Optimization of Wye-Delta-Type Quantum Hall Resistance Standard

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***Abstract* — Theoretically wye-delta transformation can be used to realize ultra-high resistances up to PΩ. For graphene-based quantum Hall array resistance standards fabricated to utilize the wye-delta transformation, a few challenges present themselves, including the unique quantized resistance in a graphene Hall bar and the limitation of the area of homogeneous high-quality graphene. In this paper, we discuss approaches to optimize the transformation for quantum Hall array resistance standard and propose a dual-output design for 1 MΩ and 100 MΩ as an alternative to other build-up techniques, shortening the path from quantum resistance standards.**

***Index Terms* — dual-source bridge, wye-delta transformation, quantum Hall array resistance standard, dual-output.**

I. Introduction

Wye-delta (or Y-Δ) transformation [1] is a well-known mathematical technique to simplify complex circuits, widely used in the analysis of three-phase electric power circuits. This technique attests the equivalence of a "Y"-shaped network (the orange schematic in Fig.1) and a "Δ"-shaped network (the green schematic in Fig.1) that connect the three terminals A, B, C. The "Y"-shaped network has an extra node in the center. The equivalence is manifested by the resistance values between each pair of the three terminals, as shown by equations (1-3).

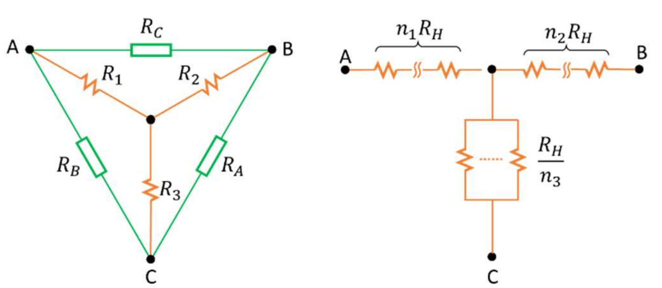

Fig. 1. (a) Schematic diagram for Y-Δ transformation. (b) Schematic diagram for quantum Hall array resistance standard based on Y-Δ transformation.

$$R_A = R_2 + R_3 + \frac{R_2 R_3}{R_1} \tag{1}$$

$$R_B = R_1 + R_3 + \frac{R_3 R_1}{R_2} \tag{2}$$

$$R_C = R_1 + R_2 + \frac{R_1 R_2}{R_3} \tag{3}$$

In the electrical calibration community, the transformation from Y-network to Δ-network is adopted to generate ultra-high resistance standard in the range of MΩ to PΩ [2-5]. As show by the equations (1-3), for large resistors, the resistance of the Δ-network is approximately proportional to the multiplication of the resistance of two arms in Y-network and inversely proportional to the resistance of the third arm. For example, $R_C = 10$ MΩ can be realized by two 100 kΩ resistors (R_1, R_2) and one 1 kΩ resistor (R_3).

Recently, Y-Δ-type quantum Hall array resistance standard (QHARS) has been demonstrated by utilizing 79 graphene Hall bars to produce $R_C \approx 20.6$ MΩ [5]. At 2.5 K and in a magnetic field above 4 T, the Hall resistance of each graphene Hall bar manifests $\nu = 2$ plateau ($R_H = R_K/2$), where $R_K = \frac{h}{e^2} = 25812.8074593 \ldots$ Ω. In the previous proof-of-concept work, two arms of Y-network consist of 39 graphene Hall bars connected in series, respectively, and the third arm has a single graphene Hall bar. Measurement of R_C with a dual-source bridge shows a 5 μΩ/Ω offset from the theoretical value that has been attributed to non-ideal bridge ground connection and errors stemming from possible imperfect graphene Hall bars.

This work will present the analysis of various Y-network designs utilizing graphene Hall bars to produce 1 MΩ, 10 MΩ, and 100 MΩ resistors, which offers a preliminary approach to optimize ultra-high resistance QHARS.

II. Wye-delta-type QHARS

To scale up the quantum Hall resistance using the Y-Δ transformation, we should choose series networks for arm 1 and arm 2 and a parallel network for arm 3 in the Y-configuration (Fig. 1b). For a symmetric Y-network, the resistances from the corresponding Δ-network are

$$R_A = R_B = \left(n_1 + \frac{2}{n_3}\right) R_H \tag{4}$$

$$R_C = (n_1^2 n_3 + 2n_1) R_H \tag{5}$$

For the nominal resistance value R_n and a chosen n_3, we can solve equation (5) to obtain

$$n_1 = n_2 = \left(\frac{R_n}{n_3 R_H} + \frac{1}{n_3^2}\right)^{\frac{1}{2}} \quad (6)$$

Equation (6) is a useful tool to search for the optimized configuration (n_1, n_2, n_3) for a symmetric Y-network with minimum difference (offset σ) between the nominal resistance R_n and the theoretical value R_C.

When adopting asymmetric Y-configuration for QHRS, equation (3) becomes

$$R_C = (n_1 n_2 n_3 + n_1 + n_2) R_H \quad (7)$$

Table 1.

$R_n(\Omega)$	n_1	n_2	n_3	Total n	$R_1(\Omega)$	$R_2(\Omega)$	$R_3(\Omega)$	$R_A(\Omega)$	$R_B(\Omega)$	$R_C(\Omega)$	σ/R_n (µΩ/Ω)
1 000 000	1	1	75	77	12 906	12 906	172	13 251	13 251	993 793	6 207
1 000 000	3	3	8	14	38 719	38 719	1 613	41 946	41 946	1 006 699	6 699
1 000 000	25	2	1	28	322 660	25 813	12 906	39 752	496 897	993 793	6 207
10 000 000	6	6	21	33	77 438	77 438	615	78 668	78 668	9 912 118	8 788
10 000 000	1	86	8	95	12 906	1 109 951	1 613	1 250 308	14 538	10 002 463	246
100 000 000	31	31	8	70	400 099	400 099	1 613	403 325	403 325	100 024 629	246
100 000 000	6	158	8	172	77 438	2 039 212	1 613	2 083 309	79 113	99 998 816	12

Table 1 shows several Y-configurations for nominal resistance values of 1 MΩ, 10 MΩ, and 100 MΩ. For $R_n = 1$ MΩ, three Y-networks, (n_1, n_2, n_3) = (1, 1, 75), (3, 3, 8), (25, 2, 1), yield similar offset $\sigma = |R_C - R_n|$ at the level of ≈ 6000 µΩ/Ω. Among these three configurations, only (25, 2, 1) allows dual-source bridge triple-series connections to all quantum Hall resistance terminals, due to the single Hall bar in arm 3 [5]. However, (3, 3, 8) is the best option for minimizing the fabrication error because it has the minimum number of Hall bar. For this configuration, arm 3 has much smaller resistance due to the parallel graphene Hall bars. We will demonstrate that this design can utilize the voltage sense capability of the two digital calibrators and a voltage detector in the dual-source bridge to eliminate the need for triple-series connections, because voltage detection has been found to be less sensitive to noise than current detection. For larger R_n, asymmetric Y-networks can also demonstrate improved precision, i.e., smaller σ.

Figure 2 shows a prototype of Y-Δ-type QHARS with dual-output of 1 MΩ and 100 MΩ, which can be realized by a total of 76 graphene Hall bar.

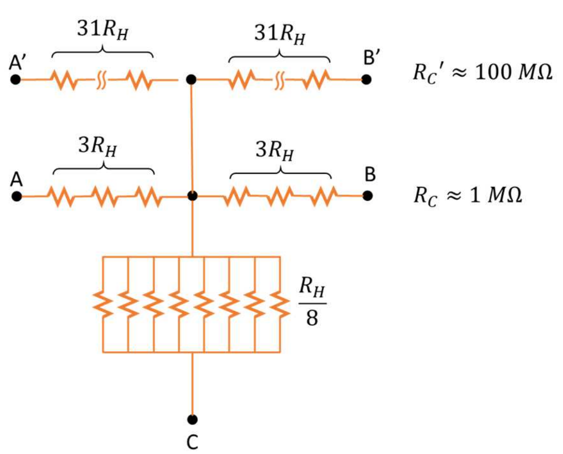

Figure 2. Schematic diagram of a Y-Δ-type QHARS with two outputs. The output resistance between terminals A and B is 1 MΩ. The output resistance between terminals A' and B' is 100 MΩ.

III. Conclusion

We applied analysis of the wye-delta transformation and compared several Y-network designs for QHARS with decade resistance values of 1 MΩ, 10 MΩ, and 100 MΩ. While symmetric Y-network shows advantages of utilizing less graphene Hall bars for a device and therefore reducing the fabrication complexity, an asymmetric design gives benefit of obtaining an output value R_C closer to a nominal resistance R_n, especially for higher R_n values. Following this work, we will fabricate and measure graphene QHARS with different Y-designs including the dual-output device shown in Figure 2.

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