THE STATISTIC AND DETERMINISTIC KETTLE

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ABSTRACT

In some accident scenarios, a nuclear reactor could be assumed to be similar to a kettle. Namely, the water inside the core, or in this case in the kettle, is experiencing heating, and at some point, for a non-boiling reactor, the safety boundary may be crossed. This paper presents two approaches, *i.e.*, statistic and deterministic, to evaluate the time it takes to boil water in a kettle as an analogy to evaluate safety criteria for a nuclear core. The two computational approaches are compared with correspondence to the uncertainty and safety margins. The performed analysis relies on the methodology proposed by the International Atomic Energy Agency technical documents, which address the essential issues to evaluate the uncertainty within a particular task. Finally, this paper assesses the safety margin criteria definition and its value in safety analysis for nuclear reactors.

1. Introduction

In some accident scenarios, a non-boiling nuclear reactor, such as a Pressurized Water Reactor (PWR) is very similar to a kettle. Namely, the water inside the core is experiencing heating due to residual heat dissipation and at some point, some safety boundaries may be crossed. The same can be said about a kettle, the water in the kettle is heated and at some point, it will cross a safety margin. In many cases, we are interested to know how long it will take to cross such a margin to evaluate the consequences of an accident, such as a loss of coolant scenario. Furthermore, we are interested to establish a margin or a safety factor to identify when the "goal is achieved". As a result, we tend to ask, "which modeling techniques should be used to determine whether this goal is achieved?"

In essence, two main types of analysis could be used: *Deterministic Analysis*, which aims to demonstrate that the analysis is tolerant to identified uncertainties that are within the "design basis", thereby defining the limits. The second type is the *Probabilistic (Statistic) Analysis*, which aims to provide a "realistic" estimate of the presented problem. This method can also be used to confirm the validity of the deterministic safety assessment.

In our case, we were interested to establish a methodology to evaluate the grace period in case of a Station Black Out (SBO) accident at a research reactor. In our analysis, the safety criteria were chosen not to cross the saturation water temperature in the reactor pool, since the reactor design is currently in the pre-conceptual design phase, detailed engineering core structure currently amorphic.

We have chosen the kettle analogy to represent our problem as shown in Figure 1. In this analogy the kettle represents the core, the kettle heater represents the residual heat that is produced from the nuclear fuel and the water in the kettle represents the heat sink which is the water in the reactor pool. The time it will take to reach the saturation temperature of the water is our safety metric and this accident will be classified as a Design Base Accident (DBA). The initial boundary conditions are given by the initial water temperature in the kettle and the safety margin is the water temperature distance to saturation. Since the detailed core structure is not yet defined it is hard to determine what fraction of the heat dissipates to the surrounding and core structure material as well as how much, if any, of the water will be evaporated till the margin is reached.

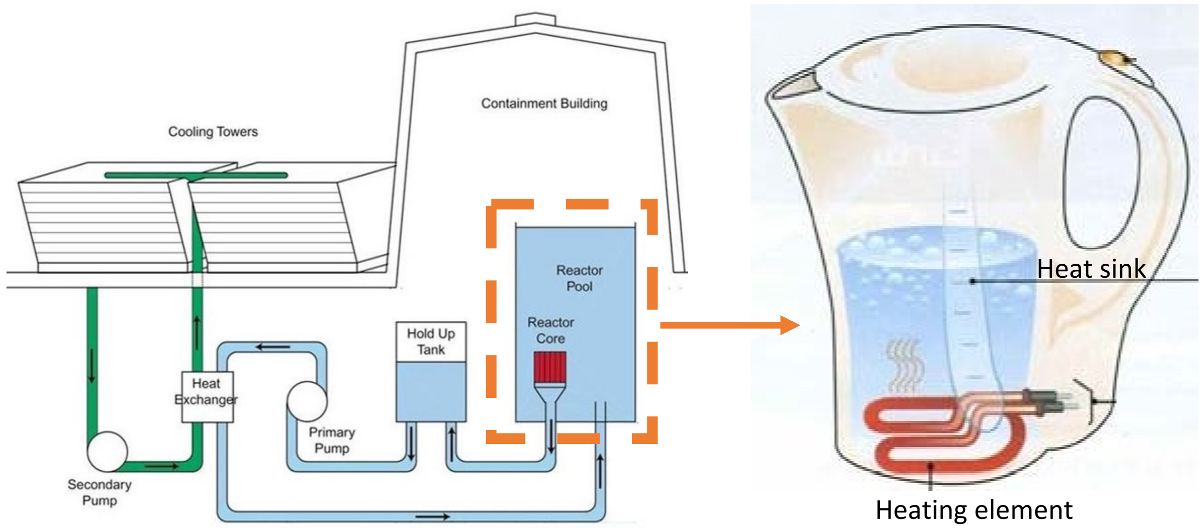

Figure 1: Analogy between nuclear core to a kettle

In the following sections we will present two main approaches, probabilistic (statistic) and deterministic, to evaluate the time it will take to boil one liter of water in an electric kettle. We will address the essential issues to evaluate the uncertainty of the results as well as the safety margin criteria definition and its value.

2. Methodology

In our analysis we will try to answer the question: "How long it takes to boil one liter of water in a kettle?" as an analogy to nuclear core. The latter question can be estimated using three methods: statistic – wisdom of the crowd (expert judgment)[1], deterministic – calculation[2], and the probabilistic – identify and understand key vulnerabilities[3].

As it is common in scientific research, we have performed literature review relying on Google as presented in Figure 2. Based on the literature review, Google suggested that it will take "about 4 minutes to boil one liter of water" in an electrical kettle.

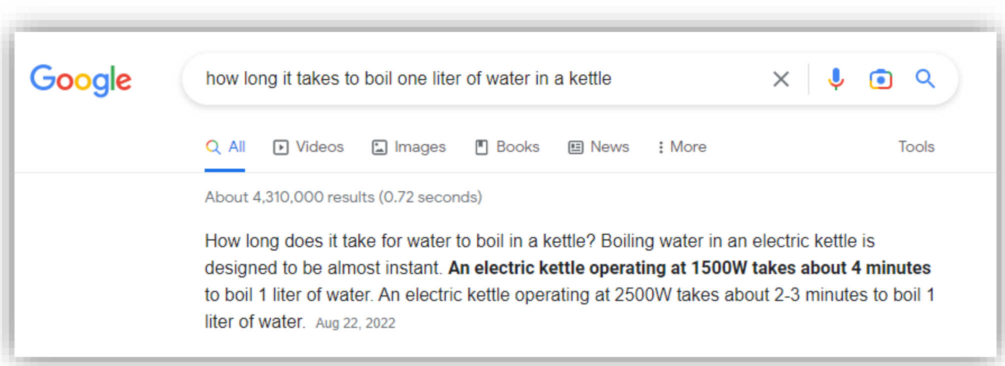

Figure 2: Google based literature review

However, the same literature review indicated that the result is not absolute and that it is dependent on the power of the kettle. Therefore, we switched to a more modern methodology based on the wisdom of the crowd as it was proposed by Sir Francis Galton[4] in 1906. To make a better decision (*collective guess*), we have performed our survey among an expert scientific community (expert judgment) with legacy experience in this field. The survey results, which included 51 top specialists, are presented in Figure 3. From these results, the average time to boil one liter of water in an electric kettle is 169 seconds with standard deviation of 82 seconds. Majority of the results were based on a pure expert judgment. Four of the reported results were based on experimental data, however, the experiments were not documented. Two results were based on calculations, but the applied mathematical model is not provided.

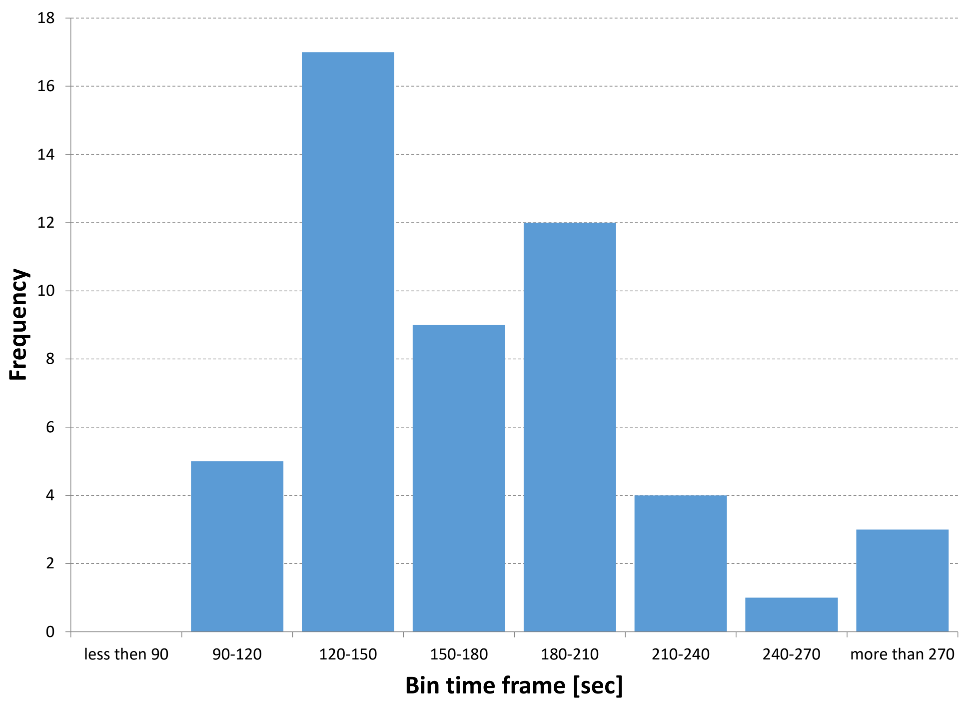

Figure 3: Referendum results histogram

Analyzing the histogram in Figure 3 we found out that the distribution is a non-normal distribution – an observation that triggered suspicion about possible proliferation of the survey. Moreover, since the standard deviation of the result was relatively high (49%), we have decided to abandon the wisdom of the crowd methodology and try an alternative approach.

2.1. The deterministic kettle model

The simplified deterministic kettle model was based on a heat capacity equation as presented in Eq. 1.

$$Q = \int \dot{q}\,dt \underbrace{=}_{\dot{q}=const} \dot{q}t = mc_p\Delta T = mc_p(T_{sat} - T_{in}) \quad \text{Eq. 1}$$

Were:

Symbol	Connotation	Units
m	Mass of the water	$[kg]$
c_p	Heat capacity	$\left[\frac{kJ}{kg°C}\right]$
T_{sat}	Saturation temperature	$[°C]$
T_{in}	Initial water temperature	$[°C]$
$\dot{q}$	Generation heat rate of the kettle	$[kW]$
t	Time	$[sec]$

Since the problem was defined as: "How long it takes to boil one liter of water in a kettle" the following assumptions were made:

a. 1 liter of water is equivalent to 1 kilogram
b. Saturation water temperature or the temperature which the water should reach is 100 ^oC
c. The initial water temperature is at room temperature of 24 ^oC
d. The water heat capacity is assumed to be constant and equal to 4.2 kJ/kg ^oC
e. The generation heat rate of the kettle is assumed to be fixed at 2kW.

Armed with the above assumptions and based on Eq. 1 we derived that the time to boil one liter of water is 160 seconds. However, several of our assumptions are questionable as both the initial conditions and the heat production rate have some uncertainty.
For instance, the initial water temperature can vary[5] from 4 ^oC (winter-time) up to 60 ^oC depending on whether the kettle is filled with cold tap or hot tap water, respectively. The expected 1L of water can also exhibit an uncertainty that can reach 200 cc (0.2L) attributed to the labeling of the scales. In general, the trigger to stop the kettle operation relies on an alternative mechanism, such as resistance of the heater, and not the saturation temperature directly. This is analog to probing the clad temperature rather than water temperature in a reactor core. As a result, "the saturation water temperature" at which the trigger will be reached can be up to 7 ^oC lower (93 ^oC) than the actual saturation water temperature. The heat generation rate of the kettle can vary[6] from 1.2 kW to 3.0 kW depending on the vendor and the model. This is alike the nuclear fuel residual heat rate variations, hence it depends on the fuel type, accumulated burnup, and decay time.
To better understand the uncertainties associated with the above assumptions we can use the guidelines recommended by the International Atomic Energy Agency (IAEA)[7]. According to the IAEA, the qualitative evaluations are typically done using two different methods, namely the conservative analysis and the best estimate with uncertainty analysis. Although different, both approaches require adopting initial parameters, various assumptions, and safety assessment techniques. Even with these tools, we are still required to implement Safety Margins to

compensate for unknown or other uncertainties. Safety Margins are the differences between the established safety limits/criteria of assigned parameters associated with a phenomenon under consideration, and the calculated values of those parameters. Therefore, for practical purposes, the safety margin is usually understood as the difference in physical units between the acceptance (regulatory acceptance) criteria and the results provided by the calculation of the relevant event/problem.
Figure 4 illustrates the safety limits and margins and the regulatory acceptance criteria as a function of the safety assessment using a deterministic methodology.

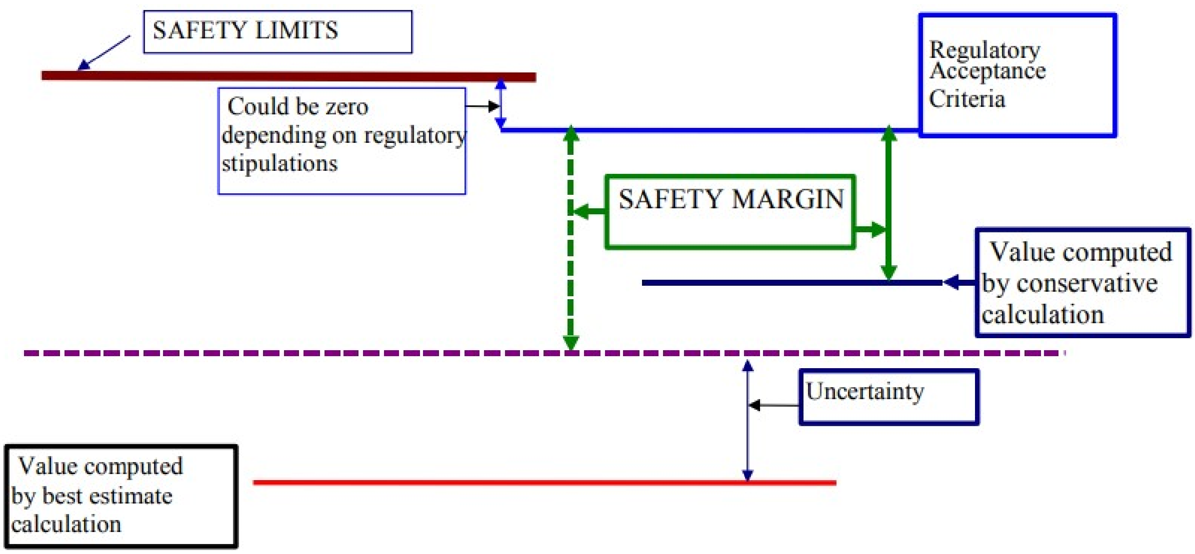

Figure 4: Deterministic evaluation and safety margin diagram (et al. IAEA-TECDOC-1332)

In the past, the margins for the acceptance criteria have been determined by conservative evaluations of a model. Recently, an increasing tendency in computational analysis is to replace these conservative calculations by “best estimate” or “realistic” calculations. In case of best estimate calculations, it is necessary to supplement uncertainty analysis when determining the safety margin. A prerequisite for this approach is qualification, applicability, and validity of the model. In practice, one can calculate the limit value based on conservative calculation, simply taking all the parameters to their worst limit, and then add a safety margin (solid green line in Figure 4) to cope only with the uncertainty of the model. An alternative method is to use the best estimate calculation which either adopts average values or relies on “as made/measurement” parameters in conjunction with safety margin added on top. The latter approach takes into consideration both the uncertainty of the model and the uncertainty of the parameters (denoted by the greed dashed line in Figure 4). The debate on which is adequate to determine the safety margin will be discussed in the results section. Since there are no specific recommendations for safety margin (Safety Factor) in nuclear industry[2], we have adopted the Safety Factor as it is used in the space industry[8] as the failure rates requirements are similar between these industries. A typical applied Safety Factor in space industry is between 1.5 up to 2.5 with a nominal value of 2.0.

2.2. The probabilistic kettle model

Modern safety assessments tend to make use of both deterministic and probabilistic techniques because of their complementary nature. Although emphasis is more focused on deterministic evaluations, these are supplemented with probabilistic analysis, technical judgement, and experiences to reach at risk informed decisions.
The probabilistic assessment may be defined as the difference between the established probabilistic targets (Regulatory Probabilistic Safety targets) to the calculated value considering uncertainties in the data, modelling etc. as presented in Figure 5.

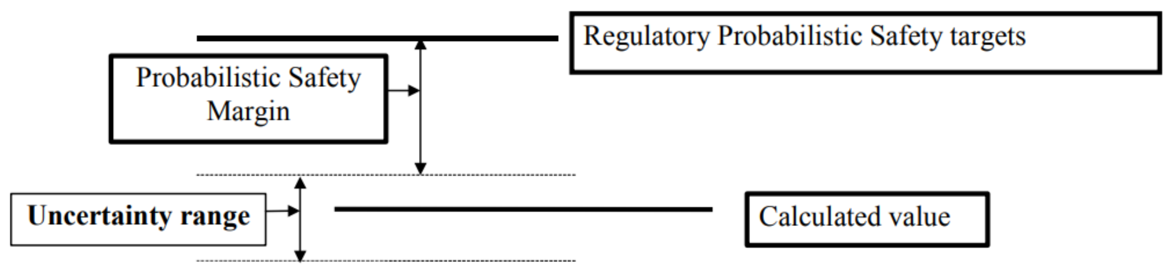

Figure 5: Probabilistic evaluation and safety margin diagram (et al. IAEA-TECDOC-1332)

The Regulatory Probabilistic Safety targets[9,10] can be set based on the categorization of the accidents and their consequences. Those are well classified and described in the International Nuclear and Radiological Event Scale (INES)[11]. Based on the accident, one can use the data presented in Table 1 to classify the Regulatory Probabilistic Safety target.

Table 1: Probabilistic safety accident classification

Deterministic approach consequences result	**Regulatory Probabilistic Safety targets**	**Accident classification**
INES level 0	Up to 10^{-2} per demand Up to 10^{-2} per R-Y	AOO
INES level 1	Up to 10^{-3} per demand Up to 10^{-2} per R-Y	AOO
INES level 2	Up to 10^{-3} per demand Up to 10^{-3} per R-Y	DBA
INES level 3	Up to 10^{-4} per demand Up to 10^{-4} per R-Y	DEC
INES level 4	Above 10^{-5} per demand Above 10^{-5} per R-Y	MHA
INES above level 4	Above 10^{-6} per demand Above 10^{-6} per R-Y	

- R-Y – Reactor Year
- Regulatory Probabilistic safety targets are based on IAEA recommendations (Safety Series No. 75-INSAG-3 and Safety Series No. 106)

The probabilistic safety margin can be calculated, or set, based on confidence intervals which is a function of the uncertainty range. For example, if we would like to classify the time to boil the water in our kettle analogy as a DBA (Design Basis Accident) the probabilistic safety target should be 10^{-3}. In other words, one needs to ensure adequate number of standard deviations corresponding to a confidence level of 99.9%. The number of standard deviations may be obtained using the z-value approach[12].

3. Results

In this section, we will present the results regarding the time it takes to boil one liter of water in a kettle. The results cover both methods described in the methodology section, respectively, at the initial conditions as presented in Table 2.

Table 2: Boundaries of the input paraments

Parameter	Upper boundary	Lower boundary
The mass of the water, kg	1.2	0.8
Water heat capacity, $\frac{kJ}{kg°C}$	4.18	4.20
“Saturation” water temperature, $°C$	93	99
Initial water temperature, $°C$	4	60
Generation heat rate, kW	1.2	3.0

In addition to the bounds provided in Table 2, a best estimate case was used in our analysis as shown in Table 3. The methodology described in section 2.1 (deterministic model) was applied to assess the conservative and the avantgarde cases as presented in Table 3. As opposed to the traditional definition of the grace period, the conservative and avantgarde cases are characterized by the longest and shortest times to boil the water in a kettle.

Table 3: Best estimate, conservative and avantgarde cases

Parameter	Best estimate	Avantgarde	Conservative
The mass of the water, kg	1.0	0.8	1.2
Water heat capacity, $\frac{kJ}{kg°C}$	4.19	4.18	4.2
“Saturation” water temperature, $°C$	96	93	99
Initial water temperature, $°C$	24	60	4
Generation heat rate, kW	2.0	3.0	1.2
Time based on Eq. 1, sec	**151**	**37**	**399**

In addition to implementing the proposed methodology as described in section 2.1 (deterministic model) a Safety Factor of 2.0 is imposed on the best estimated result (without any uncertainty). The latter product yields a boiling time of 302 seconds which is less restrictive than the time obtained based on the conservative approach, which yields to 399 seconds.

As an alternative, we can implement the probabilistic model, in this case we will assume that the input paraments are distributed uniformly, between the upper and lower boundary as given in Table 2. A Monte Carlo (MC) simulation was performed by sampling the input paraments in conjunction with using Eq. 1. Figure 6 presets the distribution of the results for the boiling time using 60,000 samples. The mean time is 136 seconds with a standard deviation of 54 seconds.

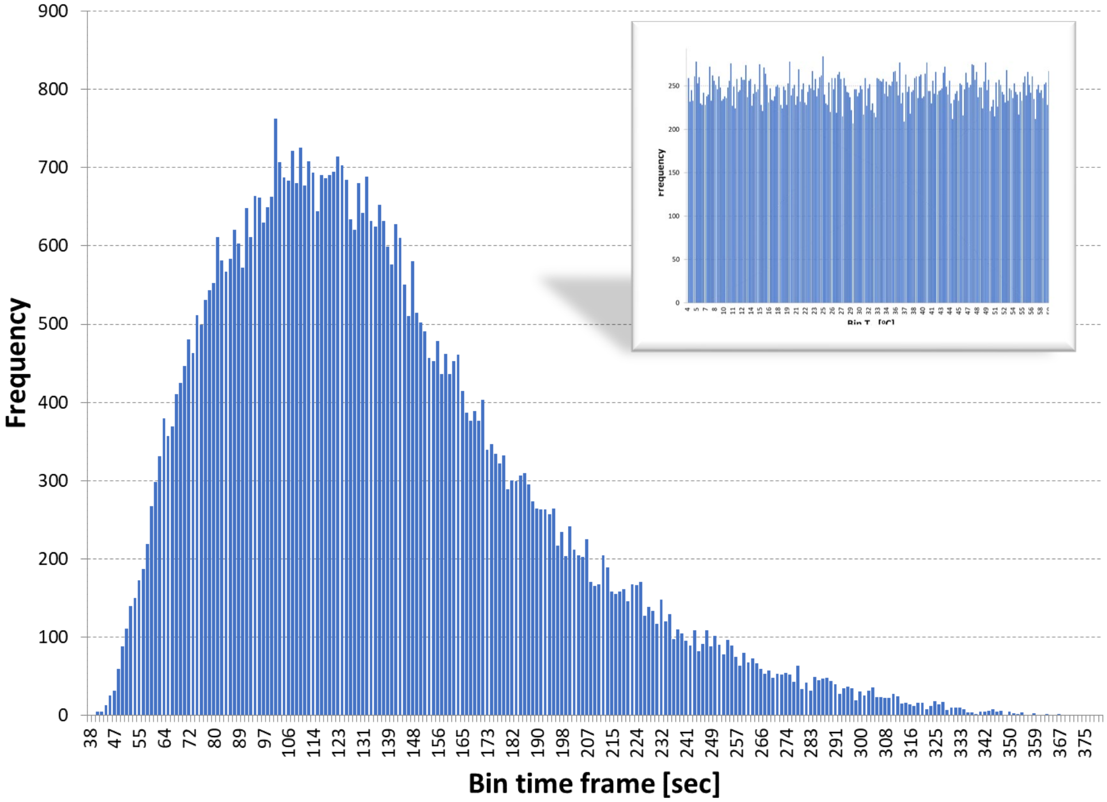

Figure 6: Time distribution based on 60,000 MC sample. The upper right corner depicts the uniform distribution sampling of initial temperature which is one of the inputs paraments

Based on the proposed methodology described in section 2.2 (probabilistic model) where a confidence level of 99.9% (DBA of 10^{-3} as classified in Table 1) is chosen, we need to multiply the standard deviation by 3.09 (based on z-table) and add it to the mean value. As a result, the "Probabilistic Safety Target" time will be 301 seconds.

All of the aforementioned results are summarized in Figure 7. No significant difference is observed between deterministic (302 seconds) and probabilistic (301 seconds) methods. However, the value for the conservative approach is considerably different (i.e., 399 seconds), and in this specific case, it is clearly the more constraining one.

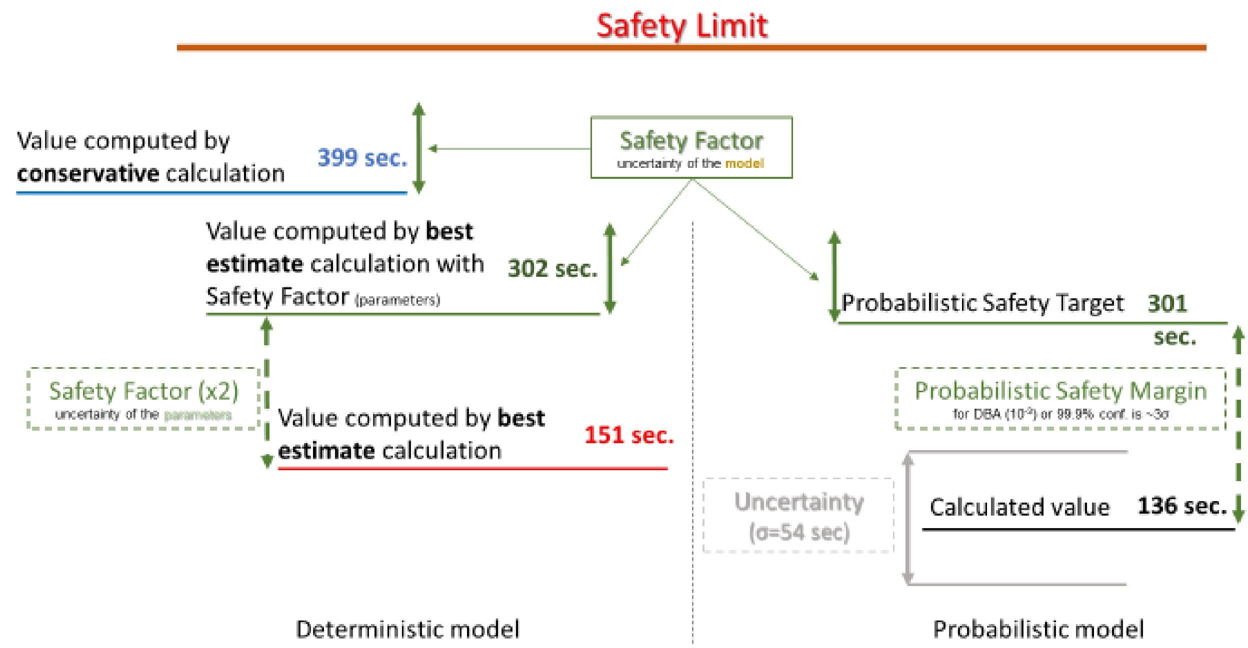

Figure 7: Summary of the results based on deterministic and probabilistic models

As shown in Figure 7, the deviation in results is relatively large. A potential way to address this deviation is by monitoring the input parameters. In the deterministic approach, this monitoring can be achieved either by directly measuring input quantities (as is) or restricting the allowed range/tolerances of control parameters (e.g., temperature measurement, heat production rate) as shown in Table 4. In contrast to the deterministic approach, the inputs are not monitored directly, but rather a data-driven distribution (e.g., normal) of values is provided when the probabilistic approach is used. For our analysis, the ranges for different input parameters are provided in Table 2, with normal distribution used for most of the inputs, excluding the inlet water temperature that adopts the Poisson distribution with a mean value of 24 °C. Best estimated and conservative results for the deterministic approach are presented in Table 4 while Figure 8 presents the results for the probabilistic approach.

Table 4: Controlled boundaries of input paraments

<table>
<tr><th colspan="2">Parameter</th><th>Upper boundary</th></tr>
<tr><td colspan="2">The mass of the water, kg</td><td>1.00 ± 0.05</td></tr>
<tr><td colspan="2">Water heat capacity, $\frac{kJ}{kg°C}$</td><td>4.19</td></tr>
<tr><td colspan="2">“Saturation” water temperature, $°C$</td><td>96 ± 1</td></tr>
<tr><td colspan="2">Initial water temperature, $°C$</td><td>24 ± 1</td></tr>
<tr><td colspan="2">Generation heat rate, kW</td><td>2.0 ± 0.1</td></tr>
<tr><td rowspan="2">Time based on Eq. 1, sec</td><td>Best estimate</td><td>151</td></tr>
<tr><td>Conservative</td><td>171</td></tr>
</table>

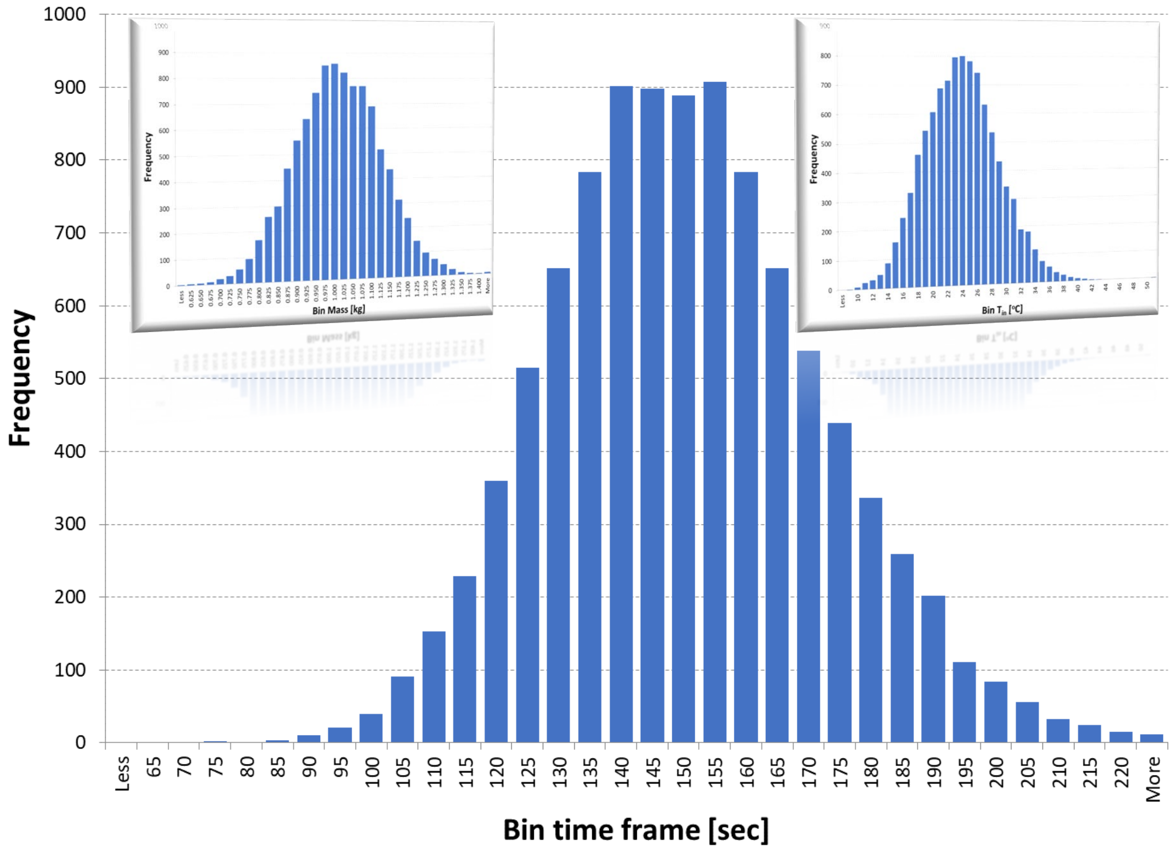

Figure 8: Time distribution based on 10,000 MC sample when the input parameters are normally distributed. The upper right corner depicts the distribution sampling of initial temperature, and the left corner depicts the distribution sampling of the mass.

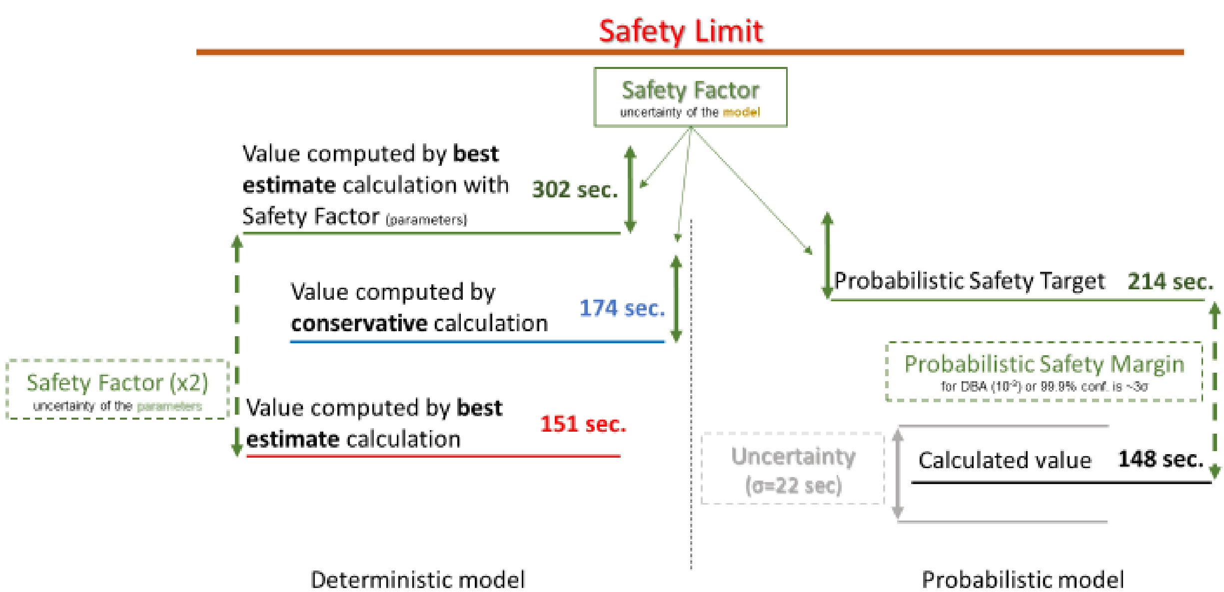

Figure 9: Summary of the results based on deterministic and probabilistic models with controlled input parameters

Figure 9 depicts the summary of the results for the deterministic as well as the probabilistic analysis where the input parameters were controlled. As it can be seen, in case of probabilistic analysis, the mean value is 148 seconds, which is close to the mean value in case of uncontrolled parameters (136 seconds) and to the "best estimate" (151 seconds) results. However, since the parameters were sampled from non-uniform distributions, the standard deviation was significantly reduced from 56 to 22 seconds, which in turn affected the "Probabilistic Safety Target" time to be 214 seconds. Similarly, the deterministic conservative value was dropped from 399 to 174 seconds, and it is no longer the most restrictive case.

4. Conclusions

In this paper, the kettle is used as analogy to a reactor core. A simple energy balance model was used to represent the underlying physics within the kettle. The main objective was to compare between two main methods, i.e., deterministic against probabilistic, used for safety analysis. The deterministic approach is simple to implement and requires multiplying the best-estimate results with pre-determined, yet somewhat arbitrary, safety factors. These factors are not necessary aligned with the magnitude of inputs uncertainties. Moreover, these factors are required to be previously determined. The deterministic approach can also be altered to rely on the most conservative calculations, and then the safety factors can be omitted.

The probabilistic approach is more convoluted to implement, as it largely depends on the quality of distributions, but it is more intuitive for determining the safety margins. This method requires no safety factors, but rather replaces them with well-defined pre-determined probabilistic targets with corresponding classified scenarios. These probabilistic targets represent the confidence level, from which the required number of standard deviations is deducted. The challenge using this approach is related to the fidelity of the distributions for all the parameters, which in some cases are difficult to obtain (especially for new systems). The latter may require significant investment, e.g., multiple experiments or measurements.

The main objective of this paper was to answer a simple question related to the time it takes to boil the water in a kettle. The results presented in this paper show that the results are strongly dependent on the data uncertainty. In the uncontrolled case, both the deterministic and probabilistic approaches yield similar times. It is important to note that a safety factor of 2 was used in the deterministic case for obtaining these results. The most restricting case was the conservative case without utilizing any safety factors. Conversely, in the case where we have controlled input parameters to reduce the uncertainty, the deterministic approach yielded similar results to the uncontrolled case, while the boiling time was drastically reduced using the probabilistic method. Furthermore, the restrictive case was not the most conservative one, but rather the one obtained using an arbitrary safety factor of 2. An additional observation is that safety factor does not have to be a ratio, but in some cases should rely on an absolute value. The utilized simple kettle as an analog to a reactor design at its preliminary stage emphasizes the need to generate a pre-determined licensing methodology for safety limit evaluations and analysis.

Disclaimer

Certain commercial equipment, instruments, or materials are identified in this study in order to specify the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.

Acknowledgment

To Prof. Dan Kotlyar for his words of wisdom and a friendly company.

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