Progress toward the Kibble Dynamic Force Reference

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***Abstract*—We present progress in the development of a dynamic force reference instrument (KDFR) based on the Kibble principle. We describe the operating principle and design of the KDFR and discuss how the design for alternating current (AC) force measurements contrasts with Kibble balances for mass measurement and necessitates additional corrections to the force calculation. We further analyze the coil impedance, a critical factor for correcting the measured voltage in a single-mode AC Kibble measurement and illustrate a scheme for calibrating its temperature dependence.**

***Index Terms*—Electromagnetic forces, force measurement, impedance measurement, measurement uncertainty, precision measurements.**

Kibble balances have seen success for primary mass metrology by equating mechanical and electrical forces in a coil and magnet system [1]. The measurement of dynamic forces could also benefit from the same principle. Many force sensors have significant frequency dependence and yet are often calibrated statically (only at DC). Accepting this limitation, statically calibrated transducers are frequently used for dynamic measurements in various fields including materials testing, automotive crash testing, and wind tunnel aerodynamics [2]. Moreover, the dynamic response of a force sensor depends on its mechanical environment, reducing the utility of *ex situ* sensor calibrations in a metrology lab [3]. One possible improvement would be an *in situ* calibration with a dynamic force reference.

NIST is developing a low-profile force generation device based on the Kibble principle to serve as a primary, portable, *in situ* dynamic force calibrator [4] with a target amplitude of 10 N and uncertainty of 1 % across the range 100 Hz to 10 kHz. This would compare favorably to current technology, an impact hammer transfer standard with uncertainty ≈2 % up to 5 kHz [3]. This Kibble dynamic force reference (KDFR) is a generic device concept with the potential for variations including as a primary source of dynamic force for modal testing and as an embedded, primary-calibrated device for critical sensing applications. In this presentation, we will describe our progress toward the KDFR. We will discuss the operating principle and current design, including initial results and a comparison with the high-precision Kibble balances for mass metrology. Finally, we will present an uncertainty analysis with implications for future designs.

The principle underlying Kibble balance operation with a coil and magnet is the equivalence between the geometric factor, BL, relating current (I) to force (F) in the Lorentz force law and voltage (U) to velocity (v) in Faraday's law [1]. The geometric factor then cancels out of the basic force calculation, given by

$$F = UI/v, \tag{1}$$

where each variable is a complex function of frequency.

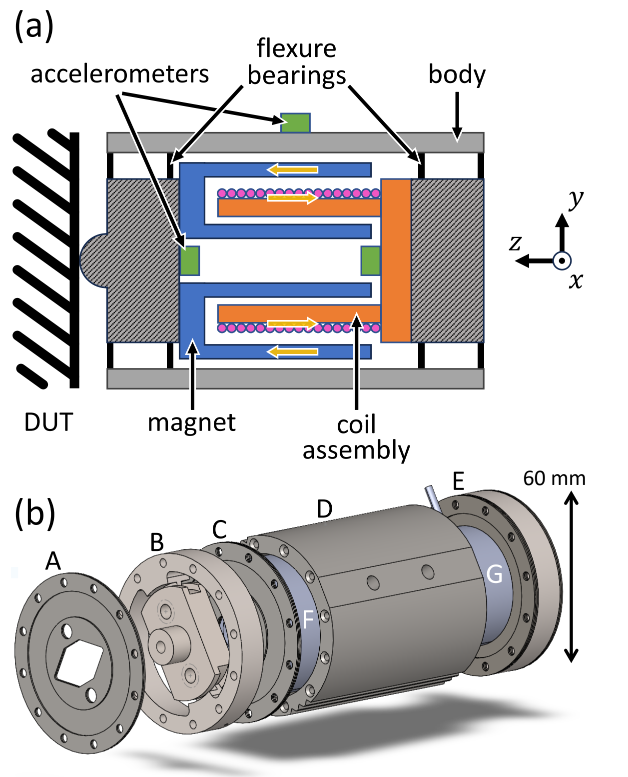

Fig. 1. (a) A cross-sectional, not-to-scale, diagram of the KDFR in contact with a sensor device under test (DUT). The magnet and coil assembly are coupled to the body via separate flexure bearings which constrain each to linear motion along the z axis. An equal and opposite Lorentz force applies to the coil and magnet (yellow arrows) and is transmitted through a steel block to the DUT. Accelerometers measure the relative motion of the magnet, coil, and body. (b) Computer-aided design image of the KDFR, exploded along the z axis, screws and accelerometers omitted. See the text for descriptions of the labeled components.

Figure 1(a) is a not-to-scale diagram cross-section of the KDFR, which in essence consists of the metal body, the coil assembly (aluminum bobbin), and the magnet assembly. Figure 1(b) is a rendered computer-aided design of the KDFR components, including the steel diaphragm flexure A, force transfer block and spacer ring B, mirrored flexure C, steel body D, coil assembly flexure E, magnet F, and coil G. An AC current is supplied and measured simultaneously with the voltage. The Lorentz force, F, drives the magnet and coil in opposite directions along z. The coil is free to move, and F transmits through the stiff force transfer block to the device under test (DUT). Meanwhile, accelerometers record the relative motions of the three components. The design reflects two basic principles to reduce uncertainty in F: (A) maximization of velocity per unit force and (B) minimization of parasitic motions and forces, including deformation and internal resonances.

The primary components are coupled with flexure bearings which serve design principles (A) and (B) in three ways: (1) to constrain the magnet and coil to linear motion along the axis of force (z), (2) to free the lightweight coil assembly for increased motion, and (3) to substantially decouple the moving assemblies from the mechanical modes of the body and mount. The flexure design indicated in Fig. 1 has simulated stiffness on the order of 10 kN/m for motion along z and stiffness > 10 MN/m for motion in other degrees of freedom.

We opt to use a one-stage measurement, reminiscent of some precision mass Kibble balances [5], in which we drive the force with an AC current through the coil and simultaneously record the current, voltage across the coil, and relative velocity between the magnet and coil assembly. In contrast with the dual-stage operation of many mass Kibble balances, this simplifies and speeds the measurement and removes the need for an external driver of motion for the velocity mode. The disadvantage is that for a single stage, the measured voltage (U_{meas}) is the sum of contributions from both Faraday's and Ohm's laws [5], requiring a correction to isolate the Faraday component: $U = U_{\text{meas}} - IZ$, where Z is the coil impedance.

The equation for the KDFR force applied to the DUT, including corrections, is

$$F = ((U_{\text{meas}} - IZ)I/\Delta\dot{x}_{\text{mc}}) - M_{\text{m}}\ddot{x}_{\text{m}} - \Xi(\Delta x_{\text{mb}}, \Delta\dot{x}_{\text{mb}}), \quad (2)$$

where $\Delta\dot{x}_{\text{mc}}$ is the relative velocity between magnet and coil, M_{m} is the magnet assembly mass, $\ddot{x}_{\text{m}}$ is the magnet assembly acceleration, and Ξ is the force applied to the magnet assembly flexure, which is a function of the relative position from neutral (Δx_{mb}) and velocity ($\Delta\dot{x}_{\text{mb}}$) between the magnet and body.

The accurate application of Eq. 2 requires precharacterization of Z and Ξ as a function of frequency and, for Z especially, temperature. For most frequencies below 10 kHz, Ξ and $M_{\text{m}}\ddot{x}_{\text{m}}$ will be relatively small corrections compared to $F = 10$ N; however, typically, $IZ/U > 1$. As an example, we expect $IZ/U \approx 8$ at 500 Hz and 10 N operation, and therefore, we aim to characterize Z with relative uncertainty lower than 10^{-3}. The challenge is that we anticipate as much as 20 W of power to be dissipated within the coil at any given time, but often less due to the strong frequency dependence of Z, leading to temperature-driven Z fluctuations on the order of 1 %. To overcome this, as exemplified in Fig. 2, we have measured Z as a function of frequency (f) and DC coil resistance (R), the most relevant proxy for temperature. This was achieved through repeated Ohmic heating cycles of the coil, clamped to suppress motion, and sampling U and I time-domain waveforms during the intermediary cooling periods for an array of frequencies using the same circuitry and instrumentation as normal KDFR operation. We have performed fits of Z vs. R (e.g. see Fig. 2(b)) from which we extract the linear slope m for each

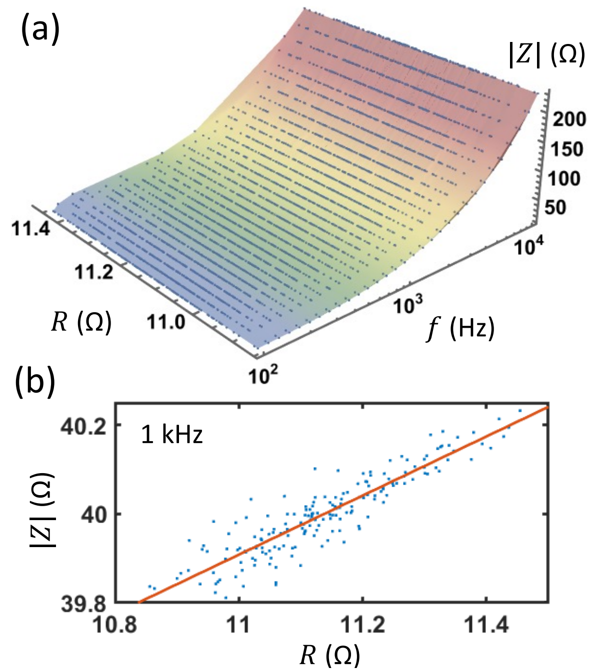

Fig. 2. (a) Absolute value of measured coil impedance plotted versus frequency and DC coil resistance. During KDFR operation, this data is analyzed and interpolated to provide a calibration for Z despite temperature fluctuations. (b) Example linear least-squares fit of data from (a) at 1 kHz.

frequency with ≈ 5 % fitting uncertainty. During operation, the KDFR will periodically sample R and apply m calibration data to extract Z with $\approx 0.5 \times 10^{-3}$ accuracy.

If this Z accuracy proves insufficient to correct U_{meas} in Eq. 2 to achieve our 1 % uncertainty goal, a possible improvement would be to use a differential voltage measurement with a nominally identical, external reference coil (impedance difference ΔZ), rigidly clamped to prevent motion. An impedance bridge technique could determine ΔZ with comparable relative accuracy to Z, and such a scheme would relax the requirement for Z uncertainty by a factor of $Z/\Delta Z \approx 100$.

Identification of limiting factors in the uncertainty such as the accelerometer calibration and the IZ correction, and exploration of trade-offs in addressing these limitations, will be invaluable next steps to realize an accurate dynamic force reference.

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