does not exceed 5.0, the lot can be confidently used to perform the indirect verification of an impact machine in accordance with ASTM E23.

Values of sample size were calculated (Table 6) to assess the variability/homogeneity of the specimens machined from the two types of bars, in each sampling location, considering each machine separately and all machines combined. Test results for all sampling location were also pooled together to obtain an overall sample size for each bar type, irrespective of the sampling location.

**Table 6. Sample size calculations.**

Bars	All data	Reference machine			Sampling location		
		SI	TO	TK	CENTER	ABOVE	BELOW
Type A ( $D = 1''$ )	1.892	3.381	2.005	1.887	2.498	2.009	2.681
Type B ( $D = 1 \frac{1}{4}''$ )	1.924	1.370	2.094	2.253	3.094	4.171	2.515

All the calculated values of sample size in Table 6 are lower than 5.0, and therefore correspond to acceptable reference specimen lots, irrespective of the specific scenario (bar diameter, test machine, sampling location). Most of the individual  $n_{SS}$  values (11 out of 14) are actually lower than 3, which points to a very low variability of test results.

These calculations clearly show that the use of larger bars of 4340 steel for the production of reference Charpy specimens, extracted from adjacent locations within the cross section, is acceptable for the practical purposes of the NIST Charpy Machine Verification program.

#### 4. Conclusions

The use of larger diameter round bars of 4340 steel (namely,  $D = 1''$  and  $D = 1 \frac{1}{4}''$ ) for the production of low-energy and high-energy NIST reference Charpy specimens was investigated, with respect to the possibility of extracting specimens from side-by-side locations within the bar cross section.

In a few cases, the influence of sampling location was found to be statistically significant at a 95 % confidence level, with respect to absorbed energy values obtained from the center of the bars. However, the sample sizes calculated in every possible scenario were all lower than the NIST threshold for acceptable test result variability, and therefore demonstrate that using larger bars and extracting multiple specimens within the cross section is a viable option.

For a practical standpoint, it is still recommended to purchase 4340 steel bars with  $D = 5/8''$ , which would only allow one specimen to be sampled from the center of bars. If these are unavailable or significantly more expensive, the second-best choice would be procuring round bars with  $D = 1 \frac{1}{4}''$ , which are equivalent to the smaller ones in terms of both estimated cost and amount of waste per specimen.  $1''$  diameter bars are slightly more expensive and produce more waste, but appear fully acceptable in terms of test result quality.

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