Quantifying Variance Components for Repeated Scattering-Parameter Measurements up to 110 GHz

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***Abstract* — We utilize a Multivariate Analysis of Variance (MANOVA) method to quantify components of variance for scattering-parameter measurements made with a vector network analyzer (VNA). A hierarchical model allows us to estimate the variance components for a device under test (DUT) over multiple calibrations, each with repeated measurements taken amongst a series of disconnections. We present results for experiments performed on a VNA involving WR-28 (26.5 GHz to 40.0 GHz) waveguide and 1.0 mm coax (200 MHz to 110 GHz). For each case, we compare our overall repeatability with the systematic uncertainties.**

***Index Terms* — Analysis of variance, measurement, repeatability, scattering parameters, vector network analyzer.**

I. INTRODUCTION

Repeatability refers to a measurement system's ability to consistently produce identical results when a particular item is measured multiple times under identical conditions. Repeatability of calibrated scattering-parameters is influenced by (1) the short-term stability of the vector network analyzer (VNA), (2) the ability to duplicate connections of a device under test (DUT) to the test ports of the VNA, and (3) the ability to replicate the calibration of the VNA, which involves connecting and measuring various standards.

In a previous publication [1], we presented a Multivariate Analysis of Variance (MANOVA) method for quantifying the three variance components within a hierarchical design, depicted in Figure 1, using maximum likelihood to obtain parameter estimates [2]. The experimental design consists of performing a series of calibrations using identical standards. Within each calibration, we repeatedly measure a DUT while disconnecting it and reconnecting it between measurements. And within each disconnect, we also perform repeated measurements without breaking the connection. In the following sections, we summarize our approach, and present results for WR-28 waveguide and 1.0 mm coax measurements.

II. APPROACH

In [1], we assume the following random effects model:

$$\boldsymbol{Y}_{ijk} = \boldsymbol{\mu} + \boldsymbol{C}_i + \boldsymbol{D}_{(i)j} + \boldsymbol{\varepsilon}_{(ij)k}\,, \qquad (1)$$

where $\boldsymbol{Y}_{ijk}$ is the multivariate response vector (i.e. calibrated magnitudes of *S*-parameter measurements) corresponding to calibration *i*, disconnect *j*, and repeat *k*. These vectors have dimension *F*, the number of measured frequency points. The mean response $\boldsymbol{\mu}$ is constant at each frequency and is the expected value of $\boldsymbol{Y}_{ijk}$. We assume the calibration effects $\boldsymbol{C}_i$, the disconnect effects $\boldsymbol{D}_{(i)j}$, and the measurement errors $\boldsymbol{\varepsilon}_{(ij)k}$ are independent random variables with means $\vec{\mathbf{0}}$, with respective variance component vectors of $\boldsymbol{\sigma}_C^2$, $\boldsymbol{\sigma}_D^2$, and $\boldsymbol{\sigma}^2$, which are estimated using a MANOVA method [3]. This approach allows us calculate the estimated mean square errors (MSE) due to the different factors, and from the MSE, we estimate the variance of the overall mean $\widehat{\mathrm{Var}(\bar{Y}_{\ldots})}$ and the variance components $\widehat{\boldsymbol{\sigma}}_C^2$, $\widehat{\boldsymbol{\sigma}}_D^2$, and $\widehat{\boldsymbol{\sigma}}^2$. As in [1], we only consider the diagonal elements of the matrices for computational ease and notational clarity.

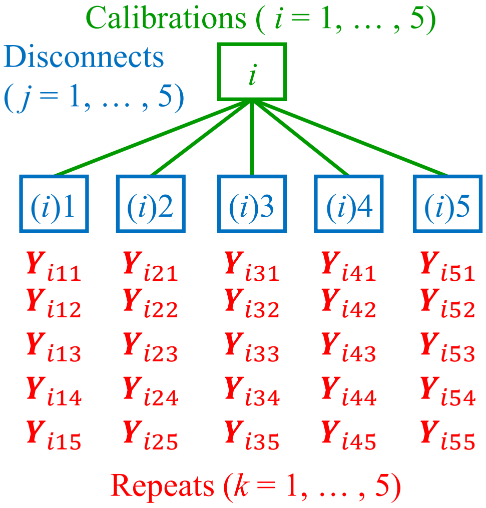

Fig. 1. The hierarchical design consisting of *I*=5 calibrations, each including *J=5* disconnects and *K*=5 repeated measurements.

III. WAVEGUIDE MEASUREMENTS

Our first repeatability study consisted of performing five calibrations (*I*=5) using a Thru-Reflect-Line (TRL) calibration kit with WR-28 waveguide connectors. Within each calibration, we connected and disconnected our DUT, a mismatched transmission line, five times (*J*=5) and made five repeated measurements (*K*=5) during each connection, as illustrated in

Figure 1. Thus, the mismatched line was measured 125 (5×5×5) times. Measurements were performed at frequencies from 26.5 GHz to 40.0 GHz in steps of 0.1 GHz (136 points) with an Intermediate Frequency (IF) bandwidth of 50 Hz. Figure 2 depicts the values of $|S_{11}|$ and $|S_{21}|$ for a single measurement.

Figures 3 and 4 plot the square root of variance component estimates, corresponding to measurement error ($\hat{\boldsymbol{\sigma}}$), disconnect ($\hat{\boldsymbol{\sigma}}_D$), and calibration ($\hat{\boldsymbol{\sigma}}_C$). For both $|S_{21}|$ and $|S_{11}|$, the variations due to error, $\hat{\boldsymbol{\sigma}}$, are much smaller than those the due to disconnect and calibration, $\hat{\boldsymbol{\sigma}}_D$ and $\hat{\boldsymbol{\sigma}}_C$. Neither $\hat{\boldsymbol{\sigma}}_D$ nor $\hat{\boldsymbol{\sigma}}_C$ consistently dominate the total variability throughout the measured frequency range, but both variances are typically higher for $|S_{11}|$ than $|S_{21}|$. This can be explained by noting reflections coefficients may be more sensitive to small misalignments between the DUT and the VNA's test ports than transmission coefficients.

It is important to verify our data does not violate the random-effects model described in Eq. 1. To do this, we calculate the residuals by taking the differences between measured values and the predicted values from the model over all 125 measurements and at all frequencies. Details on the calculation of residuals can be found in [1]. Plotting these residuals, as shown in Figure 5 for the mismatched line's $|S_{21}|$ measurements and in Figure 6 for the line's $|S_{11}|$ measurements, allows us to look for potential outliers. The figures indicate more variance from Calibration i=1, Disconnect j=1 (denoted with red plusses). Violations of model assumptions can have unintended consequences; among other issues, variance estimates may be inflated to absorb the atypical structure, potentially distorting other estimates. Despite this, we found no obvious reason to exclude this group. Notably, the deviation appeared in a single disconnect, so perhaps a slight problem with a loose connection may have been the culprit.

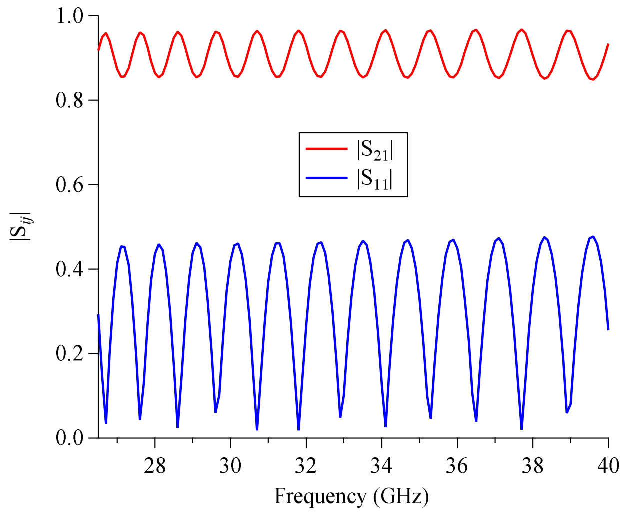

Fig. 2. A single measurement of the WR-28 mismatched transmission line.

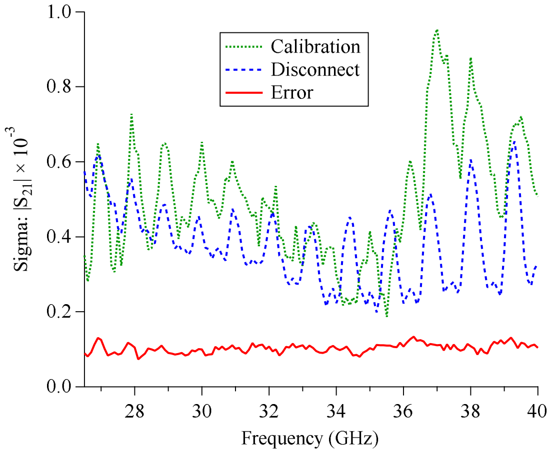

Fig. 3. Estimated values of the square root of variance components for the WR-28 mismatched line's $|S_{21}|$ values including error ($\hat{\sigma}$), disconnect ($\hat{\sigma}_D$), and calibration ($\hat{\sigma}_C$).

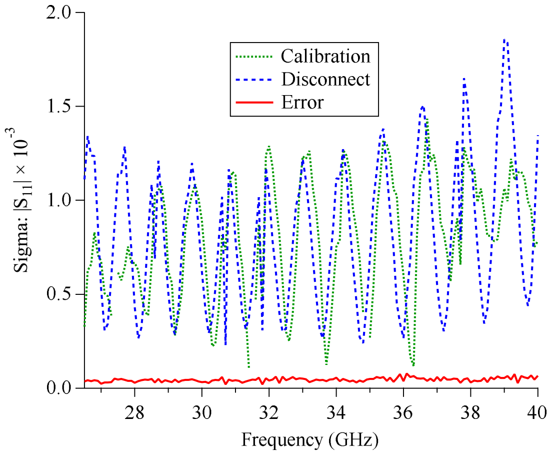

Fig. 4. Estimated values of the square root of variance components for the WR-28 mismatched line's $|S_{11}|$ values including error ($\hat{\sigma}$), disconnect ($\hat{\sigma}_D$), and calibration ($\hat{\sigma}_C$).

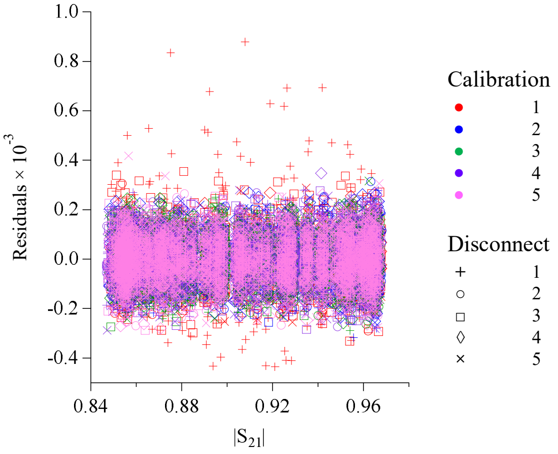

Fig. 5. Residuals versus measured $|S_{21}|$ values for the WR-28 mismatched line.

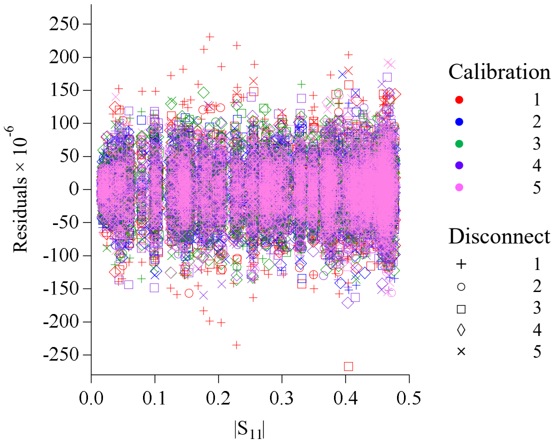

Fig. 6. Residuals versus measured $|S_{11}|$ values for the WR-28 mismatched line.

Figure 7 plots the random and systematic uncertainties for the mismatched line's $|S_{21}|$ and $|S_{11}|$ values. The random portions, $\sqrt{\mathrm{Var}\left(\widehat{\bar{Y}_{\cdots}(f)}\right)}$, were calculated as defined in [1]. The systematic portions were computed with the Microwave Uncertainty Framework [4]. Systematic uncertainties were determined by propagating uncertainties due to dimensional tolerances using models of rectangular waveguide transmission lines [5]. In our case of WR-28, the dimensions for the shim, which was used as the line in the TRL calibration, were height H = 3.556 ± 0.006 mm, width W = 7.112 ± 0.006 mm, and length L = 3.018 ± 0.013 mm, as specified by the manufacturer of the calibration kit. A direct connection between test ports was used as the thru, and the flush short was assumed to be ideal.

Systematic uncertainties not considered were junctions with unequal heights and widths, junctions with misalignments, and internal corner radii, which would normally be included in calibration service measurements, where traceable dimensional measurements are available for the calibration kit and VNA test ports.

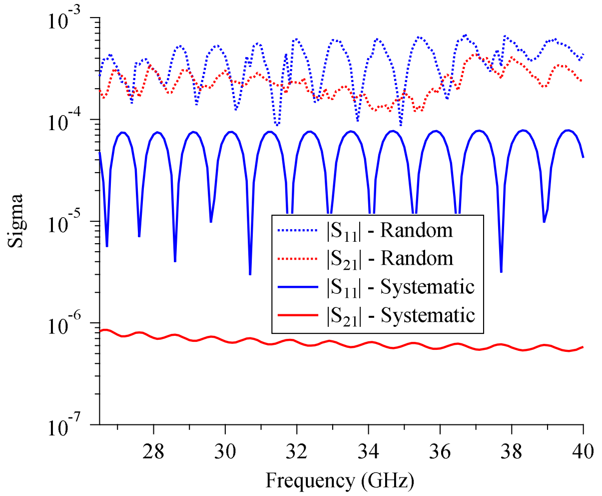

Fig. 7. Random (dashed) and systematic (solid) uncertainties for the WR-28 mismatched line's $|S_{21}|$ (red) and $|S_{11}|$ (blue) values.

Figure 7 illustrates that the uncertainties due to random effects are larger than the uncertainties due to systematic effects. Thus, it is important to take multiple measurements to account for the effects of repeatability, including multiple calibrations and disconnects.

IV. COAXIAL MEASUREMENTS

Our second repeatability study was performed with 1.0 mm coaxial connectors using a calibration kit that consisted of four offset shorts, an open, a load, and a thru connection. As with the WR-28 measurements, we performed five calibrations (I=5), and within each calibration, we connected and disconnected our DUT, a mismatched transmission line, five times (J=5) and made five repeated measurements (K=5) during each connection. Measurements were performed at frequencies from 200 MHz to 110 GHz GHz in steps of 200 MHz (550 points) with an IF bandwidth of 50 Hz. Figure 8 depicts the values of $|S_{11}|$ and $|S_{21}|$ for a single measurement.

Figures 9 and 10 plot the square root of variance component estimates, corresponding to measurement error ($\hat{\boldsymbol{\sigma}}$), disconnect ($\hat{\boldsymbol{\sigma}}_D$), and calibration ($\hat{\boldsymbol{\sigma}}_C$). For both $|S_{21}|$ and $|S_{11}|$, the variations due to measurement error, $\hat{\boldsymbol{\sigma}}$, are smaller than those the due to disconnect and calibration, $\hat{\boldsymbol{\sigma}}_D$ and $\hat{\boldsymbol{\sigma}}_C$, up to 67 GHz. At higher frequencies, the values of $\hat{\boldsymbol{\sigma}}$ are noticeably larger, presumably due to additional noise in the VNA's extenders heads, which require frequency multiplication. The variance component due to calibration, $\hat{\boldsymbol{\sigma}}_C$, is also notably larger than disconnect, $\hat{\boldsymbol{\sigma}}_D$, over most frequencies. Due to space limitations, we do not include the residual plots like we did for WR-28 but suffice it to say we found no outliers.

Figure 11 illustrates the random and systematic uncertainties for the mismatched line's $|S_{21}|$ and $|S_{11}|$ values. Systematic uncertainties were determined by propagating dimensional uncertainties in the Microwave Uncertainty Framework using models of coaxial transmission lines [6, 7]. In contrast to WR-28, the uncertainties due to systematic effects are typically larger than the uncertainties due to random effects for the 1.0 mm measurements.

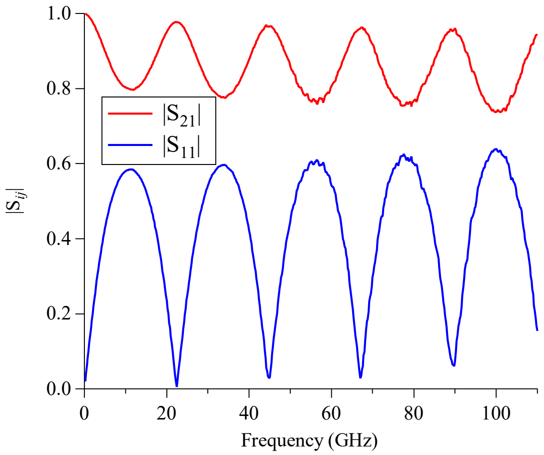

Fig. 8. A single measurement of the 1.0 mm mismatched transmission line.

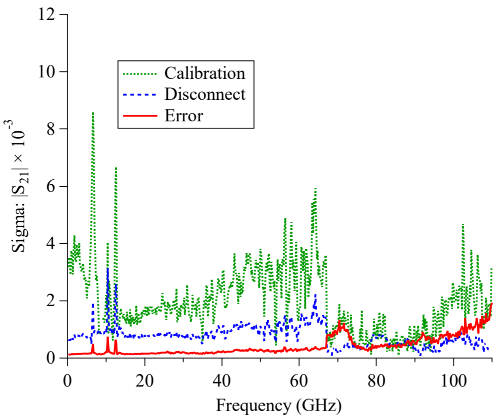

Fig. 9. Estimates of the square root of variance components for the 1.0 mm mismatched line's $|S_{21}|$ values including error $(\hat{\sigma})$, disconnect $(\hat{\sigma}_D)$, and calibration $(\hat{\sigma}_C)$.

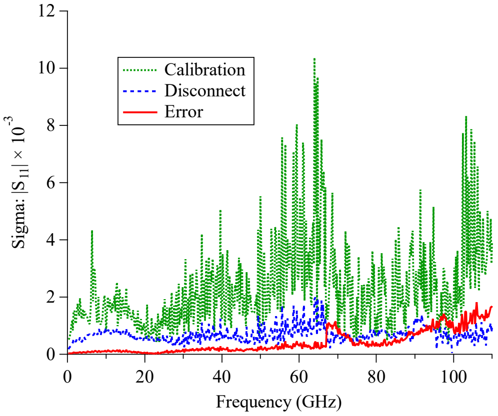

Fig. 10. Estimates of the square root of variance components for the 1.0 mm mismatched line's $|S_{11}|$ values including error $(\hat{\sigma})$, disconnect $(\hat{\sigma}_D)$, and calibration $(\hat{\sigma}_C)$.

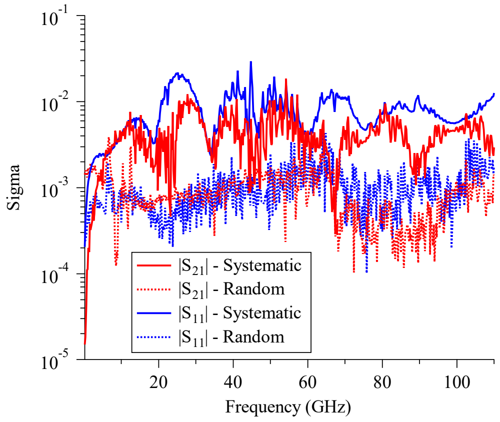

Fig. 11. Random (dashed) and systematic (solid) uncertainties for the 1.0 mm mismatched line's $|S_{21}|$ (red) and $|S_{11}|$ (blue) values.

V. CONCLUSIONS

We presented results of a MANOVA method using a hierarchical model to estimate the components of variance of DUTs with WR-28 waveguide and 1.0 mm coaxial connectors over multiple calibrations, disconnections, and repeated measurements. In the case of the WR-28 measurements, we discovered our variance estimates rose and fell with the measured magnitudes of the *S*-parameters as a function of frequency. Neither $\hat{\boldsymbol{\sigma}}_D$ nor $\hat{\boldsymbol{\sigma}}_C$ consistently dominated the total variability throughout the measured frequency range, but both variances were typically higher for $|S_{11}|$ than $|S_{21}|$. Furthermore, the square root of the variances due to multiple measurements with unbroken connections $\hat{\boldsymbol{\sigma}}$ were small compared to $\hat{\boldsymbol{\sigma}}_D$ and $\hat{\boldsymbol{\sigma}}_C$. As for the 1.0 mm coaxial measurements, $\hat{\boldsymbol{\sigma}}_C$ consistently dominated $\hat{\boldsymbol{\sigma}}_D$ throughout the measured frequency range, and variances for $|S_{11}|$ than $|S_{21}|$ were comparable. The measurement error $\hat{\boldsymbol{\sigma}}$ was negligible at frequencies below 67 GHz but became more pronounced at higher frequencies. And the systematic uncertainties were generally larger than the random uncertainties.

Because we use maximum likelihood to estimate the model parameters, relying on mean squares derived from the data to estimate variance components, there is a possibility of obtaining negative variance component estimates. When this happens, we set the estimate equal to zero [8]. There are a few points where this occurred in our WR-28 experiment, as seen in Figure 4. This is a potential drawback of the approach. The issue can be avoided by using a different estimation procedure that would prevent negative estimates, such as a Bayesian method [9]. We plan to investigate this in a future publication.

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REFERENCES

[1] A. A. Koepke and J. A. Jargon, "Quantifying Variance Components for Repeated Scattering-Parameter Measurements," *90th ARFTG Microwave Measurement Conference*, Nov. 2017.

[2] F. A. Graybill, *Theory and Application of the Linear Model*. North Scituate, MA: Duxbury Press, 1976.

[3] D. C. Montgomery, *Design and Analysis of Experiments*, 8th ed. New York: Wiley, 2013.

[4] D. F. Williams, NIST Microwave Uncertainty Framework, Available at: https://www.nist.gov/services-resources/software/wafer-calibration-software, 2020.

[5] J. A. Jargon, et al., "Physical Models and Dimensional Traceability of WR15 Rectangular Waveguide Standards for Determining Systematic Uncertainties of Calibrated Scattering-Parameters," *NIST Technical Note* 2109, Aug. 2020

[6] J. A. Jargon, D. F. Williams, P. D. Hale, and M. D. Janezic, "Characterizing a Noninsertable Directional Device Using the NIST Uncertainty Framework," *83rd ARFTG Microwave Measurement Conference*, Jun. 2014.

[7] J. A. Jargon, et al., "Physical Models and Dimensional Traceability of 2.4 mm Coaxial Airline Standards for Determining Systematic Uncertainties of Calibrated Scattering-Parameters," *NIST Technical Note* 2255, Jul. 2023.

[8] S. R. Searle, *Linear Models*. New York: Wiley, 1971.

[9] A. Gelman, J. B. Carlin, H. S. Stern, D. B. Dunson, A. Vehtari, and D. B. Rubin, *Bayesian Data Analysis*, 3rd ed. Boca Raton, FL: CRC Press, 2013.