

# Removing Bias in Dual-Comb Spectroscopy from Pipeline Analog to Digital Converter Conversion

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**Abstract:** We quantify the percent-level bias in dual-comb spectroscopy due to nonlinearities from the analog-to-digital conversion and demonstrate a compensation method to enable gas concentration measurements with 0.2% accuracy. Work of the U.S. Government and not subject to copyright.

To monitor greenhouse gas sources and sinks, a spectrometer must be sensitive to sub-percent enhancements in downwind gas concentration [1]. Dual-comb spectroscopy (DCS) can achieve that precision in open-path measurements [2], however small nonlinearities in the analog-to-digital converter (ADC) that records the interferograms from DCS can alter the fit concentration by multiple percent [2]. This bias comes from the challenge of accurately measuring both the large centerburst and the 100-1000x smaller free-induction decay tail that comprise the interferogram. If the effective ADC gain differs for these two components, there will be a bias in the retrieved gas concentration. Here we quantify this effect in terms of the ADC integrated nonlinearity (INL) [3] and then demonstrate it can be successfully compensated to achieve less than 0.2% concentration bias. We develop compensation approaches that significantly improve upon the simple sinewave dither used in the past DCS work [2,4,5].

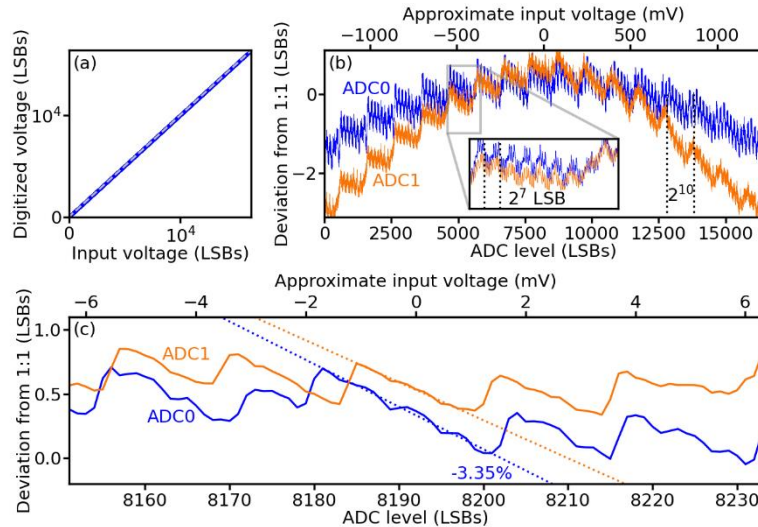


Fig. 1. Integrated nonlinearity (INL) measurements for two 200 MS/s, 14-bit ADCs. The ADC responses are nearly linear over full voltage range, but closer inspection reveals a nested sawtooth structure that presumably arises from the pipeline ADC architecture. (a) ADC transfer function. (b) Integrated nonlinearity shows structure at  $2^{10}$  LSBs and  $2^7$  LSBs. All voltages are expressed in terms of the least-significant bit (LSB) -- 160  $\mu$ V per LSB for this device. (c) Inset of (b) showing  $2^4$  LSBs structure with slopes steeper than 0.2%, suggesting that our DCS signals must exceed this voltage range to achieve sub-0.2% accuracy. For our system a typical free-induction decay tail only spans 1-4 LSBs.

The ADC INL is measured by digitizing a bandpass-filtered 10 MHz sinewave. The digitized signal is then fit to a sinewave function, allowing phase and amplitude to vary. The fitted model values at each point are taken as the true input value, and the residuals are then interpreted as ADC error. Fig. 1(a) shows the measured ADC response. The deviation of this response from a straight line, seen in 1(b) and 1(c), is the INL. The repeated sawtooth structure of the INL arises from the pipeline architecture of the ADC. While many ADC architectures exist, pipeline designs dominate for ADCs in the 12-bit to 16-bit range at speeds greater than 10 MS/s, and are commonly used for DCS.

To test ADC-imposed bias, we simulate a DCS measurement through a 5.5 cm cell filled to 130 hPa with pure CH<sub>4</sub>. We then add noise to the synthetic interferogram, apply the Fig. 1 ADC response, and average over 5000 measurements. The CH<sub>4</sub> concentration is then fit using the resulting spectrum to determine a bias.

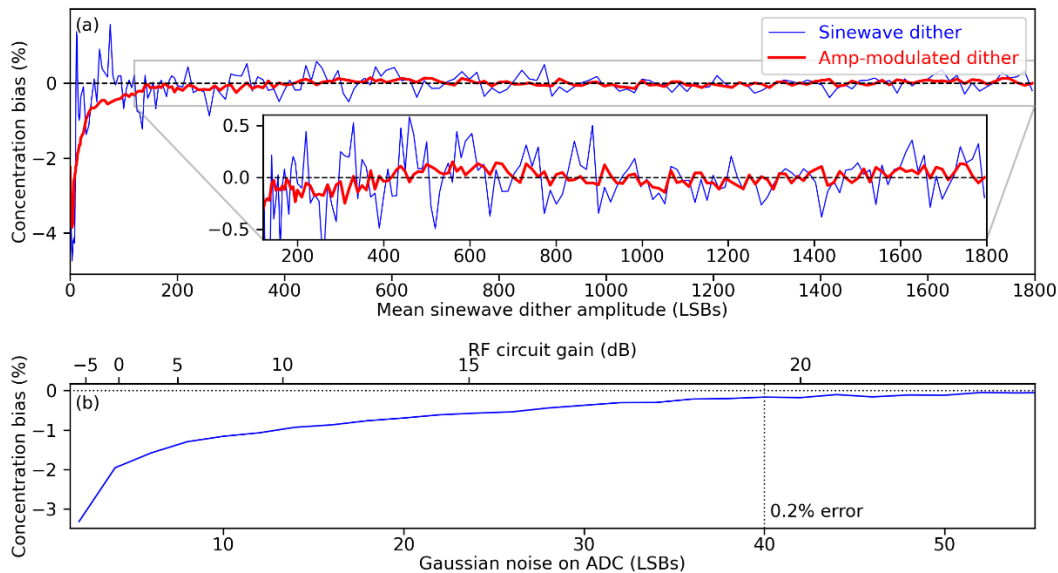


Fig. 2. Simulations showing three strategies for reducing ADC-imposed bias. (a) The bias as a function of dither amplitude for a constant amplitude sinewave and amplitude modulated sinewave. (b) The bias can also be suppressed by amplifying the signal without applying a dither. Here we apply varying levels of gain to the signal and noise level of a typical 100 MHz photodetector (PD). Significant gain is required to reach <0.2% bias and may require careful thought to avoid nonlinearities in the rf amplifiers.

Figure 2 shows the result of this analysis. The impact of adding a sinewave dither to the DCS signal is shown in Fig. 2(a). This simple sinewave can reduce the bias but a smaller oscillatory behavior continues out to very large dither amplitudes. Given the period, the oscillatory behavior is likely driven by the larger  $2^7$  LSBs structure seen in Fig 1(b). Fig. 2(a) also shows a modified approach where the amplitude of the sinewave dither is modulated by a slow triangle wave of  $\pm 64$  LSB amplitude. The amplitude modulation effectively averages over this structure and greatly reduces the bias, avoiding the oscillatory behavior. The amplitude-modulated sinewave can reliably achieve a 0.1% bias without exceeding  $\pm 800$  LSBs. For our x-bit ADC, this approach uses only 10% of the full-scale voltage range of the ADC, leaving 90% of the ADC voltage range available for the DCS signal. Because DCS signals (like most Fourier transform spectroscopy signals) can have a very high contrast ratio between the centerburst and the free-induction decay tail, this high available dynamic range is critical.

While it reduces bias, this modulated-dither approach does involve additional electronics and requires knowledge of the INL to match the amplitude modulation to the bit structure. An alternative and possibly simpler solution would be to amplify the DCS signal so that any additive white noise (either from the detector or the light) acts as the dither. Figure 2 shows variable amplification of a DCS interferogram signal with 40 dB signal-to-noise ratio (measured from the peak of the interferogram to the additive white noise). This approach converges smoothly to below 0.2% concentration bias at 19 dB amplification, where the gaussian noise extends over 40 LSBs. The amplification corresponds to a 19 dB reduction in the dynamic range and care must be taken then to avoid saturation of the centerburst. While this may be possible in the laboratory depending on the DCS parameters and differential chirp between the frequency combs, for DCS measurements in turbulent environments, such as combustion chambers or outdoor open paths, this can be a significant challenge. Additionally, the large amplification will require careful consideration to avoid introducing a nonlinear distortion by the amplification upstream of the ADC.

In conclusion we model ADC induced concentration bias in DCS based on the measured INL of our pipeline ADC and propose two solutions that support DCS concentration measurements with biases below 0.2%.

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