Uncertainty Analysis of Vector Network Analyzer Measurements of four terminal-pair air capacitors

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***Abstract*—Traditionally, vector network analyzer (VNA) measurements have been used to determine the frequency dependence of four-terminal pair air capacitors. Recently, researchers at the National Institute of Standards and Technology (NIST) have used a four-channel VNA to determine the frequency dependence of air capacitors using the locations of poles and zeros of the capacitance as a function of frequency. This contribution describes the uncertainty analysis for the measurements of the 100 pF and 1 nF. capacitors.**

***Index Terms*—Calibration, air capacitors, four terminal-pair, frequency dependence, impedance measurement.**

I. Introduction

We recently used a four-channel vector network analyzer (VNA) to measure the frequency dependency of four terminal-pair air capacitors [1]. The reported results agreed well with the standard calibration procedure used at the National Institute of Standards and Technology (NIST) for the last quarter of a century [2], but the new measurement procedure is advantageous, as it can be performed without changing leads. This is made possible by the availability of four-channel vector network analyzers that can measure the 16 elements of the scattering matrix $\mathbf{S}$ nearly simultaneously. Previously, up to eight different connections would have to be made, where each new connection can invite measurement uncertainty and operator error into the procedure. In this contribution, we evaluate the measurement uncertainties of the new method.

II. Brief Recap of the New Method

By connecting a four terminal-pair capacitor to a four-channel VNA, the S-matrix $\mathbf{S}(f)$ can be obtained for frequencies in the range of $100\,\mathrm{kHz} \leq f \leq 500\,\mathrm{MHz}$. For each frequency value, 16 matrix elements s_{ij} are obtained with $i, j \in \{1, 2, 3, 4\}$. Note, due to the symmetry, $s_{ij} = s_{ji}$, only ten unique components exist in the matrix. Figure 1 shows the absolute values of s_{ij} up to $500\,\mathrm{MHz}$ for a $1\,\mathrm{nF}$ air capacitor.

According to microwave engineering text-books [3] the impedance matrix can be obtained with the help of the identity matrix $\mathbf{I}$ using,

$$\mathbf{Z} = Z_t \cdot (\mathbf{I} + \mathbf{S}) \times (\mathbf{I} - \mathbf{S})^{-1}, \quad (1)$$

where Z_t is the load impedance, typically $50\,\Omega$. From the impedance matrix, with its sixteen components z_{ij}, the four terminal pair impedance can be obtained using

$$Z_{4TP} = \frac{z_{21} z_{34} - z_{24} z_{31}}{z_{31}}. \quad (2)$$

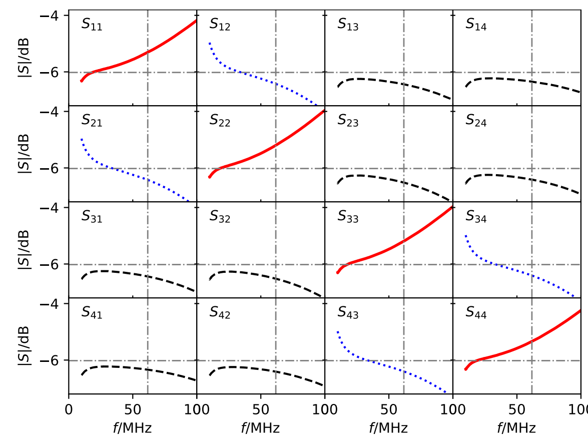

Fig. 1. The sixteen components of the scattering matrix measured on a $1\,\mathrm{nF}$ air capacitor as function of frequency. The absolute value of the complex matrix elements s_{ij} are shown. The dash-dotted vertical line indicates the pole in C at $61.8\,\mathrm{MHz}$. The colors and line types indicate the three types of curves. Shown in red (solid) are the reflection coefficients, i.e., the diagonals s_{ii}. In blue (dotted) are the transmission coefficients measured on the same side of the capacitor, i.e., excitation and measurement are on the same side. The black (dashed) curves have the excitation on one side of the capacitor and the measurement on the other.

From there, the capacitance is given by

$$C_{4TP} = \frac{1}{\omega \mathrm{Im}(Z_{4TP})}. \quad (3)$$

Callegaro and Durbiano, [5] combine equations (1) and (2) to obtain an analytic expression for Z_{4TP} from the measured s_{ij}. They find,

$$Z_{4TP} = 2 Z_t \frac{s_{21} s_{34} - s_{24} s_{31}}{d}, \quad (4)$$

where d abbreviates the denominator, which is given explicitly in [5] but is unnecessary to repeat here. For this investigation, it is sufficient to know that d evaluates to 2 for the $1\,\mathrm{nF}$ capacitor for frequencies below $100\,\mathrm{MHz}$.

We seek to obtain the frequency dependence of a capacitor relative to its value C_0 at a low frequency f_0. According to [1], this frequency dependence can be written to second order in frequency f as

$$\varepsilon := \frac{C_{4TP}}{C_0} - 1 = a_{zp} f^2, \quad (5)$$

where the coefficient a_{zp} is given by the locations of poles (p) and zeros (z) of C, i.e.,

$$a_{zp} = \sum_i \frac{1}{f_{p,i}^2} - \sum_j \frac{1}{f_{z,j}^2}. \quad (6)$$

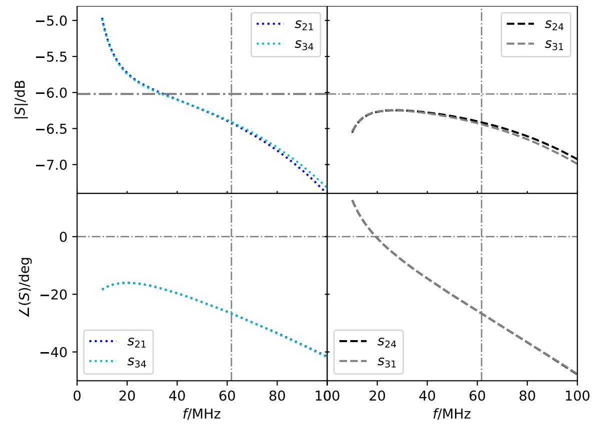

Fig. 2. Magnitude (top row) and phase (bottom row) of the four relevant matrix elements, s_{21}, s_{34}, s_{24}, and s_{31} of the scattering matrix.

For the $1\,\mathrm{nF}$ capacitor discussed here, there are no zeros and only one pole $f_{p,1}$ that occurs at $61.8\,\mathrm{MHz}$. The $100\,\mathrm{pF}$ capacitor has a dominant pole at $179.8\,\mathrm{MHz}$, a zero at $265.4\,\mathrm{MHz}$ and another pole at $253.17\,\mathrm{MHz}$. Due to the closeness of the latter pole and zero, they cancel each other according to Eq. (6) and are of no further relevance.

III. Uncertainty in the First Pole Location

Poles in C_{4TP} occur at the zeros of Z_{4TP} given in Eq. (4), i.e.,

$$s_{21}s_{34} - s_{24}s_{31} = 0 \tag{7}$$

The magnitude and phase for these four matrix elements are shown in Fig. 2.

The manual of the vector network analyzer gives the uncertainty as a function of the measured transmission coefficient. For transmission coefficients below $-20\,\mathrm{dB}$, the magnitude uncertainty is $0.04\,\mathrm{dB}$ and the phase uncertainty is $0.3°$. Both are expanded uncertainties for $k = 2$.

We use a Monte Carlo simulation with $N = 10\,000$ to simulate data used in Eq. (7). We draw the log magnitude of each s_{ij} from a normal distribution with a mean of the measured value and a standard deviation of $0.02\,\mathrm{dB}$. Similarly, the phase is drawn from a normal distribution located at the nominally measured phase and a standard deviation of $0.15°$. For every one of the $10\,000$ simulations, four magnitudes and four phases are generated independently, i.e., not correlated, from each other as described above. Using the generated data, the frequency for which $s_{21}s_{34} - s_{24}s_{31} = 0$ is calculated.

The result of the simulation is,

$$f_{p,1} = 61.8\,\mathrm{MHz} \pm 1.5\,\mathrm{MHz}, \tag{8}$$

where the uncertainty is given here and through the remainder of the text for $k = 2$. The distribution of the simulation results is given in Fig. 3. With the above result, it is

$$a_{zp} = 2.62 \times 10^{-4}\,\mathrm{MHz}^{-2} \pm 0.13 \times 10^{-4}\,\mathrm{MHz}^{-2}. \tag{9}$$

A similar calculation can be carried out for the $100\,\mathrm{pF}$ capacitor. The result is

$$f_{p,1} = 179.4\,\mathrm{MHz} \pm 0.8\,\mathrm{MHz}, \tag{10}$$

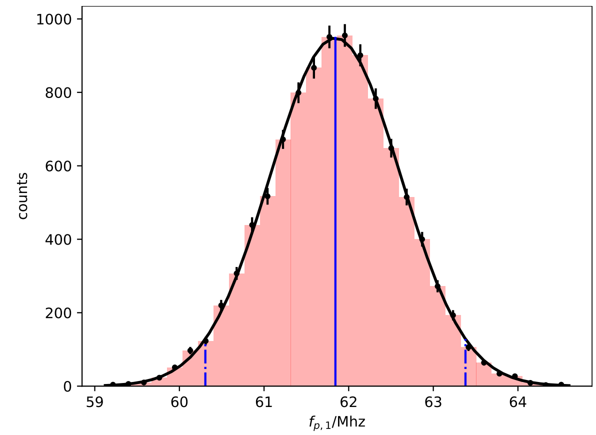

Fig. 3. Distribution of the calculated pole frequency $f_{p,1}$ for $10\,000$ simulations. The vertical uncertainties of each point centered at the bins are given by $\sqrt{n}$ where n is the number of counts in the bin. The solid line is a normal distribution centered around the mean given by the solid blue vertical line. The dashed-dotted blue lines are two standard deviations away from the mean.

which leads to

$$a_{zp} = 3.11 \times 10^{-5}\,\mathrm{MHz}^{-2} \pm 0.03 \times 10^{-5}\,\mathrm{MHz}^{-2}. \tag{11}$$

The $1\,\mathrm{pF}$ and $10\,\mathrm{pF}$ capacitors require slightly different treatment because the dominant feature is a pole and not a zero in Eq. 4. The result of this calculation will be presented at the conference. Future work will also include the additional pole and zero for the $100\,\mathrm{pF}$ capacitor.

IV. Conclusion

We have determined the uncertainty of the frequency dependence of the $100\,\mathrm{pF}$ and $1\,\mathrm{nF}$ air capacitors. By using the uncertainties provided by the manufacturer of the VNA, we could perform a Monte Carlo simulation to find the uncertainty in the pole location of the capacitance function. For both capacitors, the frequency dependency is dominated by the lowest pole in C, which is at $61.8\,\mathrm{MHz}$ for the $1\,\mathrm{nF}$ and $179.4\,\mathrm{MHz}$ for the $100\,\mathrm{pF}$. The relative uncertainties of the coefficients giving the quadratic frequency dependence, a_{zp}, are 5×10^{-2} and 9×10^{-3} for the $1\,\mathrm{nF}$ and $100\,\mathrm{pF}$ capacitor.

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