Persistent Homology of Plant-Pollinator Interactions: The Importance of Flowering Plant Surveys

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U.S. DEPARTMENT OF COMMERCE

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E. Seo, J. A. Jones, R. A. Hutchinson, and V. W. Pfeiffer, *Plant pollinator data at HJ Andrews Experimental Forest, 2011 to 2021*. https://andlter.forestry.oregonstate.edu/data/abstract.aspx?dbcode=SA026, Aug. 2022.

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- There is increasing consumer interest in plant species which are beneficial to pollinators.

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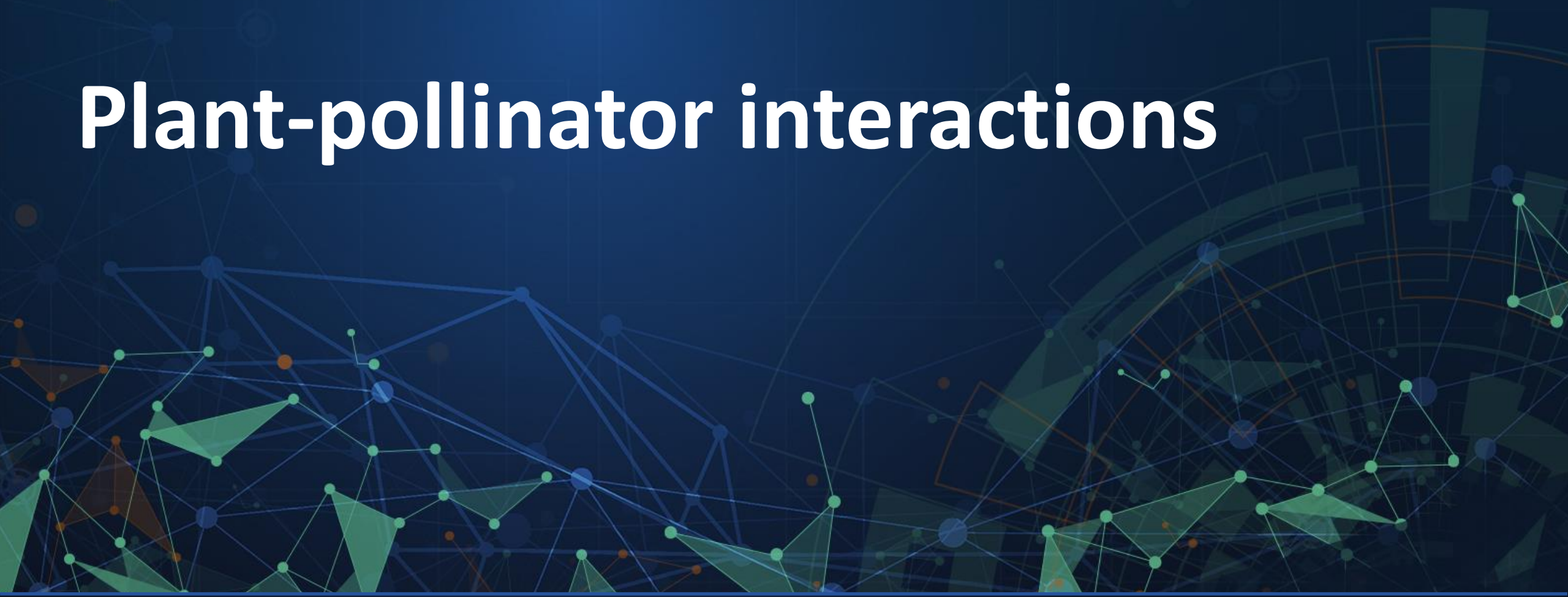
- Preference metrics provide a way to quantify how much a type of plant is favored by a type of pollinator.
- Computing preferences requires quantitative interaction data and information about floral abundance.
- Floral abundance can be estimated using interaction data or measured by performing a flowering plant survey.
- We use topological data analysis to investigate whether the use of floral survey data affects the mathematical structure of observed plant-pollinator interactions.

Plant-pollinator interactions



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Terminology

- We use flower visitation data to study plant-pollinator interactions, but not all flower visits result in pollination

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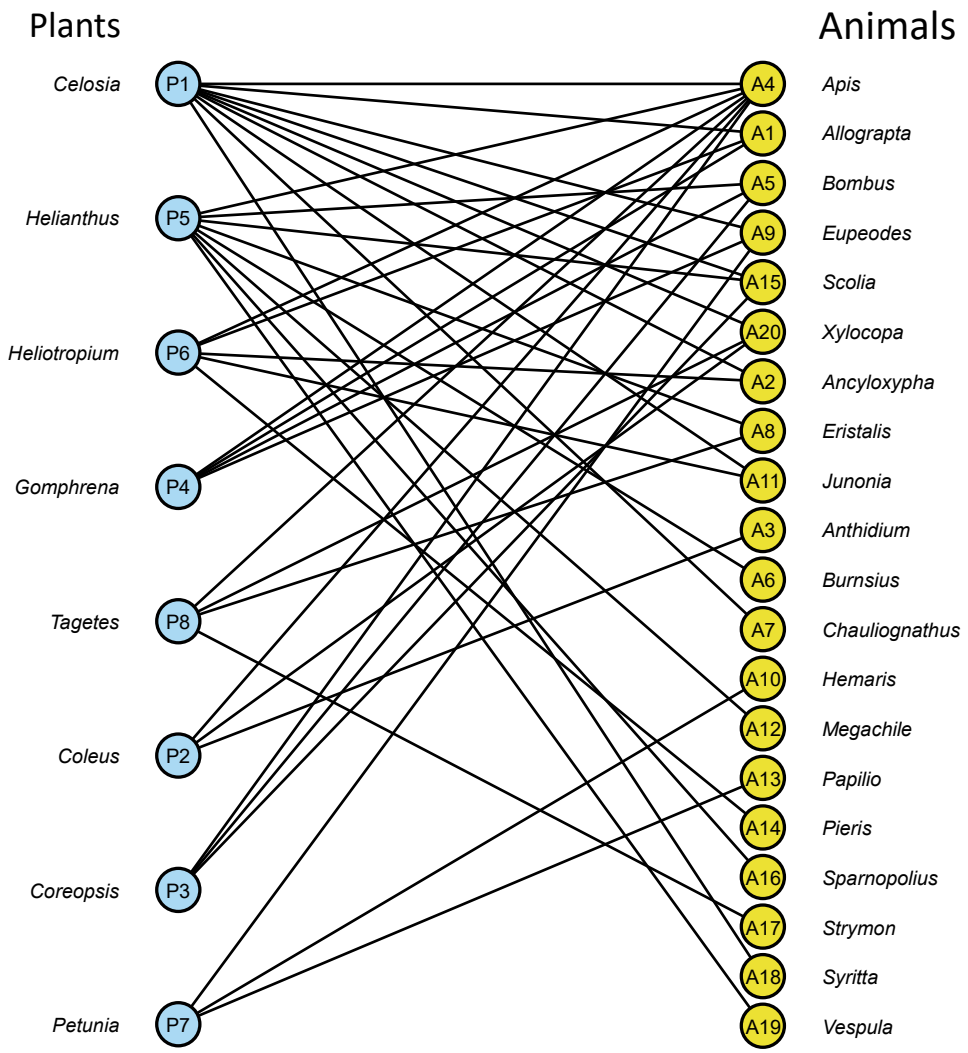
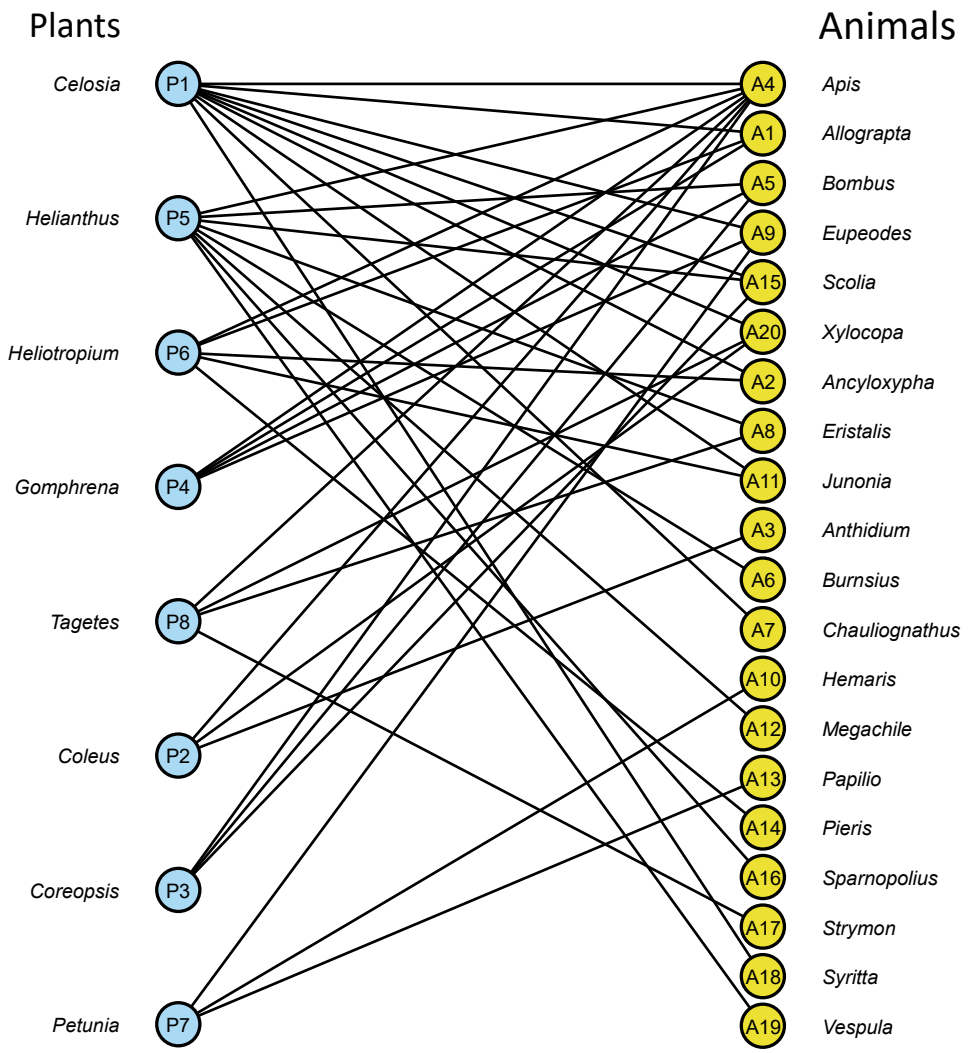
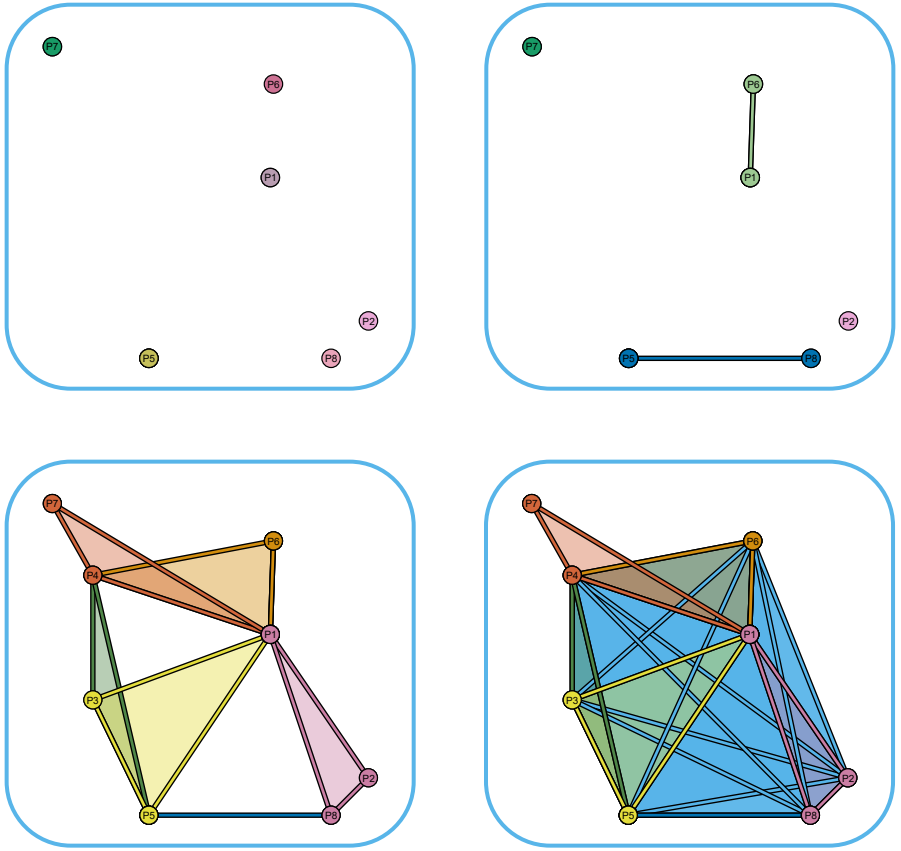
- We use flower visitation data to study plant-pollinator interactions, but not all flower visits result in pollination
- In this talk, we use the following terms interchangeably:
  - Plant / Flowering plant
  - Pollinator / Visitor / Insect / Animal
  - Interaction / Flower visitation

Binary interactions



Binary interactions





M. Kleczynski, D. Guillot, and C. Giusti, *The shape of ecological communities: From pollinators to purple martins*, 2023.

Quantitative flower visitation data

Flower visitation

Plant	Visitor	Number of interactions
Eriophyllum lanatum (common woolly sunflower)	*Apis mellifera* (honey bee)	8
Orthocarpus imbricatus (mountain owl's clover)	*Apis mellifera* (honey bee)	1
Orthocarpus imbricatus (mountain owl's clover)	*Bombus bifarius* (black-notched bumble bee)	2

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Flower survey

Plant	Number of flowers
Eriophyllum lanatum (common woolly sunflower)	30
Orthocarpus imbricatus (mountain owl's clover)	74

Preference index

$$PI_{ij}^{\text{survey}} = \frac{A_{ij}/A_{*j}}{F_i/F} \qquad PI_{ij}^{\text{no survey}} = \frac{A_{ij}/A_{*j}}{A_{i*}/A}$$

A_{ij}	total number of interactions between flower type i and visitor type j
A_{*j}	total number of interactions involving visitor type j
F_i	total number of flowers of type i
A_{i*}	total number of interactions involving flower type i
F	total number of flowers
A	total number of interactions

A. R. Kells, J. M. Holland, and D. Goulson, *The value of uncropped field margins for foraging bumblebees*, Journal of Insect Conservation, 5 (2001), pp. 283–291.

J. Fründ, K. E. Linsenmair, and N. Blüthgen, *Pollinator diversity and specialization in relation to flower diversity*, Oikos, 119 (2010), pp. 1581–1590.

Apis mellifera - *Eriophyllum lanatum* preference

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$$PI^{\text{survey}} = \frac{8/9}{30/104} = 3.1$$

$$PI^{\text{no survey}} = \frac{8/9}{8/11} = 1.2$$

Simplicial complexes

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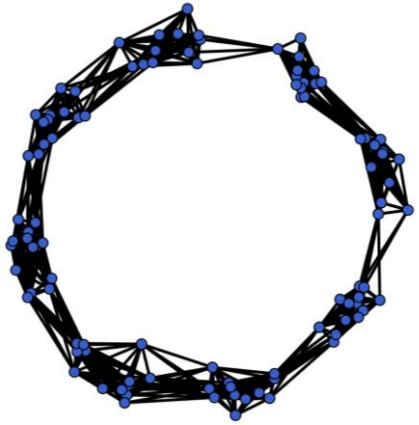
Graphs

Graphs

A graph is an ordered pair (V, E) such that V and E are sets and each element of E is a subset of V with two elements.

We call the elements of V vertices and the elements of E edges.



Graphs and simplicial complexes

Graphs

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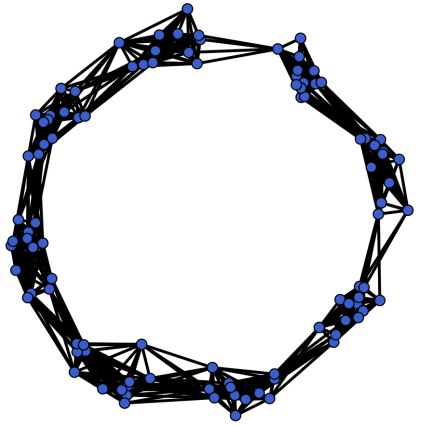
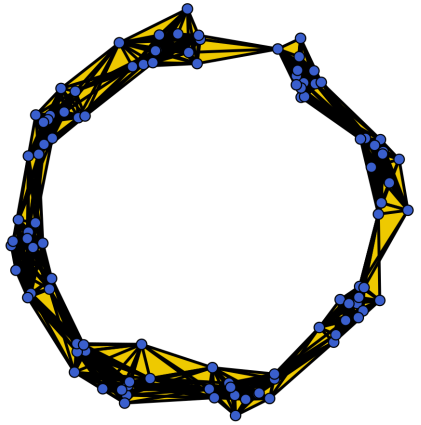


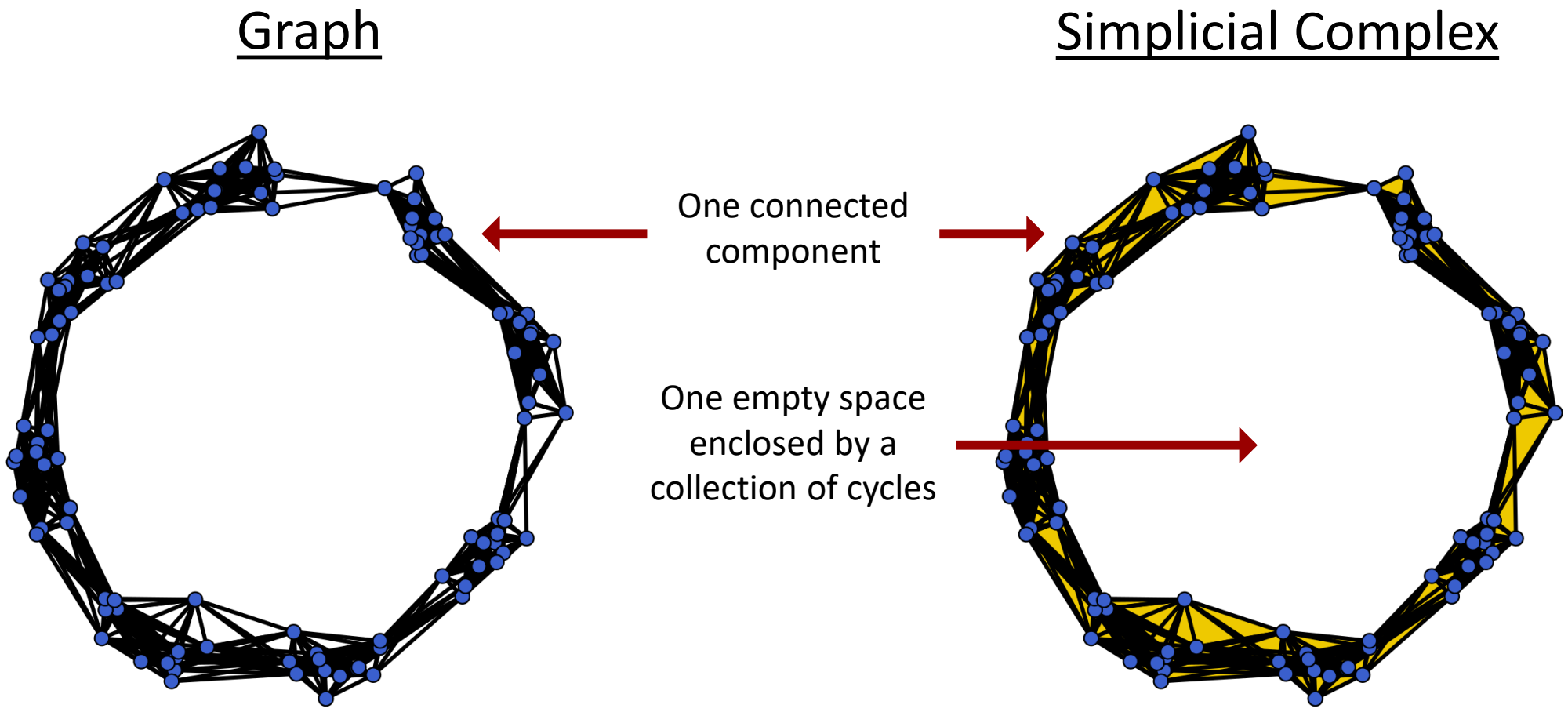
Simplicial complexes

Given a discrete set X, a simplicial complex K_X is a collection of finite subsets of X such that if $\tau \in K_X$ and $\sigma \subset \tau$ then $\sigma \in K_X$.

We call the elements of X vertices and the elements of K_X simplices. A simplex σ with $|\sigma| = k + 1$ is a k-simplex.



Structural features



Characterizing simplicial complexes

Let n be a nonnegative integer. A simplicial complex generates a homology group H_n. For coefficients over a field, H_n is a quotient vector space. Its elements are equivalence classes of cycles.

- $\dim(H_0)$ is the number of connected components.
- $\dim(H_1)$ is the number of empty spaces which are bounded by 1-simplices (like windows).
- $\dim(H_2)$ is the number of empty spaces which are bounded by 2-simplices (like bubbles).

Simplicial complexes from plant-pollinator data

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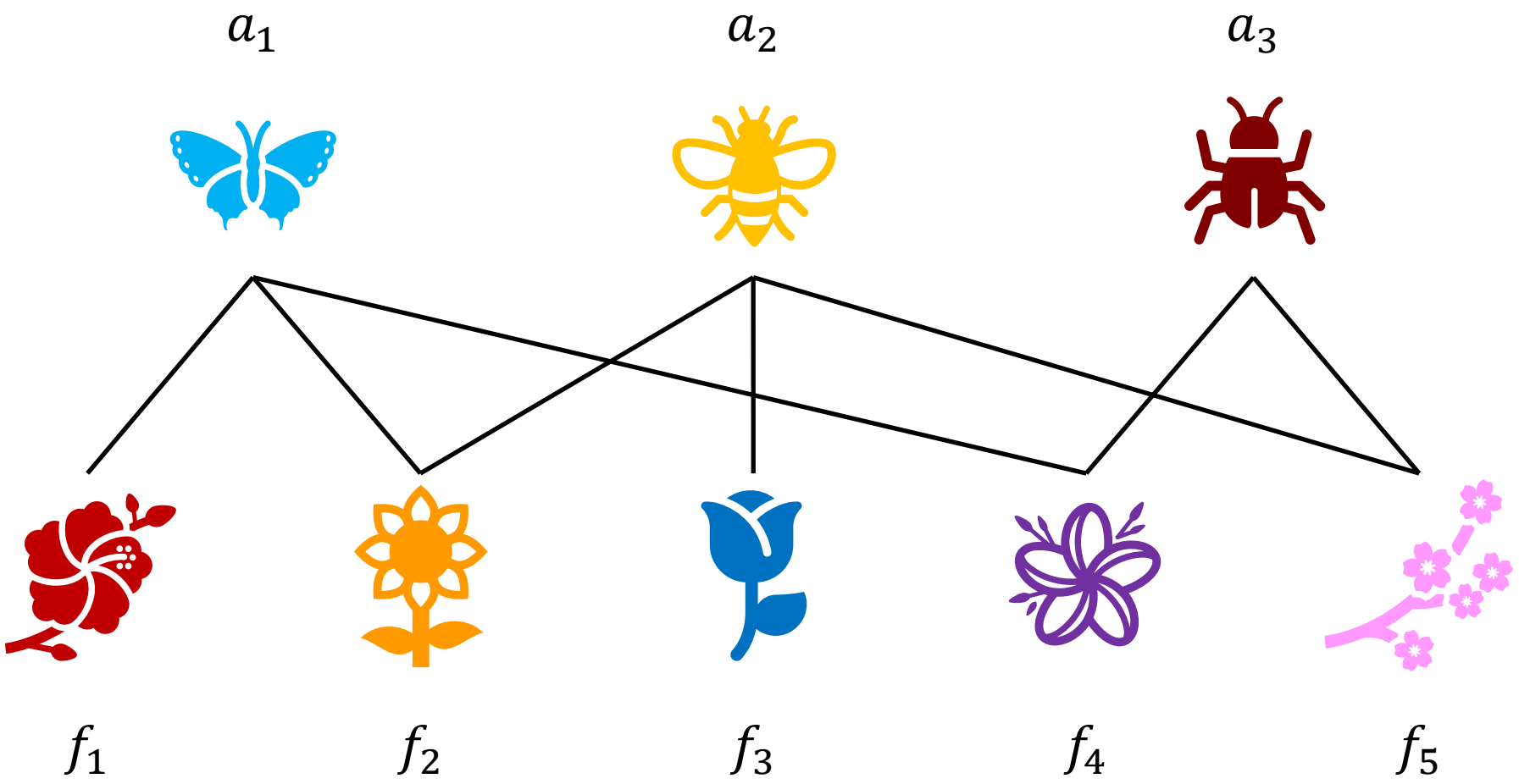
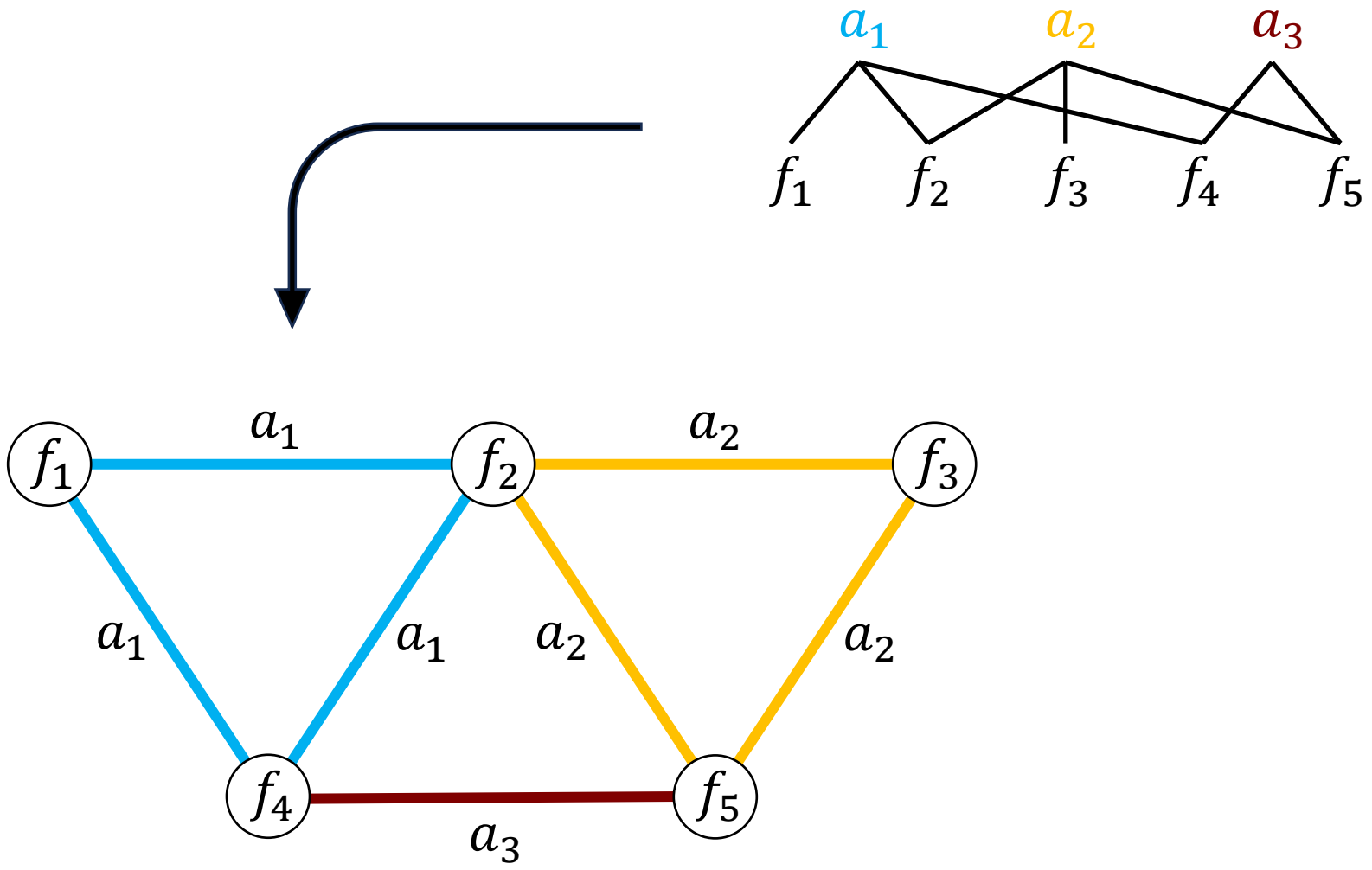
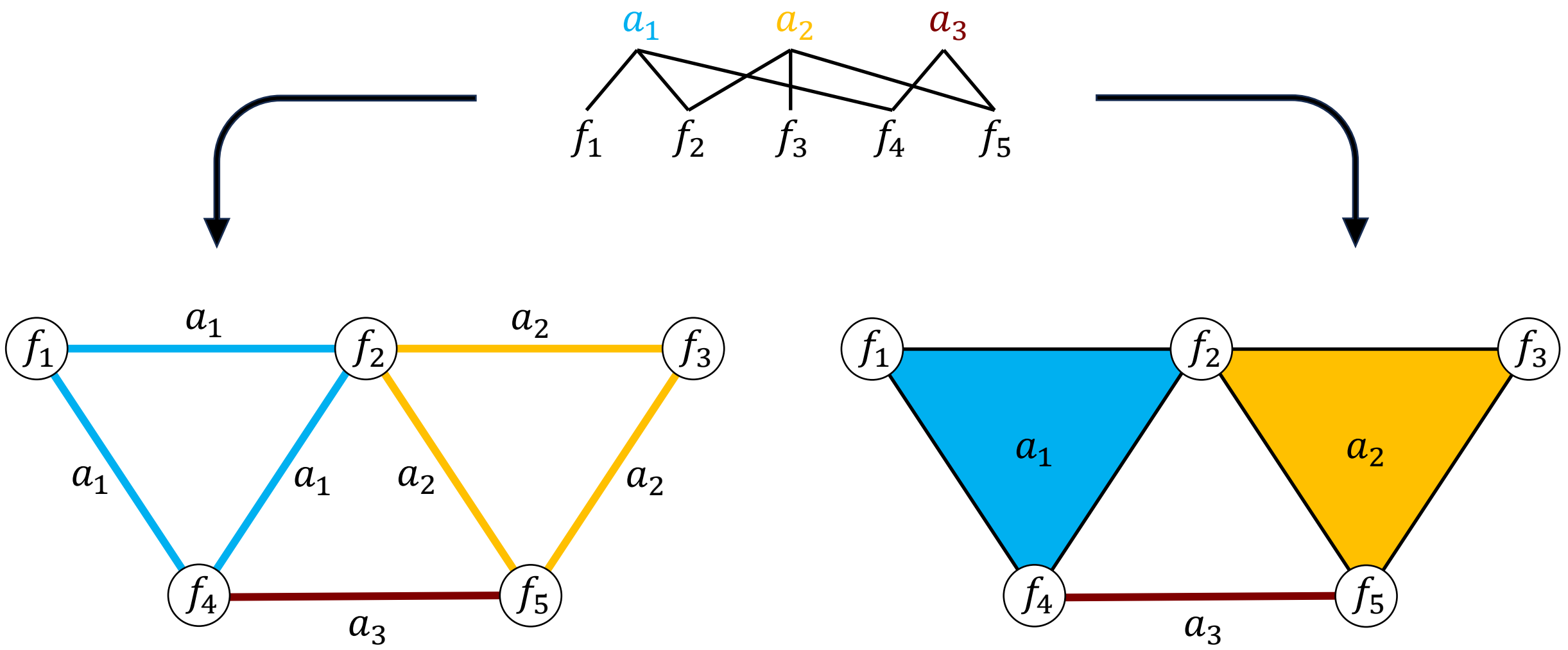
Simple flower visitation community





B. Peleg, A. Shmida, and S. Ellner, *Foraging graphs: Constraint rules on matching between bees and flowers in a two-sided pollination market*, Journal of Theoretical Biology, 157(1992), pp. 191–201.



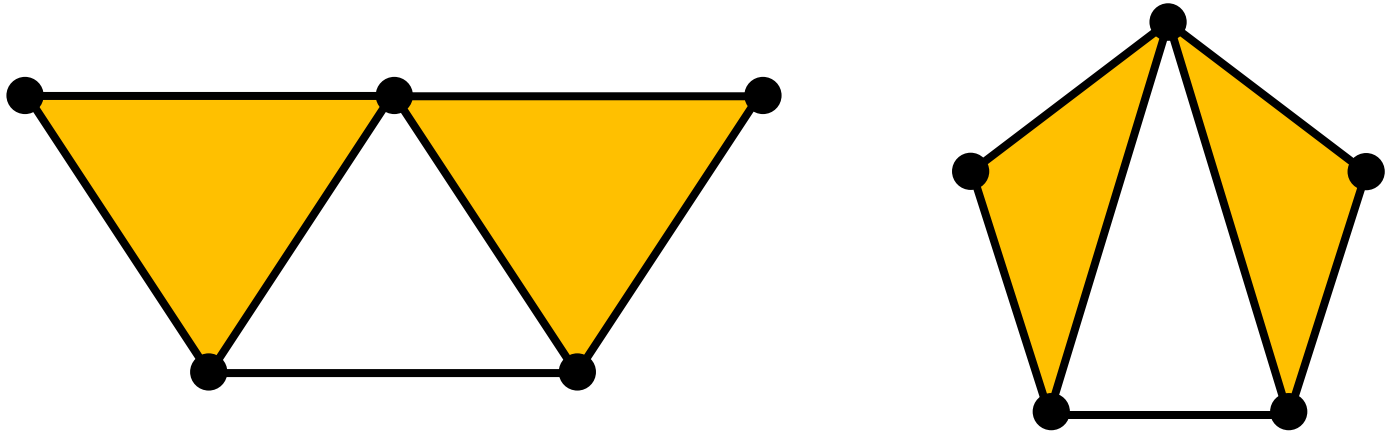
B. Peleg, A. Shmida, and S. Ellner, *Foraging graphs: Constraint rules on matching between bees and flowers in a two-sided pollination market*, Journal of Theoretical Biology, 157(1992), pp. 191–201.

C. H. Dowker, *Homology groups of relations*, Annals of Mathematics, 56 (1952), pp. 84–95.

G. Sugihara, *Niche Hierarchy: Structure, Organization, and Assembly in Natural Systems*, J. Ross Publishing, 2017.

Vertex coordinates

- The vertices of these simplicial complexes do not have defined coordinates. For example, these are the same simplicial complex.



- We may choose vertex coordinates for visualization, but we do not use them for computations.

Edge weights

Scaled preference

$$\widetilde{PI}_{ij} = \frac{PI_{ij}}{\max_{m,n} PI_{mn}}$$

	a_1	a_2	a_3
f_1	0.7	0.0	0.0
f_2	0.2	0.8	0.0
f_3	0.0	0.3	0.0
f_4	0.3	0.0	0.1
f_5	0.0	0.2	1.0

Cross-dissimilarity =
1 – Scaled preference

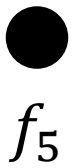
	a_1	a_2	a_3
f_1	0.3	1.0	1.0
f_2	0.8	0.2	1.0
f_3	1.0	0.7	1.0
f_4	0.7	1.0	0.9
f_5	1.0	0.8	0.0

Sequence of simplicial complexes

$$\varepsilon = 0.0$$

Cross-dissimilarity

	a_1	a_2	a_3
f_1	0.3	1.0	1.0
f_2	0.8	0.2	1.0
f_3	1.0	0.7	1.0
f_4	0.7	1.0	0.9
f_5	1.0	0.8	0.0



Sequence of simplicial complexes

$\varepsilon = 0.2$

Cross-dissimilarity

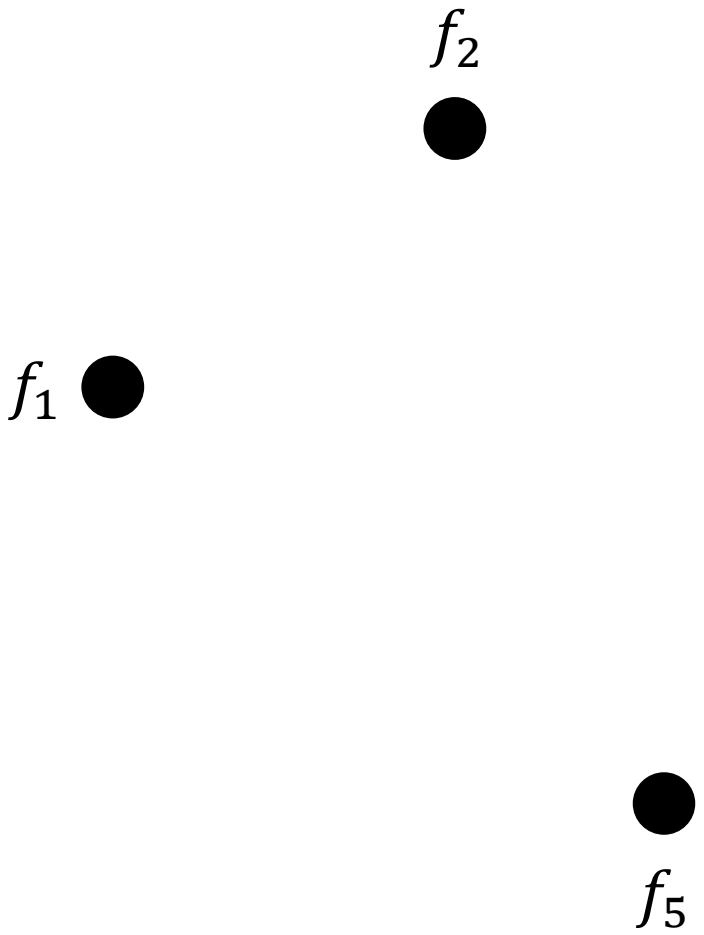
	a_1	a_2	a_3
f_1	0.3	1.0	1.0
f_2	0.8	0.2	1.0
f_3	1.0	0.7	1.0
f_4	0.7	1.0	0.9
f_5	1.0	0.8	0.0

Sequence of simplicial complexes

$\varepsilon = 0.3$

Cross-dissimilarity

	a_1	a_2	a_3
f_1	0.3	1.0	1.0
f_2	0.8	0.2	1.0
f_3	1.0	0.7	1.0
f_4	0.7	1.0	0.9
f_5	1.0	0.8	0.0

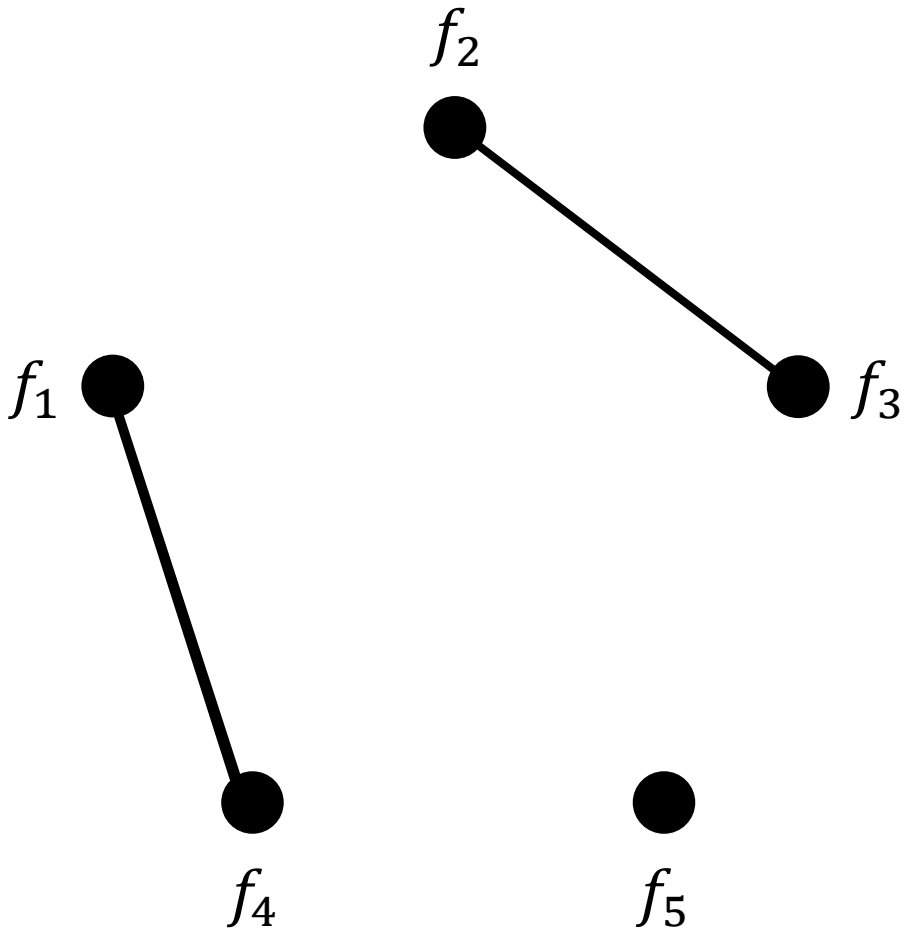


Sequence of simplicial complexes

$\varepsilon = 0.7$

Cross-dissimilarity

	a_1	a_2	a_3
f_1	0.3	1.0	1.0
f_2	0.8	0.2	1.0
f_3	1.0	0.7	1.0
f_4	0.7	1.0	0.9
f_5	1.0	0.8	0.0

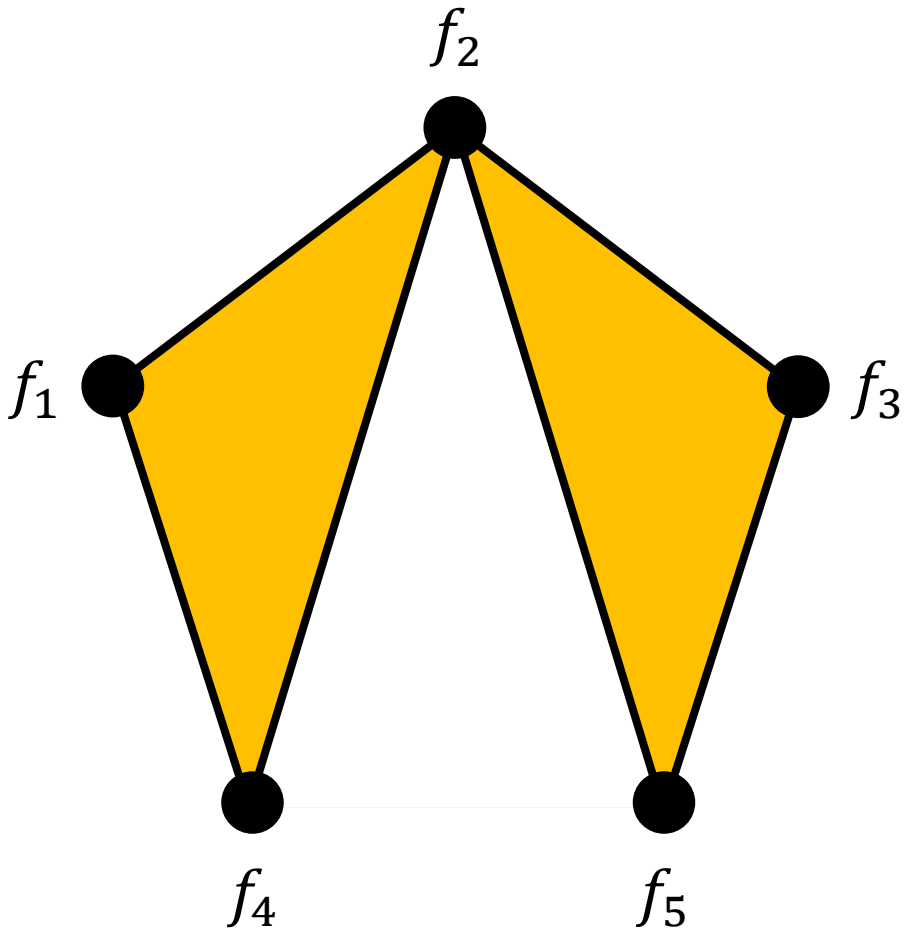


Sequence of simplicial complexes

$\varepsilon = 0.8$

Cross-dissimilarity

	a_1	a_2	a_3
f_1	0.3	1.0	1.0
f_2	0.8	0.2	1.0
f_3	1.0	0.7	1.0
f_4	0.7	1.0	0.9
f_5	1.0	0.8	0.0



Sequence of simplicial complexes

$\varepsilon = 0.9$

Cross-dissimilarity

	a_1	a_2	a_3
f_1	0.3	1.0	1.0
f_2	0.8	0.2	1.0
f_3	1.0	0.7	1.0
f_4	0.7	1.0	0.9
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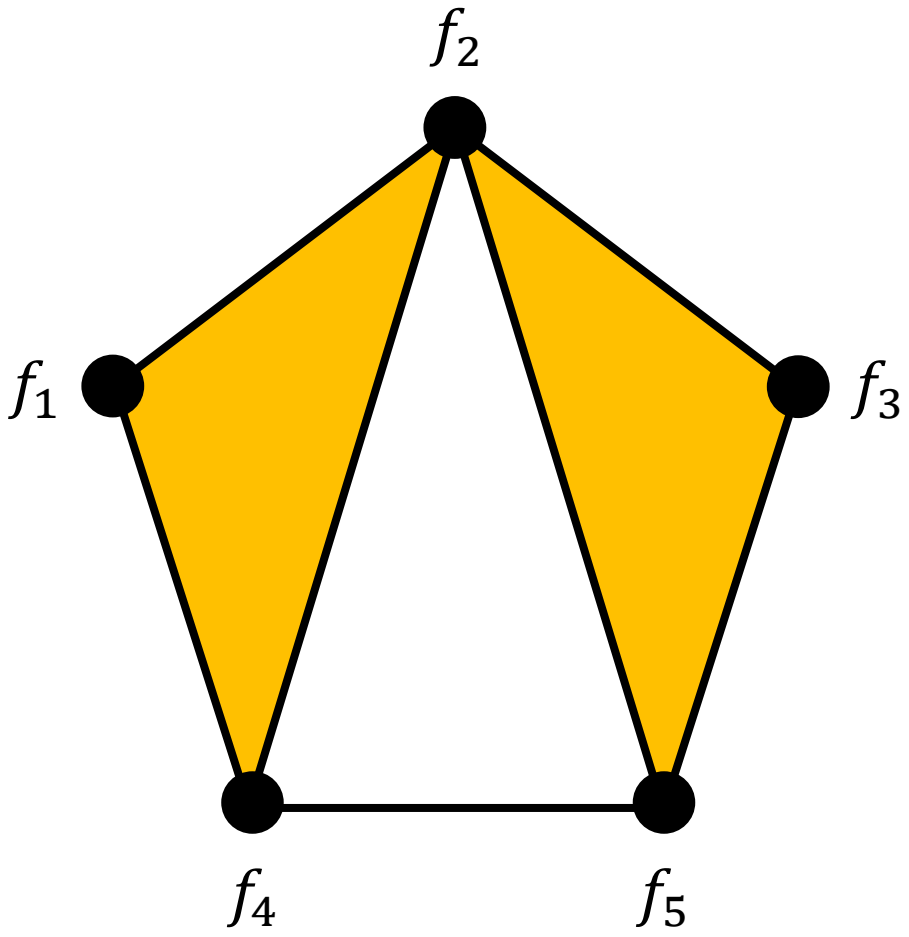

Sequence of simplicial complexes

$\varepsilon = 1.0$

Cross-dissimilarity

	a_1	a_2	a_3
f_1	0.3	1.0	1.0
f_2	0.8	0.2	1.0
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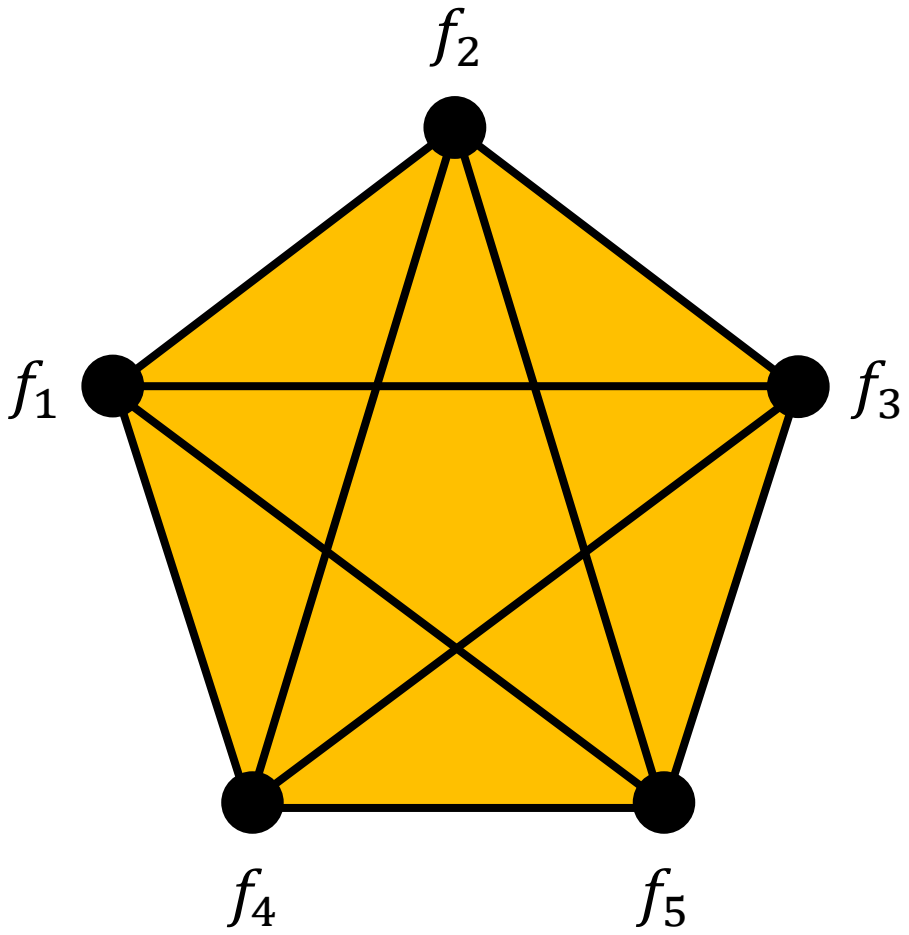

Topological summaries and distances

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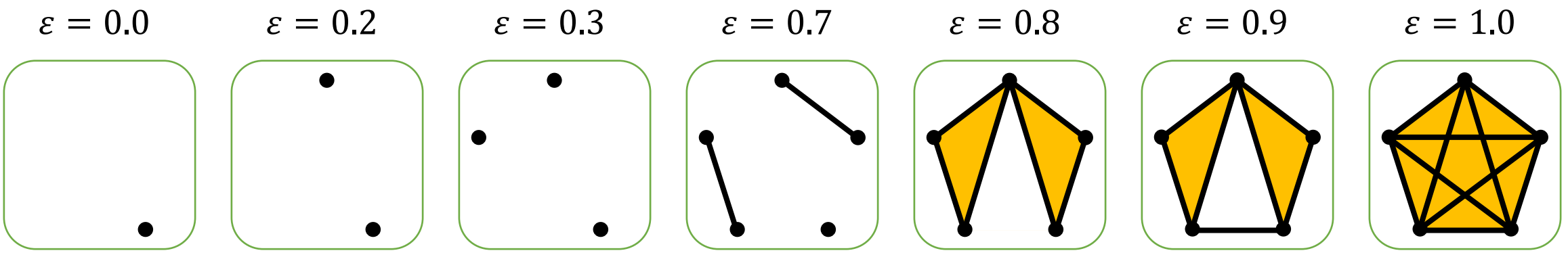
Dimension 1 barcode

Sequence of Simplicial Complexes



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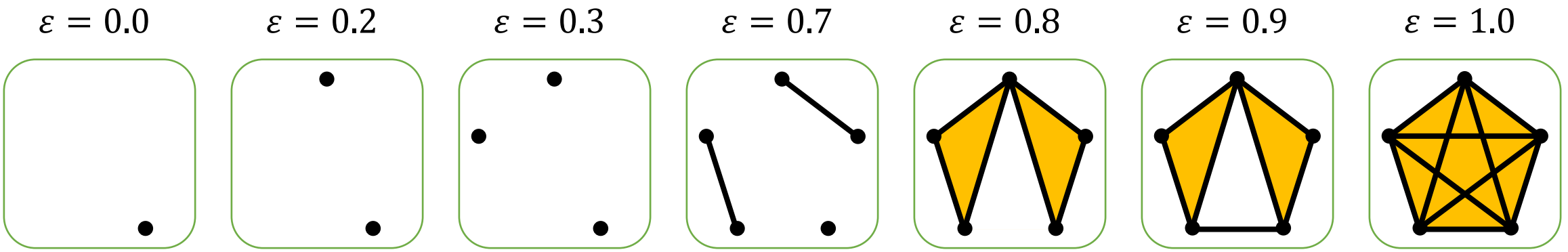
Sequence of Simplicial Complexes

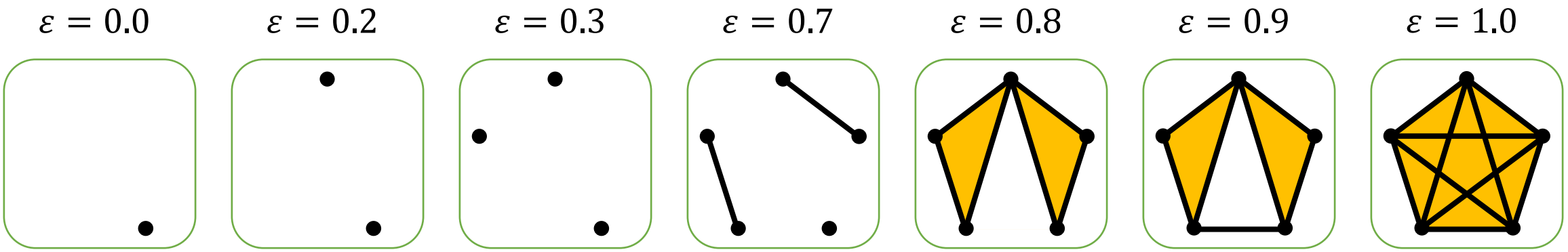


Feature
Birth
$b = 0.9$

Feature
Death
$d = 1.0$

Dimension 1 barcode

Sequence of Simplicial Complexes



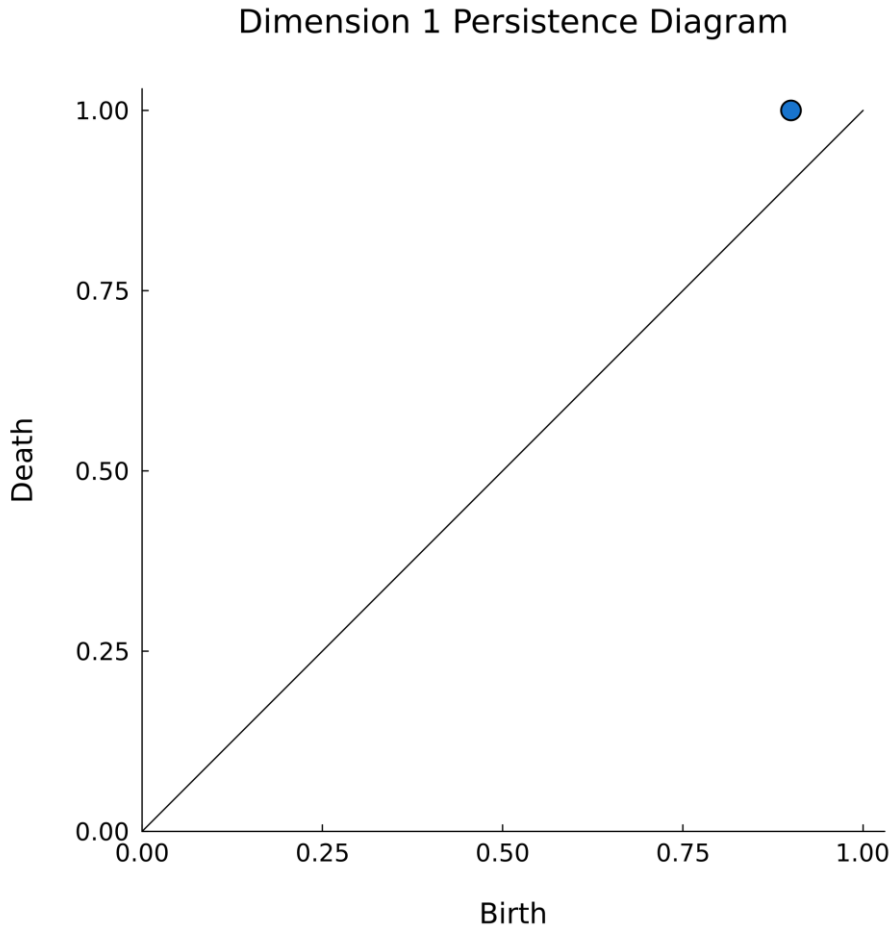
- Summarize each topological feature as an interval $[b, d)$, commonly called a bar.
- Repeat this process for each topological feature in a given dimension to obtain a multiset of intervals called a barcode.
- The dimension 1 barcode for this example is $\{[0.9, 1.0)\}$.

Persistence diagrams

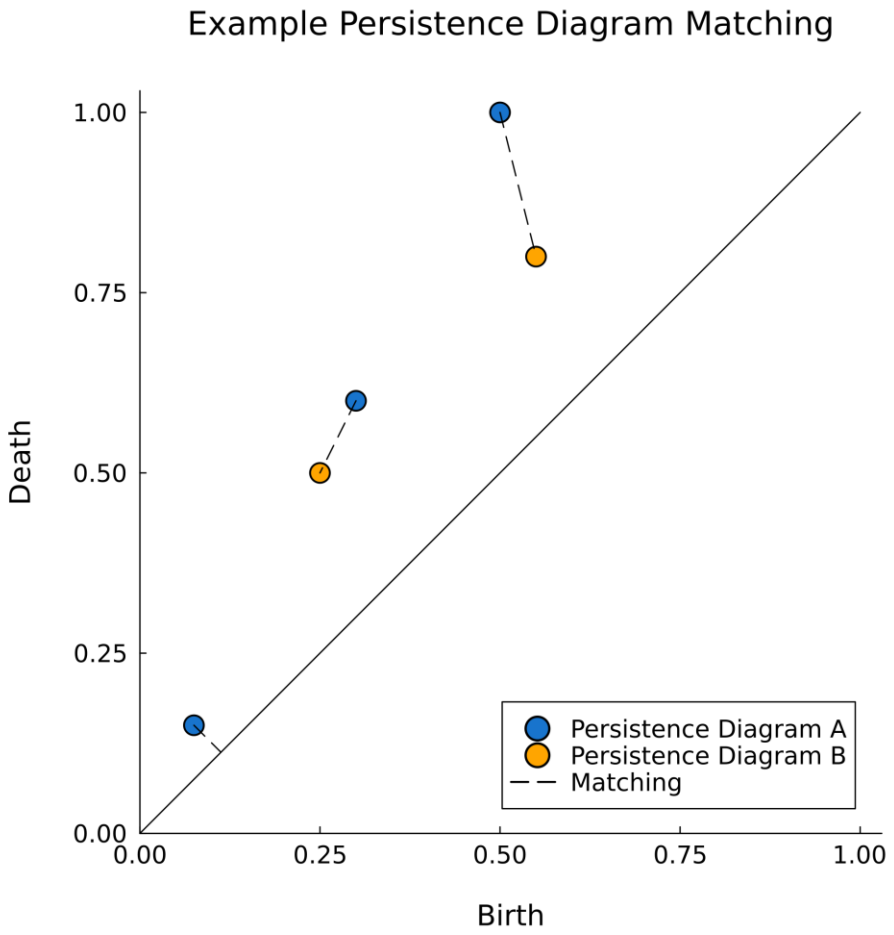
A persistence diagram is a multiset with elements

- (b, d) for each interval $[b, d)$ in the barcode (with the same multiplicity as in the barcode)
- (x, x) for all $x \in \mathbb{R}$ (with countably infinite multiplicity)

* For the current analysis, barcodes only contain finite intervals. Our definition of the persistence diagram reflects this.



(p, q)-Wasserstein distance



Let S and T be multisets of points in $\mathbb{R}^2$. Let $\text{bij}(S, T)$ denote the set of all bijections from S to T.

The (p, q)-Wasserstein distance between S and T is

$$\inf_{f \in \text{bij}(S,T)} \left(\sum_{s \in S} \|s - f(s)\|_q^p \right)^{1/p}$$

Methods

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Data processing

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- Each meadow complex contains four meadows, and there are five observation periods each year (about once a week).
- We compute preferences for each meadow-year-watch.
- We compute persistent homology for each complex-year. We use median preferences (out of all meadow-year-watches) for the main analysis and randomly chosen preferences (out of all meadow-year-watches) for bootstrap samples.

Bootstrap samples

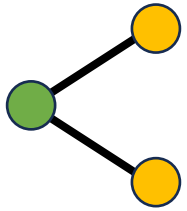
Simplified example data

Plant	Visitor	Total number of interactions	Weight
Common woolly sunflower (CWS)	Honey bee	4	0.8
Common woolly sunflower (CWS)	California bumble bee	1	0.2

Weighted subsample with replacement

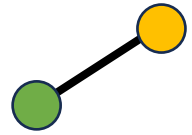
<u>Sample 1:</u>
CWS - Honey bee
CWS - Honey bee
CWS - Honey bee
CWS - California bumble bee
CWS - Honey bee

Plants Visitors



<u>Sample 2:</u>
CWS - Honey bee
CWS - Honey bee
CWS - Honey bee
CWS - Honey bee
CWS - Honey bee

Plants Visitors



Topological rankings of plant types

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- Plants with higher Wasserstein distances receive higher rankings

Preliminary results

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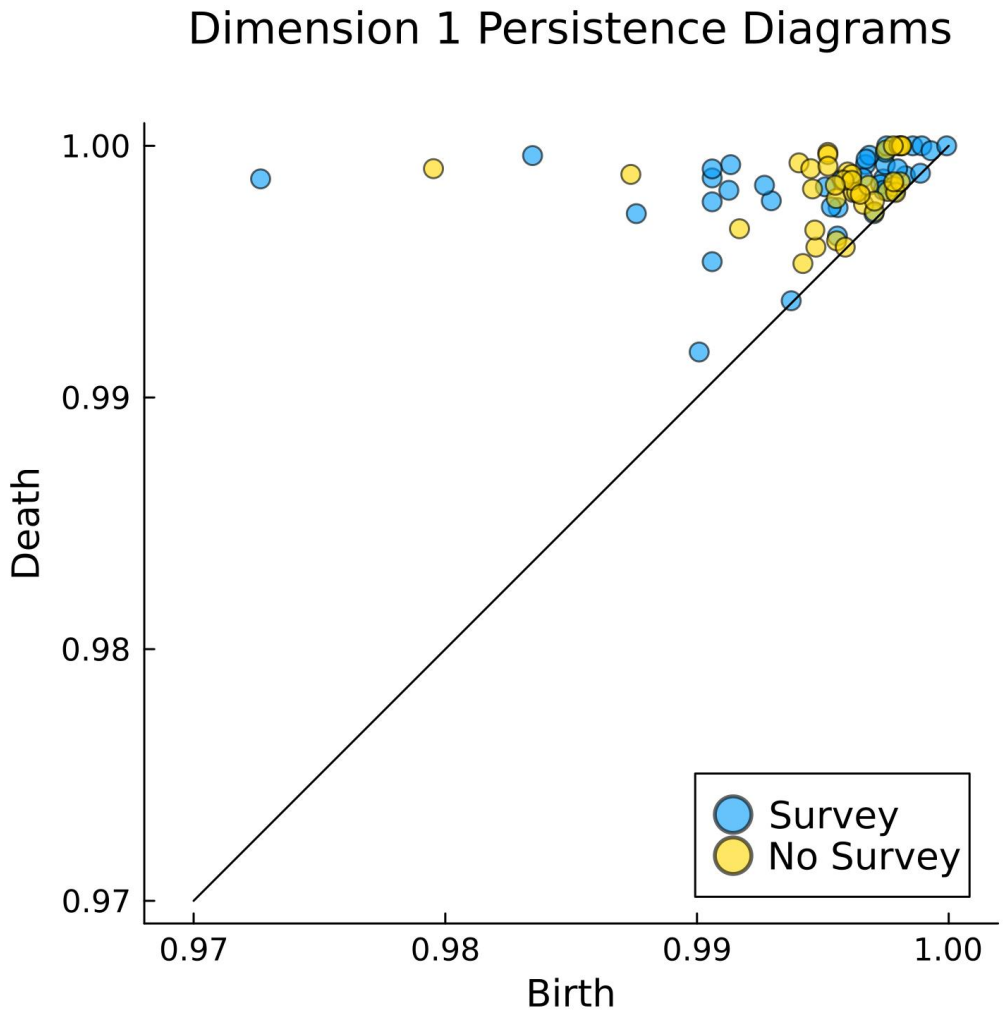
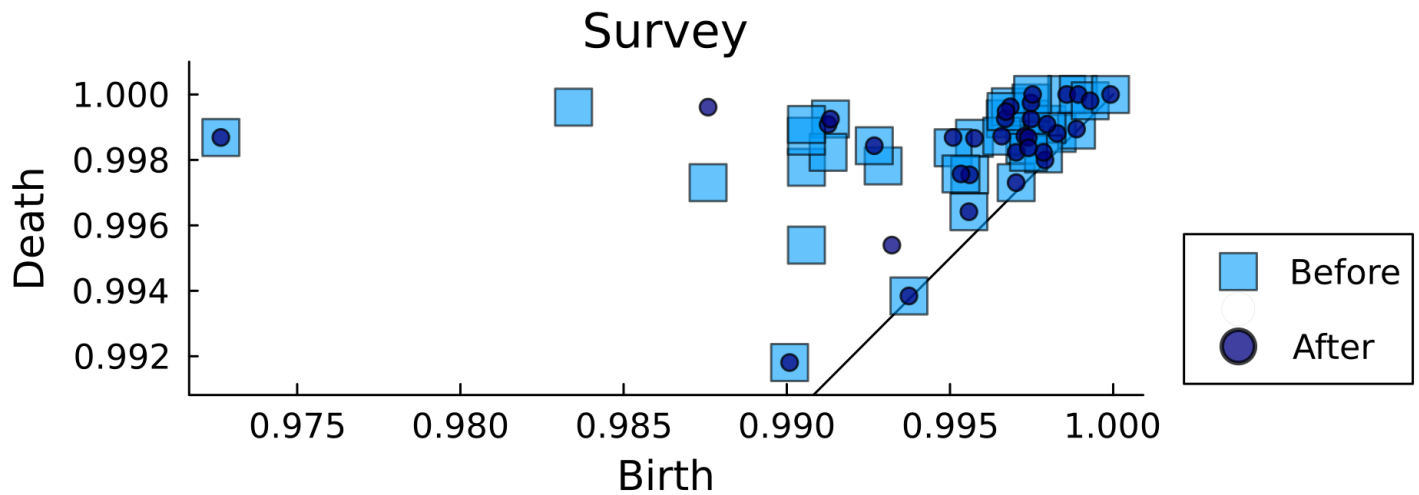
Carpenter meadow complex, 2017



Dimension 1 persistence diagrams before and after removing *Angelica arguta*



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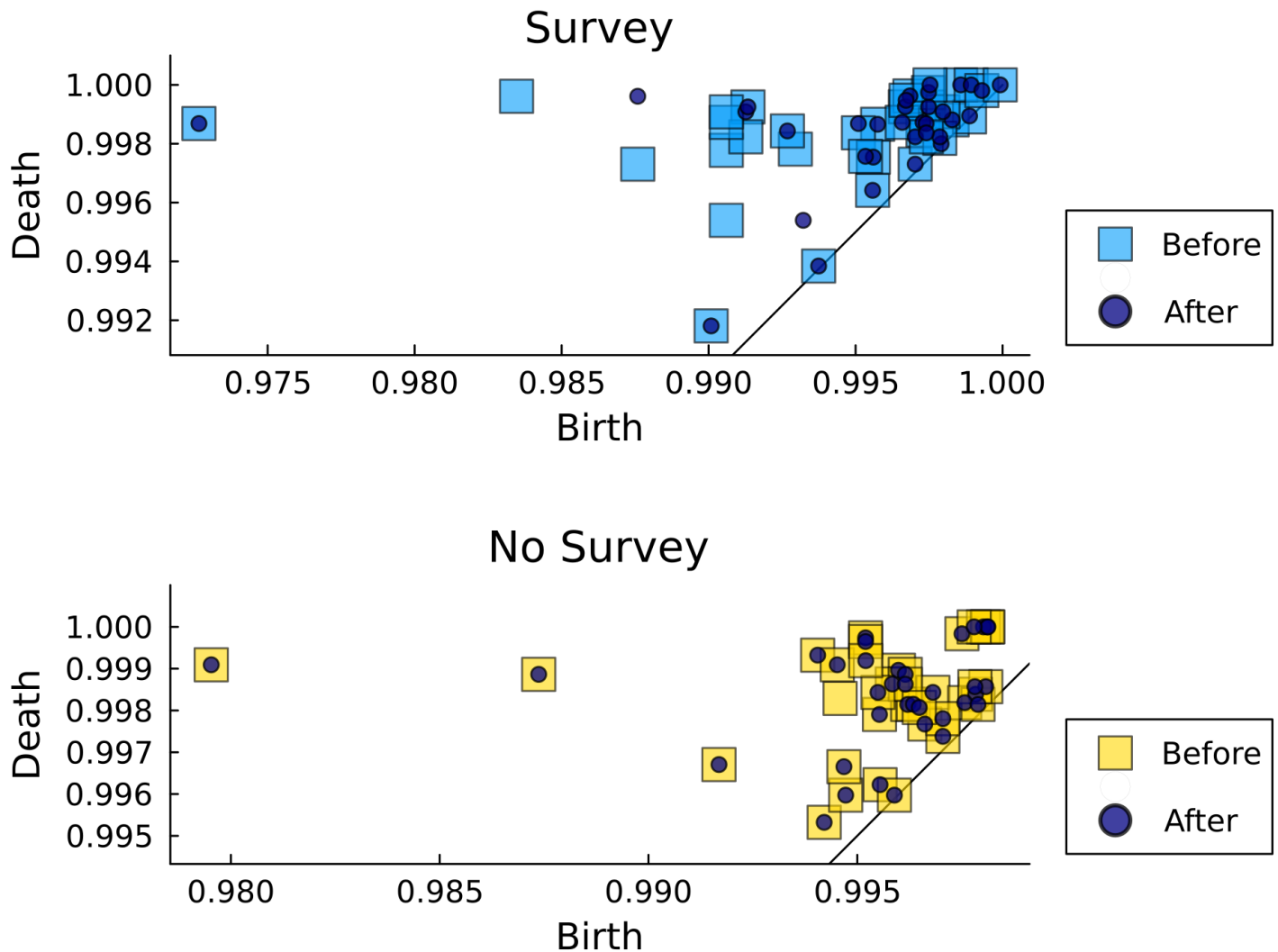

Topological rankings

Survey

Plant name	Wasserstein distance (p = q = 1)	Topological ranking
Angelica arguta	0.0427	1
Fragaria virginiana	0.0376	2
Erigeron aliceae	0.0338	3
Erysimum capitatum	0.0227	4
Ligusticum grayi	0.0178	5

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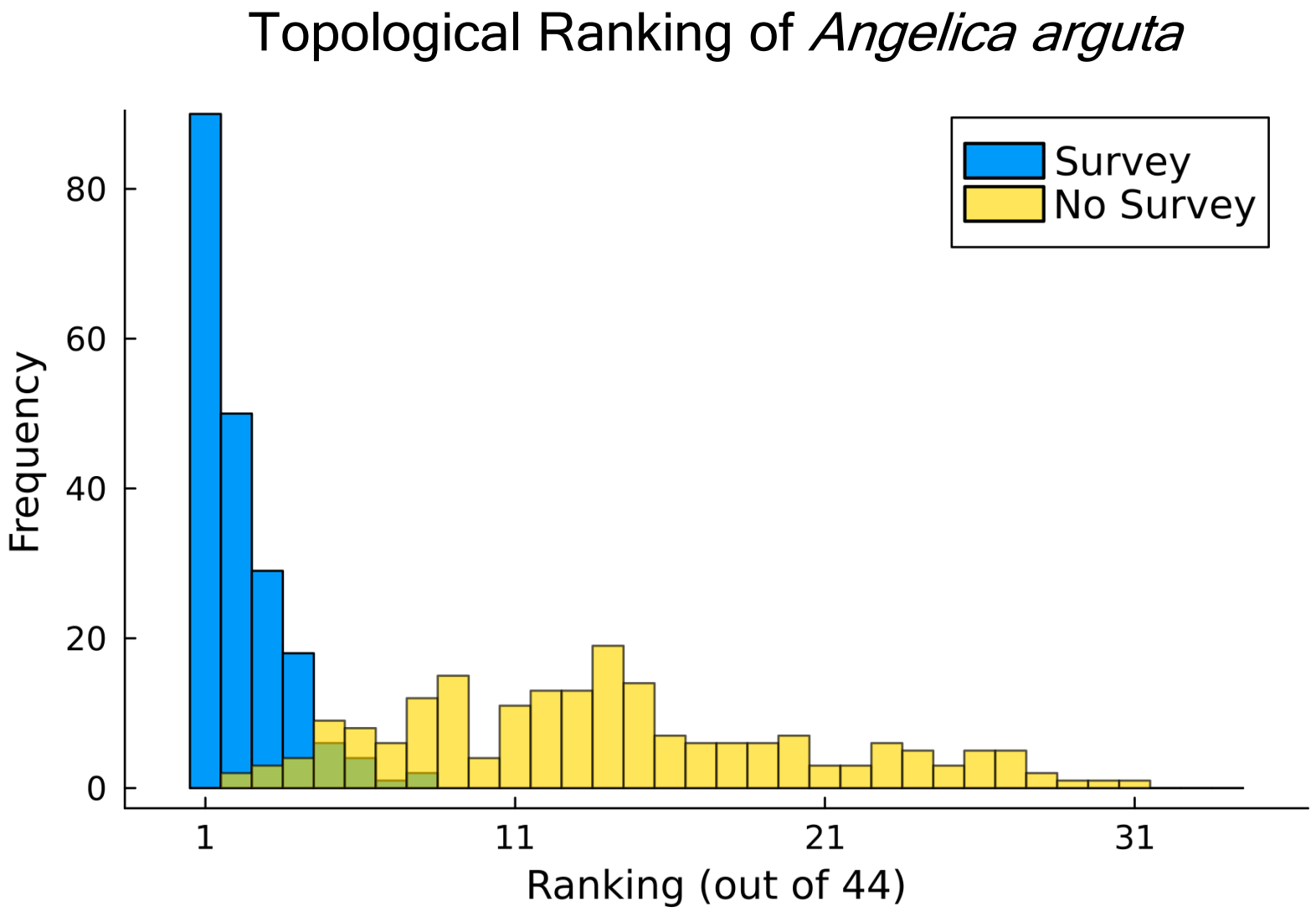
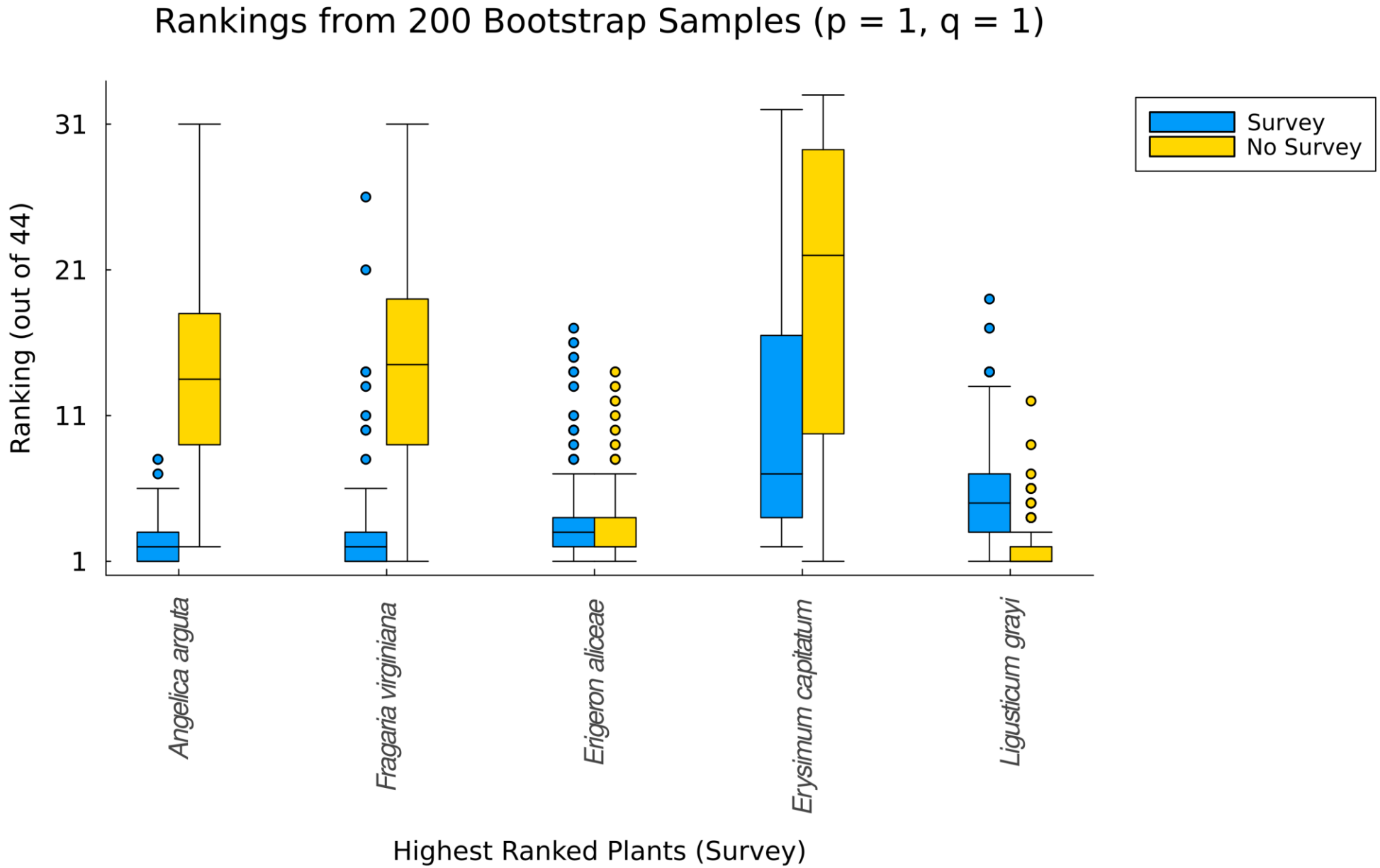
No survey

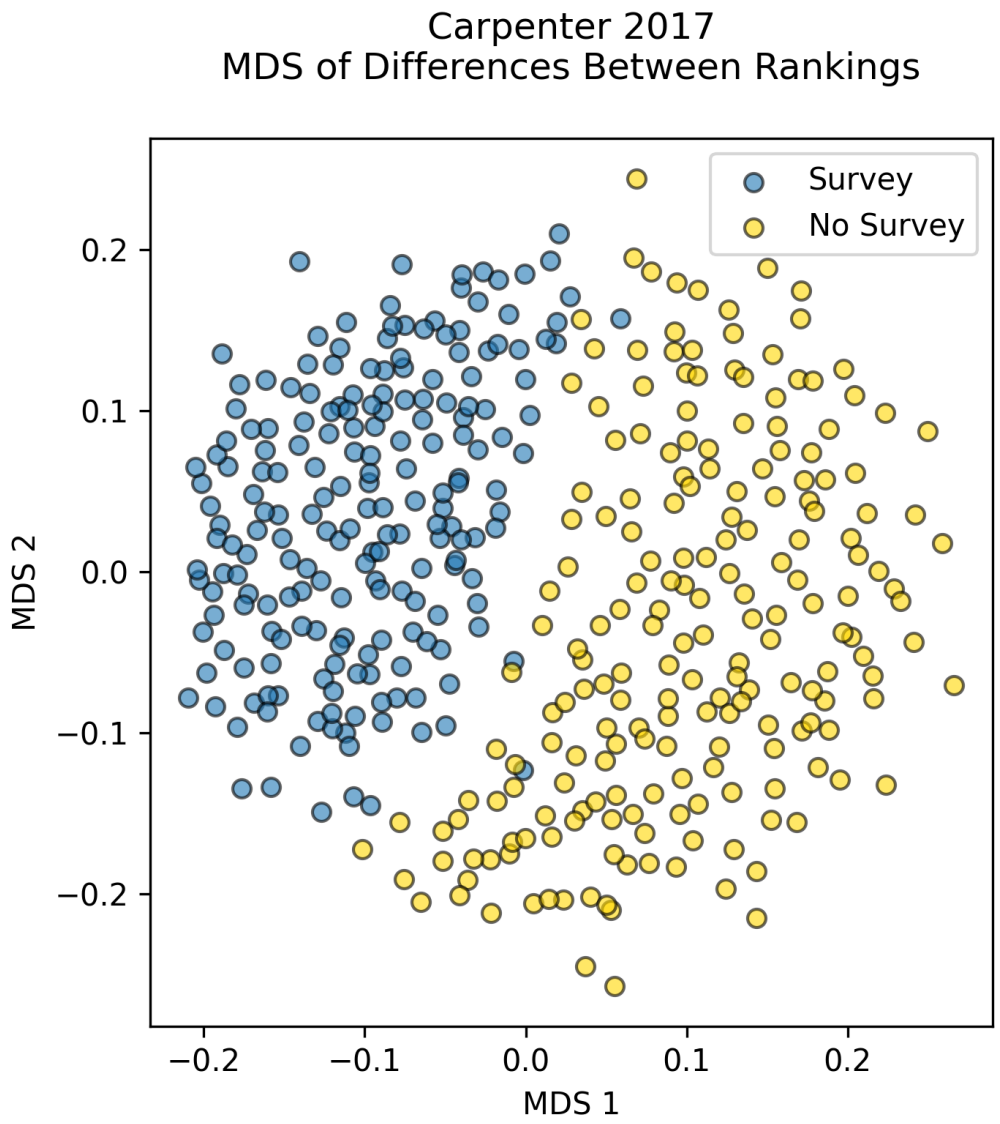
Plant name	Wasserstein distance (p = q = 1)	Topological ranking
Erigeron aliceae	0.0268	1
Erigeron foliosus	0.0214	2
Ligusticum grayi	0.0178	3
Claytonia lanceolata	0.0167	4
Erysimum capitatum	0.0164	5

Rankings from 200 bootstrap samples (p = q = 1)




Rankings from 200 Bootstrap Samples (p = 1, q = 1)
Ranking (out of 44)
31
21
11
1
Survey
No Survey
Angelica arguta
Fragaria virginiana
Erigeron aliceae
Erysimum capitatum
Ligusticum grayi
Highest Ranked Plants (Survey)

Rankings from (1, 1)-Wasserstein Distances



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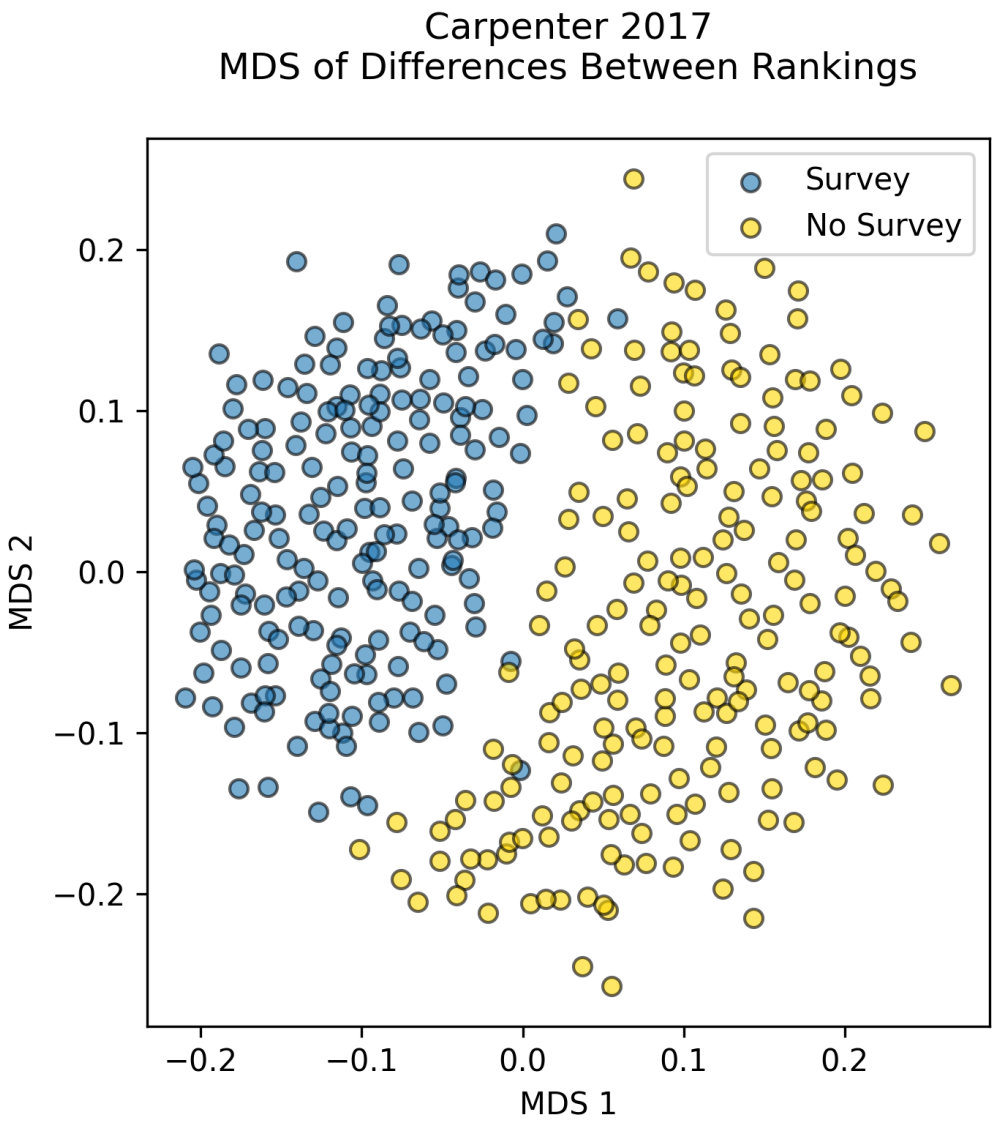
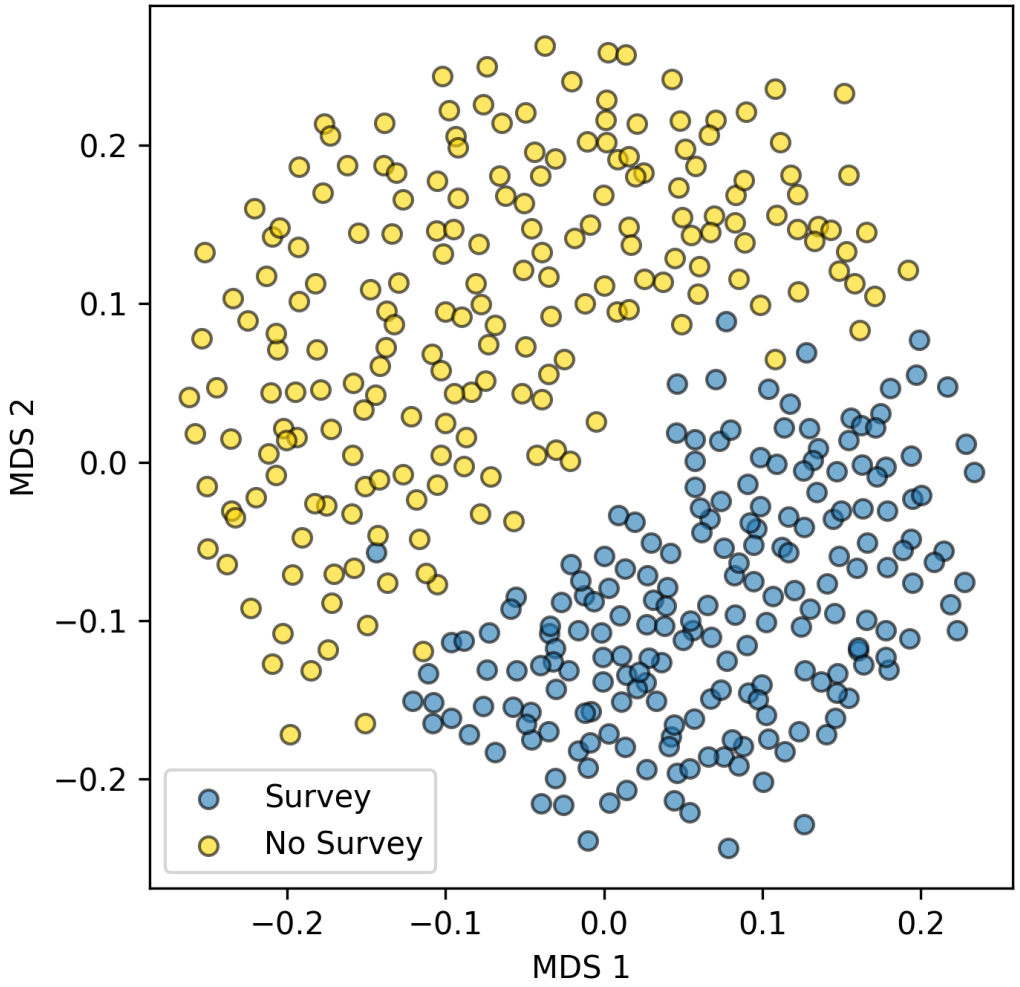


Rankings from (2, 2)-Wasserstein Distances



Conclusions

- Data-driven techniques are of interest for real-world interventions such as developing seed mixes which are beneficial to pollinators.

W. Glenny, J. Runyon, and L. Burkle, *Assessing pollinator friendliness of plants and designing mixes to restore habitat for bees*, Jan. 2022.

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- Understanding the importance of different data types helps guide optimal strategies for data collection.

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- Understanding the importance of different data types helps guide optimal strategies for data collection.
- We find that flower survey data can significantly alter the mathematical structure of plant-pollinator communities, demonstrating the importance of flowering plant surveys.

Thank you!

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