Error-Corrected Fermionic Quantum Processors with Neutral Atoms

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Many-body fermionic systems can be simulated in a hardware-efficient manner using a fermionic quantum processor. Neutral atoms trapped in optical potentials can realize such processors, where nonlocal fermionic statistics are guaranteed at the hardware level. Implementing quantum error correction in this setup is, however, challenging, due to the atom-number superselection present in atomic systems, that is, the impossibility of creating coherent superpositions of different particle numbers. In this Letter, we overcome this constraint and present a blueprint for an error-corrected fermionic quantum processor that can be implemented using current experimental capabilities. To achieve this, we first consider an ancillary set of fermionic modes and design a *fermionic reference*, which we then use to construct superpositions of different numbers of *referenced fermions*. This allows us to build logical fermionic modes that can be error corrected using standard atomic operations. Here, we focus on phase errors, which we expect to be a dominant source of errors in neutral-atom quantum processors. We then construct logical fermionic gates, and show their implementation for the logical particle-number conserving processes relevant for quantum simulation. Finally, our protocol is illustrated with a minimal fermionic circuit, where it leads to a quadratic suppression of the logical error rate.

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Introduction—Quantum computation [1,2] promises solutions to difficult problems across many disciplines of natural science, from high-energy to condensed matter physics and quantum chemistry [3–6]. While conventional quantum computers operate with qubits, many of these problems are naturally formulated with fermionic particles. Encoding the fermionic statistics with qubits, however, represents a major challenge, especially in the presence of long-range interactions [7–12]. To address this challenge, there has been a growing interest in developing programmable fermionic quantum processors, designed to naturally encode fermionic exchange statistics into their hardware architecture [13–16]. Among the most promising approaches are fermionic neutral atoms trapped in optical lattices [17–27], and, more recently, in programmable tweezer arrays [28–32]. The inherent indistinguishability of the atoms provides a direct and efficient access to fermionic simulations.

A key open question in the development of fermionic quantum processors is their compatibility with quantum error correction (QEC), a crucial ingredient to scaling quantum processors in the presence of noise [33,34]. In qubit codes, as realized in systems of trapped ions [35–39], superconducting circuits, [40–42] or Rydberg atom arrays [43–45], the strategy is to encode logical quantum information into suitably entangled states of several physical qubits. Straightforward extensions of these ideas to fermionic processors, by mapping qubit states to mode occupations, are, however, hindered by the fundamental atom number superselection rule in atomic experiments, i.e., the conservation of total atom number. Circumventing this limitation is an outstanding challenge, and existing ideas require advanced experimental capabilities [46–49], such as coherent coupling to thermodynamically large reservoirs of molecular Bose-Einstein condensates [50–54].

In this Letter, we present a novel proposal for overcoming atom number superselection in existing neutral atom setups with finite atom numbers, and design a blueprint for an error-corrected fermionic quantum processor. Our proposal liberates physical fermions from their

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number conservation constraint through a key innovation: a *fermionic reference*. This consists of an ancillary set of fermionic modes to interchange particles with system modes. We show that, with a careful design of this exchange process, the ancillary modes serve as a phase reference, allowing to create and probe coherent number superpositions in the system. In this sense, the fermionic reference is analogous to a laser that acts as a phase reference for manipulations of optical coherence [55]. Our construction thus opens up a wide range of quantum information applications to fermionic atomic systems, and we apply it here to realize fermionic QEC in neutral-atom arrays, which we show to be implementable exactly with a fixed number of fermionic atoms.

Fermionic neutral-atom arrays—The envisioned setup is based on spinless fermionic neutral atoms in optical potentials (Fig. 1). We use ground state orbitals of optical tweezers to host fermionic modes [28–32] whose occupations with atoms defines computational states [13]. Additionally, we represent the fermionic reference with a separate set of modes, e.g., realized with another tweezer array, or by the lowest-band Wannier orbitals of an optical lattice to leverage its high stability [57]. We use the dynamical programmability of tweezers to transport and manipulate the atoms [58,59], e.g., to implement tunneling operations by merging tweezers [28–31,60] or interactions via Rydberg excitations [61–66]. System and reference are interfaced with tunneling operations between tweezers and lattice sites [67–70], enabling the exchange of particles. In summary, these operations amount to the set of fermionic gates $\mathcal{G} = \{e^{i\theta n_i n_j}, e^{i\theta(f_i^\dagger f_j + \mathrm{H.c.})}, e^{i\theta n_i}\}$, where $f_i^\dagger (f_i)$ denote creation (annihilation) operators of tweezer or lattice modes i and $n_i = f_i^\dagger f_i$. Throughout this Letter, we assume that these operations are efficiently implemented at the hardware level [13]. Dominant errors arising in this setup are assumed to be phase errors from local fluctuations of the tweezer depth [Fig. 1(c)], while optical lattices are assumed to be robust, with leading errors given by homogeneous common-mode fluctuations. Focusing on such error models, we discuss how a fermionic reference enables QEC for neutral-atom fermionic processors. Additionally, our construction generalizes to more powerful codes also correcting for number-changing processes, e.g., fermionic particle loss; see Supplemental Material (SM) [71] for a construction based on the Steane code.

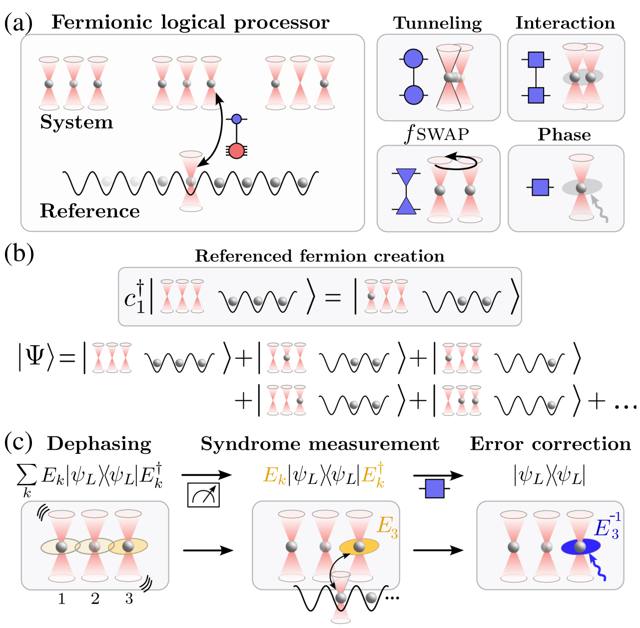

FIG. 1. Fermionic quantum processor. (a) A neutral-atom fermionic processor realizes interaction-, phase-, tunneling-, and fSWAP gates. (b) Using a fermionic reference, we construct referenced fermion operators $c^\dagger$, c. Their action creates superpositions of states with different numbers of referenced fermions but fixed physical atom number. (c) QEC employs such superposition states to protect logical fermionic states against errors. Errors are identified using syndrome measurements (involving the reference) and corrected with local operations.

Referenced fermion construction—We consider M fermionic modes with fixed total *physical* fermion number N, divided into M_s *system* modes with annihilation operators s_i $(i = 1, \ldots, M_s)$ and M_r *reference* modes with annihilation operators r_i $(i = 1, \ldots, M_r)$ [72]. Our goal is to construct new fermionic modes not constrained by a particle number superselection rule [73]. To this end, we define reference ladder operators, R, $R^\dagger$,

$$R = \sum_{j=1}^{M_r} (1 - \eta_{j+1}) r_j \eta_{j-1}, \qquad (1)$$

with $\eta_i = r_i^\dagger r_i$, $\{r_i^\dagger, r_j\} = \delta_{ij}$ [74]. For each system mode we define a *referenced* fermion mode with creation (annihilation) operators $c_i^\dagger = s_i^\dagger R$ $(c_i = R^\dagger s_i)$. These operators move physical fermions between system and reference, which lives in the relevant Hilbert space $\mathcal{H}$ [Fig. 1(b)] defined as follows: We start by defining the state $|\Omega\rangle \equiv r_N^\dagger .. r_1^\dagger |\mathrm{vac}\rangle$, $|\mathrm{vac}\rangle$ being the physical vacuum without atoms. We identify $|\Omega\rangle$ as the vacuum of the referenced fermions, since $c_i|\Omega\rangle = 0$. $\mathcal{H}$ is then spanned by the states reached from $|\Omega\rangle$ by applications of referenced fermion operators, and it has the structure of a fermionic Fock space for $N \geq M_s$.

The salient aspect of this definition is that all states in $\mathcal{H}$ have a particularly simple structure on the reference modes. To see this, note that the action of R on $|\Omega\rangle$ simply removes a fermion from the occupied mode with the largest index, such that $R^n|\Omega\rangle = r_{N-n}^\dagger .. r_1^\dagger |\mathrm{vac}\rangle$. Similarly, on these states $R^\dagger$ adds a fermion in the unoccupied mode with the smallest index. Hence, all states containing n fermions in the system have the same configuration of reference modes, with atoms in each reference mode $i \leq N - n$, and no atoms in reference modes $i > N - n$ (Fig. 1(b)) [75].

Physically, this is reminiscent of a Fermi sea, where the particle in the mode with largest index is skimmed off by application of R. The structure of reference states implies the commutation relation $\mathbf{P}[R, R^\dagger]\mathbf{P} = 0$, with $\mathbf{P}$ projecting onto $\mathcal{H}$. In combination with the fermionic property $\{s_i, s_j^\dagger\} = \delta_{ij}$ of the original fermions, this yields anti-commutation relations between referenced fermions, i.e. $\{c_i^\dagger, c_j\}\mathbf{P} = \mathbf{P}\{c_i^\dagger, c_j\} = \delta_{ij}\mathbf{P}$. Importantly, while this construction of $\mathcal{H}$ allows only $N+1$ distinct reference configurations, the configuration of fermions in the system modes is unrestricted [Fig. 1(b)].

Crucially, number-conserving operations of referenced fermions do not involve the reference, and are thus equivalent to the corresponding operations on system fermions, e.g., $c_i^\dagger c_j = s_i^\dagger s_j$. This enables implementation of number-conserving operations in $\mathcal{G}$ for referenced fermions directly at the level of physical fermions. In addition, the reference allows implementing number-changing processes, giving a fully universal gate set for referenced fermions. These latter processes do not change the number of physical particles, but require acting on the reference modes. We give an explicit decomposition of such processes in terms of the gates in $\mathcal{G}$ next.

Realization of system-reference tunneling—The main novel ingredient is a physical operation changing numbers of referenced particles. Here we focus on $D_i(\theta) \equiv \exp[i\theta(c_i^\dagger + c_i)]$, which is analogous to a Pauli-X rotation in a qubit system. While this operation allows to implement our QEC scheme, more general operations can also be obtained analogously. The unitary $D_i(\theta)$ is a generalized system-reference tunneling involving the global reference operators R, $R^\dagger$ and a single physical system mode. It is realized by a sequence of tunneling operations, where a tweezer is coupled sequentially to all reference sites, and density interactions between lattice modes. $D_i(\theta)$ is decomposed as

$$e^{i\theta(R^\dagger s_i + s_i^\dagger R)} = \prod_{k=1}^{M_r} e^{i\frac{\theta}{2}(s_i^\dagger r_k + \mathrm{H.c.})} e^{i\pi\eta_k\eta_{k+1}} e^{i\pi\eta_k\eta_{k-1}} \times e^{-i\frac{\theta}{2}(s_i^\dagger r_k + \mathrm{H.c.})} e^{i\pi\eta_k\eta_{k+1}} e^{i\pi\eta_k\eta_{k-1}}. \quad (2)$$

[See SM [71] and Fig. 2(a), and where $D_i(\theta)$ is assumed to act on $\mathcal{H}$.] The gate acts on all reference modes sequentially and therefore the number of two-mode gates is proportional to M_r.

Logical Fock space—Using superpositions of fermion number sectors, we can apply QEC to our setup. We illustrate this with a repetition code correcting local phase errors. While this example highlights the use of a fermionic reference, more general codes also correcting for number-changing errors; e.g., the Steane code [1,34] can also be implemented (SM) [71].

We start by discussing the repetition code for a single logical fermionic mode. For this, we use three physical

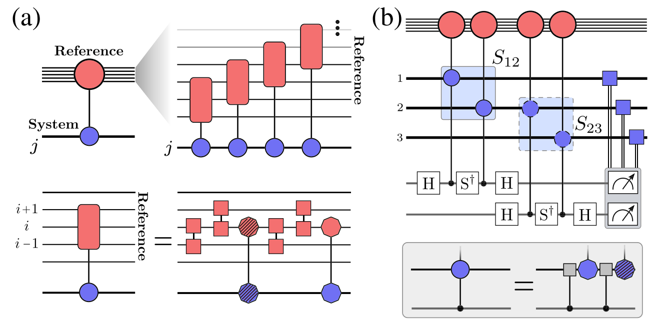

FIG. 2. Fermionic operations for error correction. (a) The fermionic reference enables operations that change the physical fermion number in the code. We show a decomposition of system-reference tunneling using $\mathcal{O}(M_r)$ elementary gates. (b) To measure error syndromes, we employ system-reference tunneling and ancilla qubits. Measurement of the ancilla reveals the syndrome. The ancilla-controlled tunneling is decomposed in terms of density interactions and tunneling (SM). Measuring syndromes in each block allows to detect and correct phase errors. Hatched gates represent the conjugate operation, circle/octagons/squares refer to tunneling- and phase gates with $\theta = (\pi/2), (\pi/4), \pi$.

fermionic modes in conjunction with the reference. Specifically, the logical fermion annihilation operator is given by $C \equiv i[c_1c_2c_3 + c_1c_2^\dagger c_3^\dagger + c_1^\dagger c_2 c_3^\dagger + c_1^\dagger c_2^\dagger c_3]$ and $C^\dagger$ is defined analogously. They act on the logical vacuum $|0\rangle_L \equiv \frac{1}{2}(1 + ic_1^\dagger c_2^\dagger - ic_2^\dagger c_3^\dagger + c_1^\dagger c_3^\dagger)|\Omega\rangle$ as $C|0\rangle_L = 0$, and $C^\dagger|0\rangle_L \equiv |1\rangle_L$ and fulfill $\{C^\dagger, C\} = 1$. The logical states are stabilized by the operators

$$S_{12} = i(c_1 + c_1^\dagger)(c_2 + c_2^\dagger), \quad (3a)$$

$$S_{23} = -i(c_2 - c_2^\dagger)(c_3 - c_3^\dagger), \quad (3b)$$

which commute with the logical operators and define the code space. The construction corresponds to Kitaev's Majorana chain [76] with six real modes (SM [71]).

The code can be generalized to multiple logical fermionic modes. For this, we partition the system into $M_L = M_s/3$ blocks of three system modes, labelled with a block index $b = 1, \ldots, M_L$. Analogously, we define logical operators C_b, $C_b^\dagger$, and the vacuum of M_L logical modes $|0_1, \ldots, 0_{M_L}\rangle_L \sim \prod_b [1 + ic_{b,1}^\dagger c_{b,2}^\dagger - ic_{b,2}^\dagger c_{b,3}^\dagger + c_{b,1}^\dagger c_{b,3}^\dagger]|\Omega\rangle$. This defines a logical Fock space $\mathcal{H}^C$ spanned by the basis states

$$|n_1, \ldots, n_{M_L}\rangle_L = (C_{M_L}^\dagger)^{n_{M_L}} .. (C_1^\dagger)^{n_1} |0_1, \ldots, 0_{M_L}\rangle_L, \quad (4)$$

with $n_i \in \{0, 1\}$. The logical states are stabilized by S_{12}^b and S_{23}^b defined analogous to Eq. (3), and are prepared by sequences of elementary gates or via stabilizer measurements. Thus, we have constructed a fermionic code with fixed physical atom number with the help of a fermionic reference common to all logical modes [77]. In total, this construction requires $M_s = 3M_L$ system modes, $3M_L$ reference modes, and $3M_L$ fermionic atoms.

Logical operations conserving the logical fermion number also conserve the number of referenced fermions, and therefore can be implemented without operating on the reference. Therefore, we restrict ourselves to Hilbert space sectors $\mathcal{H}^C_{N_L}$ with fixed logical particle number N_L next. This is relevant for quantum simulations of number-conserving interactions, as found in physically relevant fermion models [3–6]. Furthermore, the fixed number sector is especially suited for our reference construction since, as we show below, this also enables correction of errors in the reference; and it also allows us to implement our proposal with fewer resources [78].

Quantum error correction—The above construction forms an error-correcting code for the considered error set $\mathcal{E} = \{\mathbb{1}, p_i\}$ with the local parity operators $p_i = 1–2s_i^\dagger s_i$. That is, the Knill-Laflamme error correction condition [1,79] is fulfilled for the set of errors $\mathcal{E}$, i.e., $\langle i|_L E_k^\dagger E_l|j\rangle_L \propto \delta_{ij}$ for any two errors $E_k, E_l \in \mathcal{E}$. Therefore, phase errors are detectable and correctable. Specifically, p_i flips a unique combination of stabilizers, and is uniquely inferred from the syndromes.

We next detail the procedure for a single round of error correction (Fig. 2) with the help of an ancilla qubit [80]. We first measure the two stabilizers of each block independently. For example, we map the stabilizer eigenvalue S_{12} onto the state of the ancilla (index a) via the gate sequence $\mathrm{H}_a\mathrm{C}_a D_2(\pi/2)\mathrm{S}_a^\dagger \mathrm{C}_a D_1(\pi/2)\mathrm{H}_a$, where H_a is the Hadamard gate, $\mathrm{S}_a = |0\rangle\langle 0|_a + i|1\rangle\langle 1|_a$, and

$$\mathrm{C}_a D_i(\theta) = |1\rangle\langle 1|_a \otimes e^{i\theta(c_i + c_i^\dagger)} + |0\rangle\langle 0|_a \otimes \mathbb{1}, \quad (5)$$

which is followed by a projective measurement of the ancilla. A decomposition of the ancilla-controlled tunneling (5) in terms of system-reference tunneling and density interactions is shown in Fig. 2(b)—see also SM [71] for details—and a similar decomposition exists for the second stabilizer S_{23}. The errors are subsequently corrected according to the measurement outcomes using local phase gates on the physical modes (Fig. 2). One round of error correction can remove one phase error in each of the M_L blocks with a total gate depth of $\mathcal{O}(M_L)$. This follows from the sequential design of the system-reservoir operation employed for stabilizer measurements, which can be parallelized for multiple blocks. The subsequent correction step can be implemented in parallel for all blocks in constant depth.

Reference errors—We now also discuss errors on the reference, distinguishing two cases motivated by the physical properties of optical lattices. First, we consider global relative phases between reference and system, $\exp(i\epsilon \sum_j r_j^\dagger r_j)$, corresponding to global fluctuations of the lattice depth. This is trivially accounted for, since it has the same effect as the previously considered phase errors on system modes: due to physical atom number conservation, these fluctuations correspond to $\exp(-i\epsilon \sum_j s_j^\dagger s_j)$, which for $\epsilon \ll 1$ simplifies to single-mode errors p_i for all i.

Beyond global reference errors, we next discuss local reference errors $E_{Rj} = 1–2r_j^\dagger r_j$, where our construction in $\mathcal{H}^C_{N_L}$ also satisfies the Knill-Laflamme error correction condition restricted to this error set (SM [71]). By measuring the number of atoms $N_R = \sum_i r_i^\dagger r_i$ the state collapses into an eigenstate of N_R. This removes the phase error, which acts only between different reference number states. While this collapses all superpositions of atom-number sectors on the system modes, the logical quantum information is preserved, but the state needs to be re-encoded at the end with $\mathcal{O}(M_L)$ operations.

Logical gate operations—For logical computations on $\mathcal{H}^C_{N_L}$ we aim to construct the gate set

$$\mathcal{BK}'_L = \{e^{i\frac{\pi}{4}N_b}, e^{i\pi N_b N_{b'}}, e^{i\frac{\pi}{4}(C_b^\dagger C_{b'} + \mathrm{H.c.})}\}, \quad (6)$$

where $N_b = C_b^\dagger C_b$. Crucially, these logical operations are implemented without involving the fermion reference, and hence without any additional overhead due to physical particle number conservation (SM [71]).

In Fig. 3, we show explicit decompositions of logical gates in terms of physical operations, where we also involve a stable qubit ancilla. The appeal of a fermionic architecture lies in efficient implementations of fermionic exchange *f*SWAP, which is a crucial ingredient for tunneling between distant modes, as it includes all fermion phases of the in-between modes [11]. A key advantage of our hardware is a transversal implementation of *f*SWAP operations. Here, this amounts to (classically) exchanging all physical fermion modes of the two corresponding blocks [Fig. 3(a)], a process which can be highly parallelized using reconfigurable tweezer arrays [43].

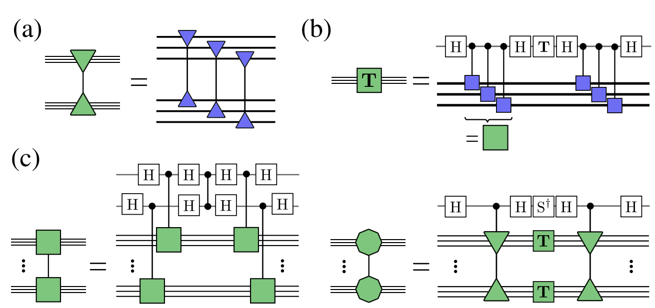

FIG. 3. Logical fermionic operations. (a) Transversal implementation of the *f*SWAP gate, which encompasses the fermionic statistics. (b) The fermionic **T**-gate $\exp[i(\pi/4)N_i]$ can be realized with qubit **T** gates on ancillas; see also Ref. [53]. (c) Similarly, two-mode density interaction $\exp(i\pi N_i N_j)$ and $\pi/4$-tunneling gates can be implemented with ancilla qubits using CZ and S gates. Ancillas are initialized in $|0\rangle$.

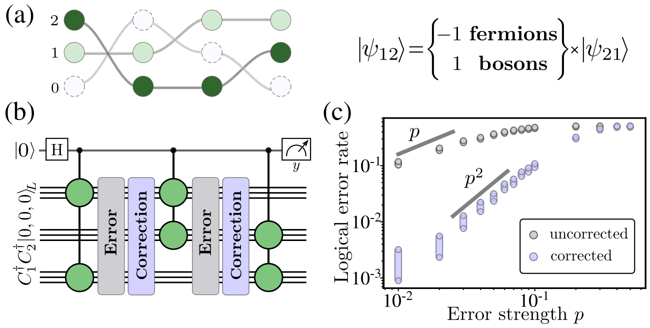

FIG. 4. Error-corrected fermionic circuit. (a) We probe the logical fermion exchange with error-corrected computation. (b) Controlled $\pi/2$-tunneling and ancilla measurements in the y basis reveal the fermionic statistics. (c) We simulate the circuit with layers of random phase errors (with single error probability p) and error correction. For small p, error correction decreases the logical error from $\mathcal{O}(p)$ (uncorrected) to $\mathcal{O}(p^2)$. Data shows 99% (Clopper-Pearson) confidence intervals for 10^4 realizations of the circuit.

Minimal error-corrected fermionic quantum circuit—To demonstrate fermionic quantum computation in conjunction with QEC, we propose an experiment to test the fundamental fermionic statistics on the logical level. We propose to initialize three logical fermionic modes with particle number $N_L = 2$ and probe their fermionic statistics under particle exchange [Fig. 4(a)]. The quantum circuit requires tunneling operations between different logical fermionic modes, which are conditioned on the state of an ancilla such that the relative phase $\exp(i\Theta)$ between the two states $|\psi_{12}\rangle = C_2^\dagger C_1^\dagger |0,0,0\rangle_L$ and $|\psi_{21}\rangle = C_1^\dagger C_2^\dagger |0,0,0\rangle_L$ can be probed [Fig. 4(b)]. This minimal setup requires nine tweezers, six reference modes, and eight fermionic atoms [78].

To showcase such an experiment, we decompose the gates of the logical tunneling operations and simulate the resulting dynamics interspersed with several layers of storage phase errors in the system modes and subsequent (error-free) error correction as shown in Fig. 4(b). For every physical mode, errors are sampled such that in each error-layer $\exp(i\pi n_i)$ phases are applied with a given probability p. Single phase errors (per block) are corrected in the error-correcting layer while higher-weight errors result in logical errors, which propagate to the ancilla and corrupt the signal. Fig. 4(c) shows the error of the resulting exchange phase $\cos(i\Theta) = -\langle Y \rangle_a$ for various error strengths p and averaged over 10^4 realizations of the circuit. While the uncorrected result deviates from the ideal result -1 with probability $\mathcal{O}(p)$, QEC reduces this error to $\mathcal{O}(p^2)$. Here, we considered a simple noise model without errors in measurements and recovery. In general, the error-correction performance is limited by the fidelities of all employed operations. A minimal circuit with refined noise models is detailed in SM [71].

Conclusion and outlook—Our Letter sets the stage for several directions of future investigations. While we demonstrated correction of phase errors with a repetition code, our reference construction allows to import the entire qubit error-correction framework [1,34] for hardware fermions, enabling generalization to more powerful codes also correcting errors such as atom loss or leakage into other motional states of the atoms [81]. It would be interesting to further extend this framework to include more general errors on ancillas and reference modes and to ultimately design fault-tolerant fermionic processors. In this context, our Letter opens up the possibility for quantitative comparisons of error-corrected fermionic and qubit quantum processors for fermionic simulations [7–12,82], and it motivates the development of novel quantum algorithms and codes to optimally leverage the fermionic nature of the microscopic particles.

The key concept of our proposal is a fermionic reference to create superposition states of different atom numbers. The crucial feature is the design of reference states from which fermions are extracted and added in a push and pop stack construction. We envision such references also for other experimental setups, beyond the concrete physical implementation proposed in this Letter. This includes alternative realizations of Fermi seas with neutral atoms in optical potentials [83], or other platforms with fermionic particles, e.g., electrons in quantum dot arrays [84,85].

Note added—In the final stages of completing this Letter, we became aware of arXiv:2411.08955, which proposes QEC for fermionic processors by coupling to molecular BEC's as reservoirs for fermion pairs. In contrast, in our Letter the fermionic reference is a finite component of the processor that is interfaced algorithmically.

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Data availability—The data that support the findings of this article are openly available [86].

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[73] We note that the entire construction here is employed for fermions, but can be applied to bosons as well.

[74] We define the boundary term in (1) via $\eta_0 = 1$, $\eta_{M_r+1} = 0$.

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