Redetermination of the Gravitational Constant using the BIPM Torsion Balance

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***Abstract*—For the past several years, we have been using the torsion balance developed at the Bureau International des Poids et Mesures (BIPM) to measure the gravitational constant G. The most notable feature of the apparatus is that it allows the measurement of G with two different methods: the Cavendish method and the Servo method.**

The systematic effects are different in several important variables, leading to an improved combined result. In the past year, we have identified a gas pressure effect that hampered previous measurements at the National Institute of Standards and Technology

***Index Terms*—measurement, measurement techniques, gravitational constant**

I. INTRODUCTION

The measurement of Newton's gravitational constant, G, is incredibly difficult because of the weakness of the gravitational force between laboratory objects. For the experiment discussed here, the gravitational torque is $31\,\mathrm{nN\,m}$, which we attempt to measure with a relative uncertainty of a few parts in 10^5. The best tool to measure these small torques is the torsion balance.

Although the torsion balance and other instruments have been used to measure G in the past 225 years, the consensus value reported by the Task Group on Fundamental Physical Constants (TGFC) of the Committee on Data of the International Science Council has a large relative uncertainty of 2.2×10^{-5}. Figure 1 shows the data reported by more than a dozen high-precision determinations of the gravitation constant carried out in the last fifty years. Noteworthy is the large scatter. The smallest and the largest measured values differ relatively by 1.4×10^{-4} , more than a hundred times the uncertainty of the best experiment.

In light of this situation, researchers at the National Institute of Standards and Technology (NIST) have decided to repeat one of the experiments. Luckily, the original researchers at the International Bureau of Weights and Measures (BIPM) agreed to make their torsion balance available to NIST. The result produced by this torsion balance [1] and a previous similar version [2] are shown as red points in Fig. 1.

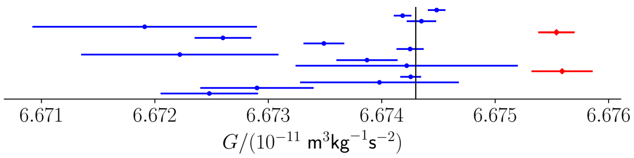

Fig. 1. The world data set of precision measurements of the gravitational constant over the past fifty years. The two measurements in red were carried out with two different versions of the torsion balance described here. The experiment is called the BIPM torsion balance.. The uncertainty bars shown in the graph are the standard uncertainty without expansion factor ($k = 1$).

II. BRIEF DESCRIPTION OF THE APPARATUS AND THE MEASUREMENT

The geometry of the BIPM mass arrangement is shown in Fig. 1. The pendulum bob consists of four copper test masses, $m = 1.2\,\mathrm{kg}$ each placed on a circle with a radius $r = 120\,\mathrm{mm}$, and a light aluminum disk to support the former. The disk supports a cylindrical tower in its middle. Mounted on top of the tower are four mirrors. Each one of them can be used to read out the torsion angle with a commercial autocollimator, which is not shown in the drawing. The torsion disk is suspended from a Copper-Beryllium ribbon labeled torsion strip in the drawing. The torsion disk, including the tower and other hardware, but not the test masses, has a moment of inertia of $I_{disk} = 7.45\,\mathrm{g\,m^2}$ It provides a restoring torque of $\kappa = 204\,\mathrm{\mu N\,m\,rad^{-1}}$. The parts described above are inside a vacuum enclosure that is also omitted from the drawing. Outside the vacuum enclosures are the source masses. Two sets of source masses are employed in the experiment. One set is made from copper weighing $M = 11.2\,\mathrm{kg}$. The masses in a second set are made from single crystal sapphire and weigh about half as much. The source masses are distributed around a circle with radius $R = 240\,\mathrm{mm}$. They are supported by a carousel that can rotate the mass assembly about the center of the torsion disk given by the strip.

One of the strengths of the BIPM torsion balance is that it can be used to measure G with two different methods: the Cavendish method and the electrostatic servo method. In each method, the source masses are parked alternatively in one of two positions, labeled clockwise or counter-clockwise. The source mass assembly is rotated by $\pm 18.2°$ against the test mass assembly. For the geometry in the experiment, the extreme gravitational torques occur at these angles.

A. The Cavendish method

In the Cavendish mode, the gravitational torque alters the equilibrium position about which the pendulum oscillates. The

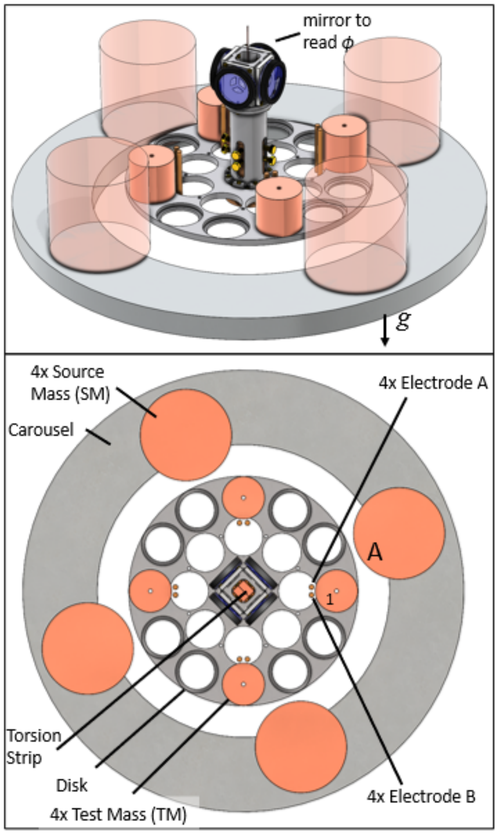

Fig. 2. Three-dimensional rendering of the torsional balance in the top frame. The lower frame shows a top view of the balance. The vacuum envelope containing the torsion balance itself (disk, test masses, and supporting elements) is omitted for clarity.

equilibrium position is given by

$$\phi_{\text{ccw}} = \phi_o + \frac{\Gamma G}{\kappa} \text{ and } \phi_{\text{cw}} = \phi_o - \frac{\Gamma G}{\kappa}, \tag{1}$$

for the two source mass positions, respectively. The mass integration constant Γ is given approximately by

$$\Gamma \approx 35 M m \frac{r^4}{R^5}. \tag{2}$$

It is calculated with high precision numerically. The spring constant κ is inferred from the measured period of oscillation $T_o \approx 120\,\text{s}$ using the moment of inertia of the disk and the test mass data as

$$\kappa = \frac{4\pi^2}{T_o^2}\left(4mr^2 + I_{\text{disk}}\right). \tag{3}$$

Combining Eq. (1)-(3) yields for the gravitational constant

$$G_{\text{Cav.}} \approx \frac{16\pi^2}{T_o^2}\frac{R^5}{70Mr^2}\left(1 + \frac{I_{\text{disk}}}{4mr^2}\right)\left(\phi_{\text{ccw}} - \phi_{\text{cw}}\right). \tag{4}$$

B. *The Servo method*

In the Servo method, an electrostatic torque is applied to counteract the gravitational torque such that the pendulum remains at a desired angle. This is accomplished with the help of two sets of electrodes, labeled A and B in Fig. 2. A full description of the servo system generating the electrostatic torque can be found in [3].

For simplicity, if we assume that the unused electrode is grounded, then the counteracting electrostatic torques are given by

$$N_{\text{ccw}} = \frac{1}{2}\frac{\mathrm{d}C_{13}}{\mathrm{d}\phi}V_1^2 \text{ and } N_{\text{cw}} = \frac{1}{2}\frac{\mathrm{d}C_{23}}{\mathrm{d}\phi}V_2^2, \tag{5}$$

where $\mathrm{d}C_{13}/\mathrm{d}\phi$ and $\mathrm{d}C_{23}/\mathrm{d}\phi$ are the capacitance gradients between the pendulum and electrodes A and B, respectively. The potential differences between the pendulum and the electrodes are given by V_1 and V_2. The capacitance gradients are measured in a separate measurement phase in which the source masses are moved to a position where they don't generate a gravitational torque on the pendulum. There, the pendulum is allowed to swing freely with an amplitude of $145\,\mu\text{rad}$ while the capacitance and the pendulum angle are simultaneously recorded. Combining Eq. (2) and Eq. (5) yields,

$$G_{\text{servo}} \approx \frac{R^5}{140Mmr^4}\left(\frac{\mathrm{d}C_{13}}{\mathrm{d}\phi}V_1^2 - \frac{\mathrm{d}C_{23}}{\mathrm{d}\phi}V_2^2\right) \tag{6}$$

C. *Combining both results*

Combining the results obtained with the Cavendish and the Servo method significantly improves the uncertainty of the experiment due to a negative correlation in the measured torsion angle, ϕ. Eq. (4) is proportional and Eq. (6) is inversely proportional to ϕ. Hence, the uncertainty of the combined result is smaller than the two individual results. Table I shows the projected uncertainties of each method.

TABLE I

PRELIMINARY TYPE B UNCERTAINTIES FOR BOTH METHODS. THE ABBREVIATION N/A IS USED FOR PARAMETERS THAT ARE NOT APPLICABLE FOR A METHOD.

Description	Symbol x	Cavendish $\frac{\sigma_x}{x} \times 10^6$	Servo $\frac{\sigma_x}{x} \times 10^6$
torsion angle	ϕ	47.0	47.0
large pitch circle radius	R	12.5	12.5
moment of inertia of disk	I_{disk}	13	n/a
small pitch circle radius	r	5.5	10
period of pendulum	T_o	1.0	n/a
mass of test mass	m	< 0.1	0.1
mass of source mass	M	< 0.1	< 0.1
voltage	V	n/a	10.0
capacitance	C	n/a	6
total		50.7	51.0

III. CURRENT STATUS

In 2023, we found a systematic effect caused by temperature gradients in the residual gas in the vacuum chamber. We have since improved the thermal environment and are taking data. It is our hope to present a new value of G at the conference this summer.

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