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Letter to the Editor

# A proposal for three categories of units within the SI

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**Abstract**
In this letter we examine the organization of the traditional base units and defining constants established in the 2019 redefinition of the international system of units (SI). For the next redefinition of the SI, which will accompany the anticipated redefinition of the second, we propose an organizational change to improve clarity while maintaining practicality. We propose three distinct categories of units: The first category comprises the four base-measurement units: The second, meter, kilogram, and ampere. The second contains physiologically-relevant derived units. The third category contains the remaining units derived from the base units.

Keywords: international system of units, SI, ampere, meter, second, kilogram, candela

## 1. Introduction to the SI

The international system of units (SI) was summarized elegantly with what might be called the 'SI wheel', shown in figure 1, following the 2019 (2018) redefinition. This highlights the seven defining constants and their relationship to ***the*** seven base units: the kilogram, meter, second, amp, kelvin, mole, and candela. That is, mass, length, time, electric current, temperature, amount of substance, and light visible to humans. *'any of the seven base units and 22 units with special names can be constructed directly from the seven defining constants […] the units of the seven defining constants include both base and derived units'* [1].

While elegant in appearance, figure 1 belies a key fact—only the four defining constants $h$, $c$, $\Delta\nu$, $e$ and the associated units, kg, m, s, A, form a coherent four-dimensional system [1]. In contrast, $N_A$ and $K_{cd}$ are isolated scale factors and $k$, though slightly different is still derived from the four-dimensional system. According to Tiesinga *et al* in the committee on data of the international science council recommended values, a constant is only fundamental as a matter of convention [2]. As we discuss later, however, there is an essential mathematical relationship between constants and units, beyond terminology that distinguish the constants, $\Delta\nu_{Cs}$, $c$, $h$, and $e$ associated with base units for time, length, mass, and electric charge and form the SI *system*, not merely independent units and constants.

The base units for time, length, mass, and charge and the corresponding defining constants, have special status in the SI, and belong in their own category apart from the mole, kelvin, and candela.

### 1.1. Background on the current SI system and why we propose a revision

Quoting from the BIPM's literature, *[…] the 26th meeting of the CGPM […] meeting introduced a new approach to articulating the definitions of the units [of measure] in general, and of the seven base units in particular, by fixing the numerical values of seven 'defining' constants.* (See table 1) *Among them*

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**Figure 1.** The *new* SI, ca. 2019.

**Table 1.** Evolution of the SI since 1790.

| Year | Change/addition | Note | Base units |
|---|---|---|---|
| 1790 | m, kg | The French metric system | 2 |
| 1889 | s | The 'MKS' system | 3 |
| 1954 | A | Proposed by IEC 1939 | 4 |
| 1960 | Cd, K | The 'International System' (SI) | 6 |
| 1970 | mol | The 'Old SI' | 7 |
| 2018 | 7 fixed constants | The 'New SI' | 7 |

*are fundamental* [defining] *constants of nature such as the Planck constant and the speed of light, so that the definitions are based on and represent our present understanding of the laws of physics. For the first time, a complete set of definitions is available that does not make reference to any artefact standards, material properties or measurement descriptions. […] Thus, this redefinition marks a significant and historic step forward* [1]. Table 1 summarizes the milestones of the SI, and an impressive history of improvements and practicality.

*1.1.1. The base units of 2018.* Since 2018, the seven base units in figure 1 are categorized apart from the twenty-two derived units. A significant feature of this classification is that each of the base units is expressed in terms of the defining constants, which the world agreed to be invariant and exact. The SI brochure states; 'The distinction between base and derived units is no longer fundamental, but is maintained mainly for historical continuity and pedagogical purposes' [1]. One might argue therefore that the modifiers, 'base' and 'derived' are irrelevant, so why use them at all? But, we note that the first paragraph of section 2.3 of the Brochure states: 'Nevertheless, the concept of base and derived units is maintained because it is useful and historically well established, noting also that the ISO/IEC 80000 series of standards specify base and derived quantities which necessarily correspond to the SI base and derived units defined here' [1]. Brown recently discussed the distinction between base and derived units, concluding 'it seems sensible to retain the distinction between SI base units and SI derived units into the future' [3]. We go beyond this to propose what the base and derived units could be, how they can be categorized, and why. We do not go as far as Quincey and Burrows who introduce angle as a base quantity but whom we applaud for the goal of removing 'ambiguity and confusion' [4]. Mohr and Phillips discuss angular units with respect to fundamental constants (read defining constants) but without explicitly requesting angle as a base quantity or unit [5].

## 2. The mole, the kelvin, the candela

While we advocate that only four base units are required, it is reasonable to ask what harm would arise from keeping all seven? If the units and constants have a symbolic and causal mathematic relationship, why is $K_{cd}$ given the same status as Planck's constant and the speed of light? $K_{cd}$ is a human-selected numerical coefficient while $h$ in contrast is associated with natural units. The value of $K_{cd}$ was picked in order to make 1 Cd visually similar to a candle flame. It is a number selected for convenience, much as a foot, which is *exactly* 0.3048 m or a pound, which is *exactly* 0.453 592 37 kg [6]. We recommend that the candela be moved out of the set of base SI units. And if the candela should be moved, why not the kelvin and mole as well? Let us first consider the kelvin and the mole and then revisit the candela.

The kelvin is convenient. It is a non-standard energy unit, much like the calorie. The corresponding Boltzmann constant ($k$) is defined as $1.380\,649 \times 10^{-23}$ J K$^{-1}$ exactly. Science does not require this unit, but it is a useful size for discussing thermal vibrational energy on the atomic scale. The SI system is *already* well set up for dealing with very small (and very large) units by means of decimal prefixes and/or scientific notation. For example, the yoctojoule, which is $10^{-24}$ J, is equivalent to about 0.07 K. Such a small unit of energy enables discussion about atomic-scale thermodynamic phenomena without using scientific notation. In this sense, it is like the inch—which is exactly $2.54 \times 10^{-2}$ m. While the inch is exactly defined in terms of the SI system, and is still in common use, no one would propose that it should be a base unit in the SI. Since the kelvin shares those characteristics, how can it be viewed as a base unit?

The Avogadro constant ($N_A$) is the exact dimensionless number $6.022\,140\,76 \times 10^{23}$. Historically the use of this number can be traced to chemistry, where stochiometric ratios assist understanding and controlling chemical reactions in solution. In that field, a convenient fact was the mass of a given atom is nearly proportional to its number of nucleons—to an accuracy of several significant figures, which was well within the error of most chemistry experiments. Historically, $N_A$ was the number of atoms in 0.012 kg of carbon-12. Today, $N_A$ is a similar and now exactly defined dimensionless number. The SI already has a unit for number density, which is m$^{-3}$, though in everyday practice it is common to use the expression mole/litre (also known as molarity, with unit M.) If there were a desire for an SI unit approximating a mole, the prefix yotta (Y) (for $10^{24}$) would be suitable. The constant $N_A$ has no special status in modern physics, but like the kelvin, it causes no harm.

In contrast, the candela is a very different concept. To understand this, it is helpful to read its definition, which focuses on 'luminous efficacy', which is an intrinsically physiological and highly variable and qualitative attribute of human beings. This may sometimes go unnoticed because of the sense of precision associated with the definition's reference to photon frequency of $540 \times 10^{12}$ Hz. (This is roughly the photon frequency at which an average person finds light most visible. But that is a very approximate idea, in several ways—the value depends on the person, the light intensity, the viewing conditions, and the precise details of the experiment used for this purpose.) The SI describes $K_{\mathrm{cd}}$ as the luminous efficacy of monochromatic radiation at the exact frequency of $540 \times 10^{12}$ Hz, and assigns it the exact quantity 683 lm $\mathrm{W}^{-1}$. From a scientific perspective, the value 683 could have been any other number. True, it was chosen to yield numerical results that were similar to those obtained with a unit of light based on standard candles. But it would arguably have been better to make this obvious by making the peak value unity—and in that case the candela could have been described as a measure of 'visually averaged radiant intensity'. There is another key challenge with the definition of the candela: Its value was selected to match a non-scientific, albeit useful, historic measure of light (the candle). This concept has limited scientific significance, but parenthetically we note that it was the search for a standard candle not based on the whale-oil artefact candle in the 1800s that led to studies if blackbody radiation, which in turn catalysed the inception of the quantum revolution.

The world can and will continue to use the kelvin, mole, and candela regardless of any re-categorization. They are practical and familiar. Their presence in the SI wheel is symbolically important and can remain, while still being re-categorized.

## 3. The logical and practical core of the SI and why the base set of units should be the kg, m, s, A

### 3.1. The international system of units (SI)

The international system of units (SI) provides a coherent and consistent set of units for the measurement of physical quantities. The BIPM and international committee for weights and measures periodically updates the SI to reflect improvements in measurement technology and our understanding of the physical constants that underlie the whole system.

*3.1.1. Goals for the advancing the SI system.* There is not a single principle to guide a revision to the SI but according to the most recent revision was described in Resolution 1 of the 26th CGPM meeting (2018) as follows: *'the essential requirement for an International System of Units (SI) that is uniform and accessible world-wide for international trade, high-technology manufacturing, human health and safety, protection of the environment, global climate studies and the basic science that underpins all these'*. And that, '*the SI units must be stable in the long term, internally self-consistent and practically realizable being based on the present theoretical description of nature at the highest level*', in other words, the SI must optimize multiple ambitious principles: *who* is important, *what* is important, and *how* it is defined. We suggest that it is important to consider six goals drawn primarily from BIPM literature. In alphabetical order, those goals are as follows.

1. **Accessibility:** The SI should be accessible to all people, regardless of their location or field of work. It should provide a common language for scientists, engineers, and people in all fields to communicate and collaborate globally. It should be applicable to all fields of science, technology, and industry. Thus, it provides a standardized framework for measurement that can be universally adopted and understood. Its goal is to continuously improve, through transparent processes, these additional principles:
2. **Clarity:** The SI units should be interrelated in a simple, clear, logical, manner.
3. **Fairness:** The SI should be equally useful and beneficial in all places and for all people.
4. **Reliability:** The uncertainty in SI base units should be as small as reasonably possible.
5. **Practicality:** The SI units should be practical in the widest possible range of circumstances.
6. **Stability:** The definitions of all SI units should be consistent as reasonably possible at all times and places.

Collectively the above ideas are consistent with the following statement of the purpose of the SI: to facilitate the quality of measurement, ensuring that scientific and technological advancements can be accurately communicated and understood across borders and disciplines (what, who, and how).

The *current arrangement* of base and derived units is *not fully consistent* with these principles. It lacks sufficient *clarity*, because it does not provide a self-consistent definition of what constitutes base units and their relationship to defining constants. The definition of a base unit should include only those units and the associated constants that form a coherent four-dimensional system [1]. Designating only the kg, m, s, A as base units meets the criteria of clarity because the units and constants are interrelated in a logical manner and form the basis of derived units. Units that are the product of powers of the base units are *'coherent'* [7]. This is formalized in section 2.3.3 of the SI Brochure [1]. But furthermore, the mathematical relationship among four base units and the defining constants is nondegenerate. We illustrate this objective distinction between our *proposed* base units and the remaining three legacy units in appendix A.1. The defining constants collectively quantify the realizations and the base units give dimension to the constants [7].

For a considerable time the kg, m, s, and A, were viewed as the base units of the SI system. This raises the question of how the candela, the mole, and the kelvin ever came to be added to that core set. We have found no citable record for the basis of these changes or evolution, but we know it was a lengthy discussion [1, appendix 4, part 2]. In informal discussions, the authors have encountered opinions on this topic that bear consideration: 'There is no absolute rule for identifying the number of base units, so this must be a matter of opinion, and all opinions should have equal weight, and the current seven were chosen by democratic means. It had the side effect, desired by some, of making lighting and chemistry no less important, and causes no harm, so why not just leave it just as it is?' There is no absolute rule for practicality. Arguably, keeping all seven defining constants associated with the respective units (base or otherwise) is entirely practical. But this is outweighed by matters of clarity and stability of the system, particularly with respect to physiological units.

*3.1.2. Why choosing a different set of four base units would be less practical.* Theoretical physicists have pointed out that a unit for charge, and a unit for mass, could be defined purely by kinematics, along with gravitation and electrodynamics [8]. For example, consider two free objects in space that are stationary and spaced 1 m apart at $t = 0$ (where $t$ is time). The unit of mass could be defined as that which, if present for both objects, would cause each to have a gravitationally-induced acceleration of magnitude 1 m $s^{-2}$. Similarly, the unit of charge could be that which, if present for both objects, would cause an electrostatically-induced acceleration of magnitude 1 m $s^{-2}$. In effect though, this is equivalent to setting a fixed value for respectively, $\epsilon_o$ and $G$, so the number of defining constants remains four. The current SI, however, uses quantum phenomena to more precisely define units of mass and charge, achieving a significant reduction in uncertainty and relies upon the four base units we describe.

*3.1.3. Can we be sure that we have selected the optimal set of four units?* The four base units described above do not constitute a unique set. The base set could consist of any four units that (a) are defined as a product of powers of the kg, m, s, A, or multiples of them; and (b) are not realized by multiplying exponentiated versions of the other three (Cd, mol, K).

There are many rigorous explorations on this topic. For example, Davis has written in detail that *'there is a set of SI units that corresponds to the four basic dimensions'*. Furthermore, the relations among the units '*mirror the equations of science with no extraneous factors required'* [7]. As a counter example, we note that it would be possible for the set to comprise the newton, inch, year, and coulomb, because they are exactly defined units for force, length, time, and charge, and those four measures are dimensionally independent. In other words, changing to that base set would have no *fundamental* impact on science—other than introducing confusion. Some will argue for eliminating the amp as a base unit, or that the volt is a preferred choice rather than the amp as a base unit. The volt is directly realized by national metrology institutes whereas the amp is not. Electric charge, however, is a readily identified and widely used defining constant. The cesium hyperfine frequency $\Delta\nu_{\mathrm{Cs}}$ is unique among the four by being based on a substance and therefore subject to revision for realizing lower uncertainty of the second. We recognize that *clarity* and *simplicity* do not occur automatically nor is there a single solution to the system.

## 4. A proposal

A revision to the SI is anticipated within the next five to ten years, motivated by the emerging possibility of a better method for quantifying the second [9]. In addition, there is growing interest, within the international commission on illumination, in improving the definition of the candela, which could require a change in what is currently called the defining constant for the candela, $K_{\mathrm{cd}}$. We view this as a rare and valuable opportunity to discuss the topography of the SI and to propose a thoughtful return to its previous simplicity.

Toward that end, we propose that there be three categories of SI units as follows: (i) the four base physics units, (ii) derived units for characterizing physiological information, and (iii) all the other derived units recognized in the SI Brochure. We are not suggesting that the mole and kelvin should *change*, rather that they should be recategorized because *they* are *not true base units*. The SI brochure of the future might describe the mole, kelvin, and candela with a term such as '*primary derived units*' or '*legacy units*' within the larger set of derived units, in recognition of their history of elevated status. Accordingly, their corresponding defining constants could be described with a term such as *fixed* constants. Such discussions are beyond the scope of the present publication.

To summarize, we propose the following categorization within the SI:

### 4.1. Four base units

Category 1—Four basic physical units: time (s), length (m), mass (kg), and current (ampere), all based on quantum mechanics and special relativity. These are exclusively the defining constants and the selected corresponding set of base units. These units and the associated constants satisfy the requirements for a coherent four-dimensional system.

### 4.2. Physiological derived units

Category 2—*Non-base unit*s that are a subset of the derived units and are useful in quantifying human perception, and for practical and/or historic reasons there has been a compelling reason to standardize them: candela, lumen, lux, sievert, and possibly others. This was described in detail by Nelson and Ruby [10].

### 4.3. Everything else

Category 3—SI units derived from the four base units through simple linear combinations of the SI base units. In other cases, they include a standard SI decimal prefix, and in special cases they employ a non-standard numerical multiplier selected for historical reasons and convenience—with two important examples being the mole and the kelvin.

## 5. Conclusion

The next redefinition of the SI is an opportunity to organize the SI units in a manner that respects their evolution while demonstrating our understanding of their significance. We believe the base and derived units can be categorized to optimize clarity. The basic set of units should be the kg, m, s, A. As we recall the words of the 2019 redefinition being *an historic step forward,* let us keep stepping.

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Authors (JL, AM, JS) note: These opinions, recommendations, findings, and conclusions do not necessarily reflect the views or policies of NIST or the United States Government.

## Appendix

*A.1. A summary of the scientific core of the proposed SI system*

The 'new SI', which went into effect in 2019, introduced a transformative approach for defining the base units of the scientific system of measurement. Its fundamental concept is that the base units of kg, m, s, A can be defined fundamentally without human-maintained primary reference standards, but rather by reference to a *selected set of four defining constants*. A critical point is that these constants can be *defined* as having exact fixed numerical values, which are selected to generate today's SI units according to a specific framework. In the current framework, these exactly-designated constants are (i) the Cs-133 hyperfine splitting frequency (here depicted as $\Delta\nu_{Cs}$), (ii) the speed of light, c, (iii) the Planck constant, h, and (iv) the charge of an electron, e, all depicted in table A.1.

As described next, those values determine the kg, m, s, A. The selected set need only fulfill three requirements:

(1) It must be practical to measure the constants with sufficient accuracy.
(2) For each, the SI unit of measurement is the product of integer powers (including zero) of the kg, m, s, A. In other words, for each, a specific vector comprising four integers represents its dimensionality. For the current set, they are depicted in the 4 × 4 matrix in table A.1.
(3) The determinant of that 4 × 4 matrix is non-zero, therefore it is invertible, and its inverse defines the kg, m, s, A in terms of products of powers of the constants, as shown in table A.2. Thus the relationship between the proposed base units and defining constants is a *nondegenerate* system and the basis of the derived units is *coherent*.

With the current basis, the resultant values for the four base units, in terms of the multipliers presented in table A.1, are presented exactly in table A.2. Also shown there is the slightly less accurate truncated decimal version of the same multipliers.

The three remaining legacy units can be expressed by means of the proposed four base units along with the legacy defining constants. The legacy constants are distinct: (i) $k$ is an energy unit—(about $1.38 \times 10^{-23}$ J K$^{-1}$) which is approximately 2% of the difference between the mean thermodynamic energy per degree of freedom of boiling water versus freezing water, now defined exactly. It is useful, but not necessary to derive other units. (ii) $N_A$ is approximately the number of C-12 atoms in 0.012 kg of carbon, but not necessary to derive other units. (iii) $K_{cd}$ is an approximate value for the number of 'standard candles' needed to reproduce the visibility of 1 W of light at a wavelength of 555 nm, but not necessary to derive other units.

Consider table A.3, (in the same format as the basic 4 SI units shown above), which represents the current definition of these additional concepts in the current SI. When expressed in the table A.3, (in the same format as the basic four SI units shown above), these extra units are unrelated to the primary SI units and are not related to one another.

**Table A.1.** Summary of defining constants for s, m, kg, A.

| Defining constant | Number label | Exact defined numerical value | SI units | Dimensionality | | | |
|---|---|---|---|---|---|---|---|
| | | | | s | m | kg | A |
| $\Delta\nu_{Cs}$ | $N_1$ | $9.192\,631\,770 \times 10^9$ | s$^{-1}$ | **−1** | **0** | **0** | **0** |
| $c$ | $N_2$ | $2.997\,924\,58 \times 10^8$ | m s$^{-1}$ | **−1** | **1** | **0** | **0** |
| $h$ | $N_3$ | $6.626\,070\,15 \times 10^{-34}$ | s$^{-1}$ m$^2$ kg | **−1** | **2** | **1** | **0** |
| $e$ | $N_4$ | $1.602\,176\,634^{-19}$ | S A | **1** | **0** | **0** | **1** |

**Table A.2.** Summary of defining formula for s, m, kg, A.

| Base unit | Exact multiplier | Approximate multiplier | Full defining formula | Concise defining formula |
|---|---|---|---|---|
| s | $N_1$ | $9.192\,631\,770 \times 10^9$ | $\Delta\nu_{Cs}{}^{-1}\,c^0\,h^0\,e^0$ | $\Delta\nu_{Cs}{}^{-1}$ |
| m | $N_1{}^{-1}N_2$ | $3.066\,331\,899 \times 10^1$ | $\Delta\nu_{Cs}{}^{-1}\,c^1\,h^0\,e^0$ | $\Delta\nu_{Cs}{}^{-1}c$ |
| kg | $N_1\,N_2{}^{-2}N_3$ | $1.475\,521\,340 \times 10^{40}$ | $\Delta\nu_{Cs}{}^{1}\,c^{-2}\,h^1\,e^0$ | $\Delta\nu_{Cs}\,c^{-2}\,h$ |
| A | $N_1{}^{-1}N_4{}^{-1}$ | $6.790\,902\,147 \times 10^8$ | $\Delta\nu_{Cs}{}^{1}\,c^0\,h^0\,e^1$ | $\Delta\nu_{Cs}e$ |

**Table A.3.** Summary of defining constants for Cd, mol, K.

| Defining constant | Number label | Exact defined numerical value | SI units | Dimensionality Cd | Dimensionality mol | Dimensionality K |
|---|---|---|---|---|---|---|
| $k$ | $N_5$ | $1.380\,649 \times 10^{-23}$ | $\mathrm{J\,K^{-1}}$ | **0** | **0** | **0** |
| $N_A$ | $N_6$ | $6.022\,140\,76 \times 10^{-23}$ | $\mathrm{mol^{-1}}$ | **0** | **0** | **0** |
| $K_{cd@555nm}$ | $N_7$ | $6.83 \times 10^{-2}$ | $\mathrm{lm\,W^{-1}}$ | **0** | **0** | **0** |

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