Single-cryostat integration of the quantum anomalous Hall and Josephson effects

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***Abstract*—By directly integrating a quantum anomalous Hall resistor (QAHR) and a programmable Josephson voltage standard (PJVS) into a single cryostat, we have implemented a unified quantum electrical instrument that provides a realization of the volt, ohm, and ampere in accordance with the revised International System of Units (SI). The quantum voltage output from this prototype ranged from (0.24 to 6.5) mV with combined relative uncertainties ($k = 1$) down to 3 μV/V. The colocated QAHR provided a realization of the ohm at zero magnetic field with uncertainties near 1 $\mu\Omega/\Omega$ at $R_{yx} \approx$ 25.9 kΩ. For the ampere, a longitudinal current applied to the QAHR is converted to a quantized Hall voltage, which was directly compared to the Josephson voltage, providing measurements of the ampere that are directly traceable to the revised SI. We determined currents in the range (9.33 to 252) nA with uncertainties of (41 to 4.3) μA/A, respectively. Limitations and improvements are discussed to aid the reproduction of similar instruments at other national metrology institutes.**

***Index Terms*—Measurement, measurement uncertainty, International System of Units, measurement techniques, voltage, current, resistance**

I. INTRODUCTION

Under the modern SI, the ohm and the volt are now practically realized from the von Klitzing constant $R_K = h/e^2$ and the Josephson constant $K_J = 2e/h$. Likewise, a practical realization of the ampere is now possible through application of Ohm's law to the quantum Hall and Josephson effects. The harmonization of the primary electrical units suggests that an integrated, quantum-based instrument—capable of realizing all three units—should be feasible. Indeed, earlier works on laboratory-level integration, in which the two standards are interconnected outside of their respective cryostats, have demonstrated remarkable performance for realization of the ampere [1], [2].

However, a significant challenge must be overcome to simultaneously realize those two effects in a shared cryostat: A quantum Hall resistance standard (QHRS) requires a substantial magnetic field (>5 T) for proper quantization, which is incompatible with quantum-locked operation of, for example, a programmable Josephson voltage standard (PJVS). Yet, recent progress in zero-field precision measurement of the quantum anomalous Hall (QAH) effect [3], [4] suggests a potential workaround: the colocation of a PJVS with a QAH-based resistance (QAHR) standard. Here we report initial results from a prototype 'quantum electrical instrument' based on direct integration of a QAHR and a PJVS within a single cryostat.

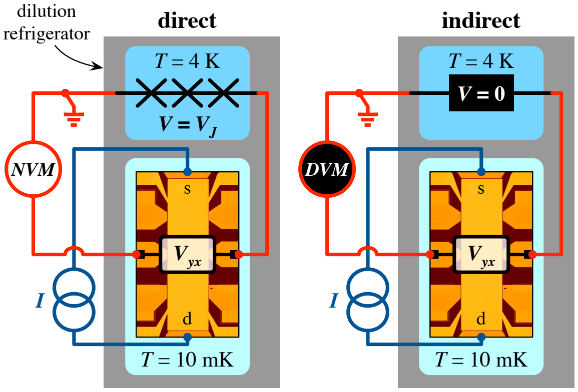

Fig. 1. Schematic of the quantum electrical instrument when measuring current. The current under test I biases the QAHR, resulting in the transverse Hall voltage $V_{yx} = IR_K$. The QAHR is mounted to a cold finger at a nominal temperature of 10 mK. In the 'direct' mode of operation (at left), the voltage V_{yx} is compared directly to the PJVS voltage V_J (at 4 K stage) to yield a null at the room temperature nanovoltmeter, NVM. For 'indirect' mode (at right), the PJVS is disabled, but still part of the circuit, and V_{yx} is instead measured with a digital voltmeter, DVM (again at room temperature).

II. EXPERIMENTAL SETUP

A dry ^{3}He/^{4}He dilution refrigerator housed both the PJVS (at 4 K) and QAHR (mounted to a cold finger at 10 mK). Direct current (DC) wiring to the QAHR and to the PJVS consisted of copper twisted pair embedded within a woven loom, thermalized to each stage of the cryostat. A schematic depicting the conceptual design of the quantum electrical instrument is shown in Fig. 1 (for the case of current measurement). The PJVS output voltage V_J is placed in a series circuit with the Hall voltage V_{yx} of the QAHR and a nanovoltmeter (external to the cryostat). An additional pair of leads (not shown) runs directly from the PJVS to an external breakout box. These leads were used for performance verification of the PJVS and would, in a more mature version of the instrument, be used for voltage calibration of external equipment. The nanovoltmeter port was also used for evaluation of QAHR quantization with a separate cryogenic current comparator (CCC). As with the case for voltage, in a future, more robust design, this port could be used for calibration of external resistors against the QAHR through a resistance bridge.

For realization of the ampere, our prototype operates by having a current-under-test I bias the QAHR to generate a transverse voltage V_{yx}. In our experiment, the current was supplied by the CCC's digital current source. Under idealized conditions, the transverse voltage and the current are related by $V_{yx} = IR_K$. Simultaneously, N junctions of the PJVS are biased at a microwave frequency f to be on their first Shapiro steps, yielding a voltage across the array $V_J = NfK_J^{-1}$. V_J is tied to the QAHR such that the potential difference $V_{\text{null}} = V_J - V_{yx}$ can be measured by a nanovoltmeter at room temperature (see Fig. 1 left panel). The current can then be computed as,

$$I_{\text{direct}} = \frac{NfK_J^{-1} - V_{\text{null}}}{R_{yx}}, \tag{1}$$

where $R_{yx} \simeq R_K$ is the QAHR Hall resistance, as determined by comparison to established standards. Here, the subscript 'direct' is used to indicate that this measurement is a 'direct realization' of the ampere — i.e., the measurement of I is done through the integration of two quantum standards in a manner consistent with the revised SI.

An independent validation of the prototype's ampere was unavailable at the time of measurement. However, as a consistency check, we also operated the instrument in an 'indirect' mode in which the PJVS is disabled and the Hall voltage is measured by a high-resolution digital voltmeter (DVM), which was calibrated separately (Fig. 1 right panel).

III. RESULTS AND DISCUSSION

To validate the PJVS, we compared its output voltage against the same DVM referenced above. For voltages near 1 mV—chosen because of the nanoamp-scale currents we expected to measure in current mode—the discrepancy between the calculated PJVS voltage and the DVM reading was less than 2 μV/V and within the combined uncertainty of the DVM's calibration. This is an encouraging result, considering the rather small voltages and the fact that this check was done after cycling the magnetic field. The mean of QAHR quantization measurements over a period of six months demonstrated relative deviations $(R_{yx} - R_K)/R_K < 1\ \mu\Omega/\Omega$ for bias currents from 9.3 nA to 252 nA.

Measurements of current in direct and indirect mode were performed to check for systematic errors in our prototype. Discrepancies in the computed value for current could indicate a lack of quantization in one or both methods or long-term instability of the source. A comparison of the two techniques (see Fig. 2) indicates disagreement ranging from -1 μA/A to -35 μA/A for currents between (9.3 to 252) nA, which is comparable to calibration and measurement capabilities (CMCs) from a number of national metrology institutes. We note that even after separately measuring (via CCC) and correcting for shifts in R_{yx} due to the PJVS's RF bias, discrepancies of approximately -25 μA/A remain for some currents. Follow-on studies suggested that the lack of agreement may be due to insufficient stability of the current source. Future efforts will include improved mitigation of the RF bias and a more stable current source.

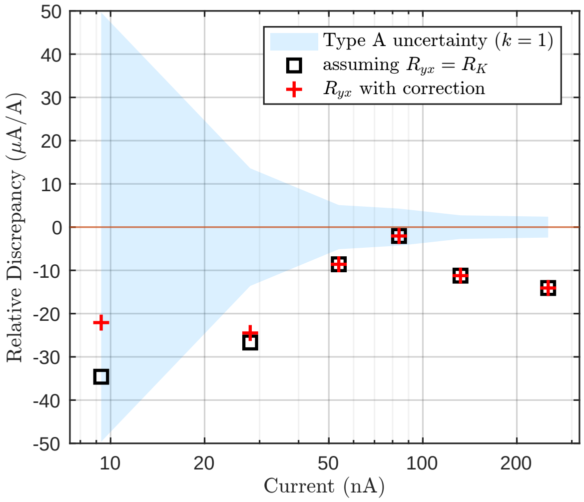

Fig. 2. Relative discrepancy $(I_{\text{direct}} - I_{\text{indirect}})/I_{\text{indirect}}$ from multiple measurements. Black squares depict the values assuming ideal quantization of the QAHR (i.e., $R_{yx} = R_K$). Red crosses show results if a correction is applied to account for RF-induced shifts in QAHR quantization for direct measurements. The shaded area shows the combined Type A uncertainty ($k = 1$) of the direct and indirect measurements.

IV. CONCLUSION

We have demonstrated quantum realizations of SI volt, ohm, and ampere from a single cryostat. The unified instrument exhibits $k = 1$ uncertainties comparable to NIST's CMCs for low voltages (2 μV/V) and to recent precision measurements of the ohm on other QAHR devices (1 $\mu\Omega/\Omega$). For realizations of the ampere between (9.3–252) nA, combined relative uncertainty was (41–4.3) μA/A, respectively. Further details of this work will be presented at the conference.

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