Impact of the Exposure Time and Distance Thresholds on the Performance of Automatic Contact Tracing

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Abstract— **Automatic exposure notification apps operate based on guidelines that specify distance and time thresholds to measure the occurrence of contacts with infected individuals. In the United States 2 m/15 min thresholds are used to determine exposures due to a close contact. In this paper, using a simplifying approximation for the probability of infection versus exposure time, we provide a mathematical framework to investigate the impact of these thresholds on the performance of the contact tracing system. We consider the number of unnecessary quarantines and the undetected infections as metrics for the system performance. This study is done through an agent-based simulation platform which emulates automatic exposure notification with Bluetooth-based proximity detection. The platform allows for a comprehensive analysis with several user-controlled parameters that can specify different scenarios. Our analysis highlights the trade-off between unnecessary quarantines and undetected infections and would pave the way for the optimization of the exposure thresholds based on factors such as the surrounding environment (e.g., indoor vs. outdoor), an individual's health, the severity of the outbreak in the community, transmissibility of the virus, etc. The ultimate objective is to enhance the effectiveness of automatic contact tracing during pandemics.**

Keywords- Proximity Detection, Bluetooth, Exposure Determination, COVID-19, Contact Tracing

I. Introduction

COVID-19 infections are primarily spread through exposure to respiratory fluids (i.e., droplets and aerosol particles) carrying the SARS-CoV-2 virus. These respiratory fluids are released during breathing, coughing, sneezing, speaking, etc. Droplets typically clear from the air surrounding the infectious source within seconds to minutes; however, aerosol particles (i.e., very fine droplets) can remain in the air for minutes to hours. Inhalation of the air containing these droplets or aerosols could cause transmission of the virus to an individual who is in proximity of an infected person [1]. The infection risk of the individual inhaling this air depends on the amount of virus to which he is exposed. The critical amount of virus that could lead to infection once entered the exposed individual's body is referred to as the infective dose [2].

Contact tracing is a well-established technique used by public health professionals to trace and identify "contacts" of a known infectious person. In the United States, a "close contact" is defined by the Centers for Disease Control and Prevention (CDC) as someone who was within 2 meters of an infected person for at least 15 minutes within a 24-hour period starting from 2 days before appearance of symptoms (or, for asymptomatic cases 2 days prior to positive specimen collection) until the time the person is isolated [1]. The total exposure time needed for determination of this "close contact" is a cumulative total of 15 min or more over a 24-hour period. The World Health Organization (WHO) also has a similar definition for close contact through proximity and duration of exposure, except that the proximity threshold is 1 m, instead of the 2 m considered by CDC [3]. Based on the distance and exposure time thresholds specified by health organizations, one can consider two zones in a two-dimensional time/distance space, i.e., exposure and safe zones (see Fig. 1).

During a pandemic, an efficient implementation of contact tracing is critical to limit an outbreak. However, resource limitations in cases of large outbreaks could create many challenges for executing manual contact tracing. Automatic contact tracing (also known as Automatic Exposure Notification) is an electronic notification protocol based on a proximity detection mechanism such as Bluetooth ranging. The availability of Bluetooth (or Bluetooth Low Energy (BLE)) technology in today's smart phones have prompted governments and industry to also consider automatic exposure notification as a tool to complement manual contact tracing in combating the spread of the virus during COVID-19 pandemic [4, 5]. Usage of this technology involves installing an app developed through collaboration between industry and government agencies and published by authorized health authorities.

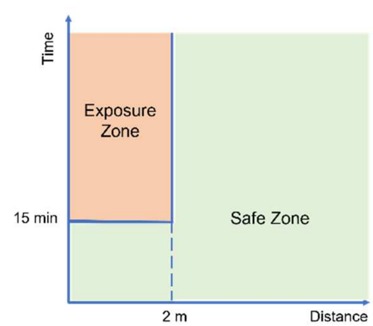

Fig. 1: Exposure zone according to the CDC definition

The automatic exposure notification apps operate based on the distance/proximity threshold outlined by the health organizations (e.g., 2 m by the CDC or 1 m by the WHO) to determine exposures as a result of close contacts. The origin of these distance thresholds is based on the work by Jennison et al. [6] who took photos of a sneeze of an adult male using a high-speed camera. Their objective was to visualize the particles expelled from the mouth during sneezing. Due to technical limitations at the time, only the movements of particles with diameters large enough to fall to the ground at speeds of milliseconds were captured. Based on that research, it was concluded that the maximum distance traveled by the droplets was less than a meter. However, in a recent study [7], researchers reviewed the evidence for distance traveled by droplets/aerosols and the health organization guidelines on respiratory protection for COVID-19 and found that available data do not generally support 2 (or 1) meters distance guideline used by CDC (or WHO). The research indicated possibility of exposure in distances well beyond 2 meters [8, 9, 10].

In this paper, we analyze the performance of automatic exposure notification with BLE-based proximity detection by investigating the impact of different distance and time thresholds as shown in Fig. 2. To the best of our knowledge, there are no comprehensive studies in the literature on the impact of these thresholds. Varying these thresholds effectively changes the size of the exposure zone and that will impact the number of individuals who could be identified as exposed. For example, a larger exposure zone potentially covers more people who could be in the vicinity of an infected person. This leads to higher number of exposure notifications from the contact tracing app. However, the number of people who are really exposed to the infective dose does not necessarily increase. Assuming that all individuals who receive an exposure notification through the contact tracing app are required to isolate or quarantine for sufficient amount of time, this will then lead to higher number of "*unnecessary quarantines*". These quarantines will incur economic cost for the individuals and the society. On the other hand, larger exposure zone means smaller safe zoon. This automatically reduces the chance of potentially exposed people who were undetected by the app. Since there is a probability that a percentage of these people are indeed exposed and possibly infected, larger exposure zone could help to limit "*undetected infections*" and avoid further retransmission of the virus throughout the society, especially during a high pandemic period.

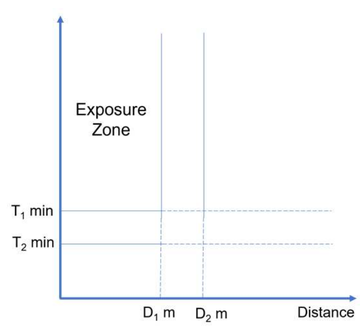

Fig. 2: Varying exposure zone by changing exposure time and distance thresholds

On the other hand, a smaller exposure zone would lead to a smaller number of exposure notifications and therefore lower number of unnecessary quarantines. However, a larger safe zone could translate to a larger number of people who had some exposure and received the infective dose through short contacts but not detected through Bluetooth proximity detection mechanism. This means there could be a greater number of undetected yet infected individuals who are unknowingly spreading the virus in their community.

This qualitative argument points to a trade-off between *unnecessary quarantines* and *undetected infections* based on the size of the exposure zone or equivalently the values of the exposure distance and time thresholds. In this paper, we provide an analysis that mathematically highlights this trade-off. The study is done through an enhanced agent-based simulation platform which was originally presented in [11]. Our analysis would allow optimization of the exposure thresholds based on factors such as the surrounding environment (e.g., indoor vs. outdoor), an individual's health, the severity of the outbreak in the community, transmissibility of the virus, etc.

The rest of this paper is organized as follows. A mapping of an individual's exposure time to the probability of his infection is proposed in Section II. Section III describes the agent-based simulation platform that has been developed to study potential exposures using the BLE-based proximity detection. Simulation results and trade-off analysis are provided in Section IV. Finally, conclusions and plans for future work are described in Section V.

II. MAPPING EXPOSURE TO INFECTION

An exposure notification by the contact tracing app doesn't necessarily mean infection for the exposed individual. To the best of the authors' knowledge, there are not sufficient statistical data to reliably estimate the probability of infection by an exposed individual (identified through automatic contact tracing). However, it is reasonable to assume that longer duration of exposure leads to higher probability of infection. This probability depends on several factors that ultimately determine whether the amount of inhaled virus by the exposed individual is greater than the infective dose. The "infective dose" is the amount of virus (viral particles) that an individual needs to be exposed before being infected. There are several studies to determine the approximate infective dose of COVID-19, although that number could vary due to the nature of a specific mutation or general health and body immunity (or genetics) of the exposed individual [12]. Additionally, it is extremely difficult (if at all possible) to determine the exact viral exposure through BLE-based proximity detection with an infected individual. The distribution of the viral particles may not be the same (i.e., uniform) in all directions around the infected person and could depend on his orientation and initial jetting of exhalations and also whether the person is wearing a mask.

To simplify our analysis, we assume a parameterized sigmoidal-shaped function as the probability of infection based on the duration of exposure. In practice, even a short encounter (i.e., contact) with an infected person (e.g., strong cough directly to the face) could indeed lead to the transmission of the virus to the exposed individual. The sigmoidal function can model such events with very low probability of occurrence. In addition, it captures high probability of infection through long exposure time. The monotonic increase in the probability of infection can be adjusted by parameters that control the slope of the rising function. The sigmoidal-shaped function is merely used to highlight the impact of the distance/time exposure thresholds on the trade-off between the numbers of unnecessary quarantines and undetected infections. We conjecture that the exact shape of this function doesn't impact our analysis and the general trade-off described in the rest of the paper.

Assume that the probability of infection ($P_{infect}(i,t)$) for individual i at time t based on his accumulative duration of exposure by time t can be expressed by the following equation:

$$P_{infect}(i,t) = \frac{1}{1+\left(\frac{Exp_i(t)}{K-Exp_i(t)}\right)^{-\lambda}} \quad (1)$$

where $K > 0$ is the duration of exposure that results in infection with certainty, $Exp_i(t) < K$ is the total time of exposure experienced by individual i up to time t, and $\lambda > 0$ is a parameter that controls sensitivity of the infection probability versus exposure time. Fig. 3 displays this function for various values of λ and K. To account for people who have natural immunity to the virus, a maximum value for this probability can be used. In other words, $P_{infect}(i,t)$ can be expressed as follows:

$$P_{infect}(i,t) = Min\left(\frac{1}{1+\left(\frac{Exp_i(t)}{K-Exp_i(t)}\right)^{-\lambda}},\ L\right) \quad (2)$$

where $L < 1$ denotes the maximum probability of infection due to any amount of exposure.

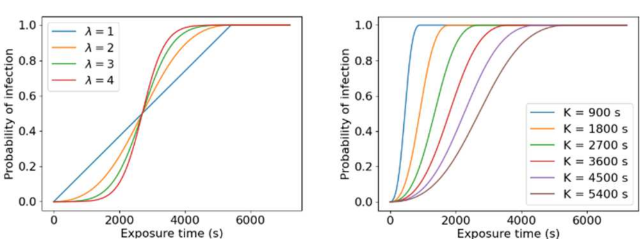

Fig. 3: Infection probability versus exposure time

To calculate the total time of exposure experienced by individual i in equation (2), the following methodology is used. Assume that the BLE signal measurements are done at a rate of $1/\Delta$, a non-stationary individual within the exposure zone may pick up incremental exposures every Δ seconds. With the hard distance/time exposure thresholds, the values of these incremental exposures depend on whether the individual is located within the distance threshold of the infected person(s). The total exposure time at $t+\Delta$ (i.e., $Exp_i(t+\Delta)$ depends on how the proximity of the individual i with respect to the infected person changed from time t to time $t+\Delta$. In general, this can be approximated by the following equation:

$$Exp_i(t+\Delta) = Exp_i(t) + \beta \times \Delta \quad (3)$$

where $0 \le \beta \le 1$ is a constant parameter indicating the impact of the possible individual movement from time t to $t+\Delta$. If β is close to one, it means that the individual stayed mostly within the exposure distance threshold during the time interval $[t, t+\Delta]$. Likewise, a β value close to zero means that the individual stayed mostly outside the exposure range during consecutive proximity measurements. Since no information is typically available on the movement of the individual from time t to $t+\Delta$, then in practice β is assumed to be 1 when both proximity measurements at times t and $t+\Delta$ indicate that the individual was within the exposure distance. Likewise, β is set to 0 when both proximity measurements indicate that the individual was outside the exposure range. When the two consecutive proximity measurements indicate that the individual has crossed over the exposure distance threshold, then the following three approaches can be followed to calculate the accumulative exposure: A) an aggressive approach where β is considered to be zero if the proximity measurement shows that the individual at time $t+\Delta$ is outside the exposure distance, or B) a conservative approach where β is considered to be 1 if the proximity measurement shows that the individual at time $t+\Delta$ is within the exposure distance, and C) an intermediate approach where β is taken to be 0.5.

For a moving individual if Δ is small, the difference between these approaches is again minimal. The three strategies could differ significantly when Δ becomes large relative to the speed of the individual. In practice, a small Δ means higher measurement and processing rate by the contact tracing application on the mobile phone. This could result in a higher battery consumption and therefore higher frequency of recharging which might not be desirable in practice.

III. Simulation Platform

In [11], we proposed an agent-based simulation platform to better understand the impact of Bluetooth proximity estimation error on automatic exposure determination. The platform considered people walking in a plaza, campus area, or neighborhood. The basic dynamics in the agent's mobility were based on the algorithms in [13]. In order to more accurately reflect agents' traffic in an open area, we have incorporated two enhancements in our platform. In the basic platform, all agents within the simulation would simply randomly walk towards a given goal. So, the first enhancement is to overcome the possibility of the agents becoming jammed or trapped against one another with no way to proceed during the simulation [14]. Jamming can occur when all the agents in a simulation have similar goals, such as all trying to reach the same area within the simulation field. To avoid jamming in our simulation and thus biasing the results, our platform periodically randomizes the goals of each agent.

The second enhancement is to allow some interaction between agents within the simulation. In other words, we allow an agent to start a "conversation" with another agent, which effectively immobilizes both agents for a random period of time. If two (or more) agents are within a certain proximity of each other, then, with a specified probability, a conversation is started. The length of this conversation is random variable with a truncated Normal distribution. In our simulations, we have chosen the standard deviation of this distribution to be 1 min while the mean is considered as a parameter. This enhancement enables us to better consider the possibility of exposure with face-to-face interactions among the agents.

Using this enhanced platform, we can track the true and estimated distances between any two moving agents at fixed time interval of size Δ. The estimated distance is calculated as the summation of the true distance plus an error which is due to the BLE proximity detection mechanism. In the BLE-based proximity detection mechanism, there is an underlying error in the process that converts the signal strength into distance. This error is due to the variation in propagation of the Bluetooth signal. The variation is caused by many factors such as the surrounding environments, phones positions and orientations relative to the individuals carrying them, antenna gain patterns of the phones, etc. Assuming a Lognormal pathloss distribution for the BLE channel with a Gaussian distributed shadowing and fading component with standard deviation σ, the distribution of the error in the estimated distance would be a function of (σ), pathloss exponent (n) and the true distance between the agents. This distribution is shown in equation (4) [11].

$$f_Y(y) = \frac{1}{\sqrt{2n}} \frac{10n}{\sigma\, log_e(10)} \frac{1}{(d_0 + y)} e^{-\frac{1}{2}\left(\frac{10n}{\sigma\, log_e(10)}\right)^2 \left(log\left(\frac{Y}{d_0}+1\right)\right)^2} \tag{4}$$

During a simulation, each healthy agent i maintains two parameters: (a) True Exposure Time ($T\_Exp_i(t)$), and (b) Estimated Exposure Time ($E\_Exp_i(t)$). The true exposure parameter keeps track of the total exposure time based on equation (3) using the true distance from infected agent(s). Likewise, the estimated exposure parameter shows the total exposure time when the estimated distance from infected agents is used to calculate exposure in equation (3). The true and estimated exposure parameters are updated every Δ seconds after incorporating the population dynamics in the simulation platform. These parameters are used to make exposure determination for all healthy agents during the length of a simulation. Comparison of the values of these parameters to the time and distance thresholds discussed in the previous sections will lead to 4 possible states for each agent including two types of errors in exposure determination.

A false positive exposure error occurs when the true exposure time of a heathy agent is less than the time threshold T_T shown in Fig. 1 while its estimated exposure time is higher than the time threshold T_T. Conversely, a false negative exposure error occurs when the agent's true exposure parameter is higher than T_T while the estimated exposure parameter shows the accumulated exposure time is less than T_T. The diagram shown in Fig. 4 describes possible states for an agent during the simulation and conditions for transitioning from one state to the other.

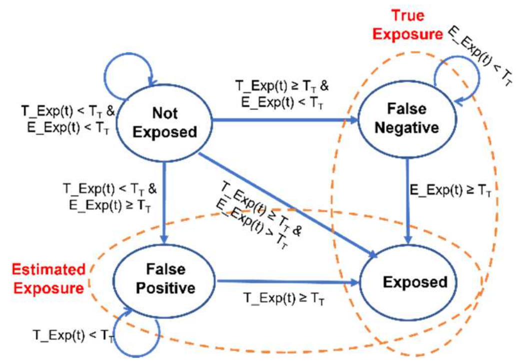

Fig. 4: State diagram of the agents in the simulation platform

IV. Simulation Results and Analysis

Assuming a fixed pathloss exponent of $n = 2$ for the BLE signal propagation, a fading standard deviation of $\sigma = 0,4$, and a proximity measurement frequency of $1/\Delta = 1$ Hz, extensive simulations have been done using the platform discussed in the previous section to investigate the impact of exposure thresholds (T_T , D_T) on the performance of the automatic contact tracing system. As discussed in previous sections, the metrics that we use to indicate this performance are the numbers of unnecessary quarantines versus undetected infected agents. The results presented in this paper consider a population of 135 agents moving within an area of size 162 m × 35 m for 8 hours (i.e., typical length of a workday). These numbers are chosen based on a standard laboratory building inside the campus area of the National Institute of Standards & Technology where the authors work. The number of infected individuals at the beginning of the simulation is set to 2% of the population. In addition, the probability of two agents starting a conversation once they are within one meter distance of each other is assumed to be 0.01 with an average conversation length of 3 min.

To simplify the analysis, we consider the impact of the exposure thresholds T_T and D_T separately as shown in Fig. 5. In our analysis, we consider the time threshold $T_T \in \{2, 3, 4, \dots, 23\}$ minutes while D_T is kept constant at 2 meters and distance threshold $D_T \in \{2, 2.25, 2.5, \dots, 4\}$ meters while T_T is set to 15 min. For each pair of T_T and D_T, simulations are executed using the platform discussed in the previous section.

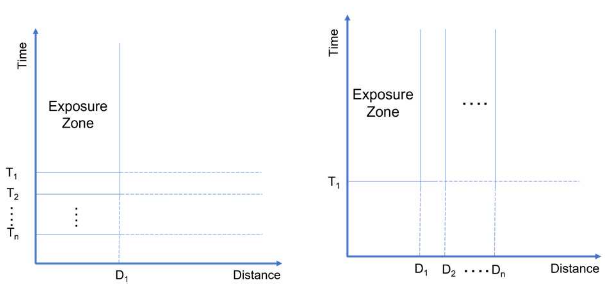

Fig. 5: Varying exposure thresholds T_T and D_T

The simulation platform provides the number of exposed and safe individuals that fall within the exposure and safe zones (as shown in Fig. 1) as well as the total exposure time for each agent at any given time. Based on this information, the average number of unnecessary quarantines (i.e., $N_{UQ}(t, T_T, D_T)$) and undetected infected agents $N_{UI}(t, T_T, D_T)$ at time t and under exposure thresholds T_T and D_T can be obtained by the following equations:

$$N_{UQ}(t, T_T, D_T) = N_{Exp}(t, T_T, D_T) - \sum_{i \in Exp.Zone} P_{infect}(i, t)\ , \quad (5)$$

$$N_{UI}(t, T_T, D_T) = \sum_{i \in Safe\ Zone} P_{infect}(i, t)\ . \quad (6)$$

Here $N_{Exp}(t, T_T, D_T)$ is the average total number of exposed individuals (i.e., the average number of people who have received exposure notifications by the contact tracing app) by time t, and $P_{infect}(i, t)$ is the infection probability of individual i at time t as defined in equation (2). Average numbers are obtained through multiple runs of simulations for each scenario.

It should be noted that for a given time t and distance threshold D_T, $N_{UQ}(t, T_T, D_T)$ is a decreasing function of T_T while at the same time, $N_{UI}(t, T_T, D_T)$ is an increasing function of T_T. Increasing the time threshold T_T will decrease the size of the exposure zone. A smaller exposure zone means a tighter constraint on exposure (or contact) identification which in turn leads to lower number of exposure detections (i.e., $N_{Exp}(t, T_T, D_T)$). However, the percentage of infections among the exposed individuals will be higher due to the higher average exposure time experienced by those individuals. This directly results in lower number of unnecessary quarantines or isolations as obtained by equation (5). On the other hand, a smaller exposure zone means a larger safe zone which translates to higher average exposure time for the individuals inside this larger zone (equivalent to higher probability of infections for those individuals). This results in higher number of potentially infected people that are unidentified or undetected by the automatic contact tracing app as indicated in equation (6). Figures 6 and 7 show these results for various time threshold $T_T \in \{2, 3, 4, \ldots, 23\}$ min while D_T=2 meters and $\sigma = 0$ and 4 respectively. Curve fitting with quadratic function has been used to obtain the solid curves representing the data points obtained through simulations. Also, a confidence interval of one standard deviation has been considered for the results presented in this section.

For $\sigma = 4$, the error in the estimated distance (due to Bluetooth proximity detection and modeled by equation (4)) will be considered for exposure calculation process. As mentioned in the previous section, this error leads to false positive and false negative exposure identification errors. False positive errors will lead to an increase in the number of unnecessary quarantines as observed in Fig. 7. At the same time, false negatives contribute to the number of undetected infections as these individuals have indeed been in contact with infected individuals for more than 15 minutes; however, they have not been detected by the Bluetooth proximity detection mechanism to be in the exposure zone. For the mobility pattern chosen for these simulations, and at $\sigma = 4$, the increase in the number of false negatives is small due to the reasons explained in [11].

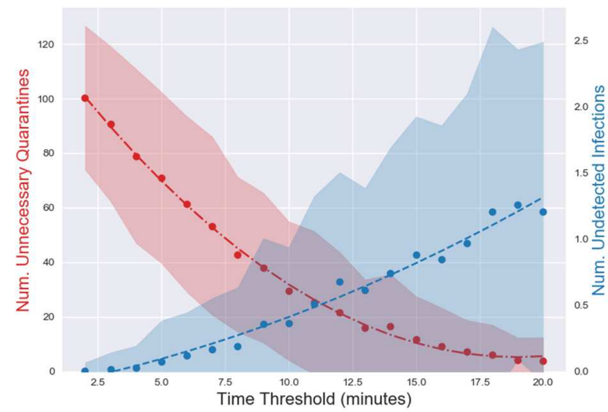

Fig. 6: Number of unnecessary quarantines and undetected infections for various time thresholds (K=5400 s, L=0.9, σ=0)

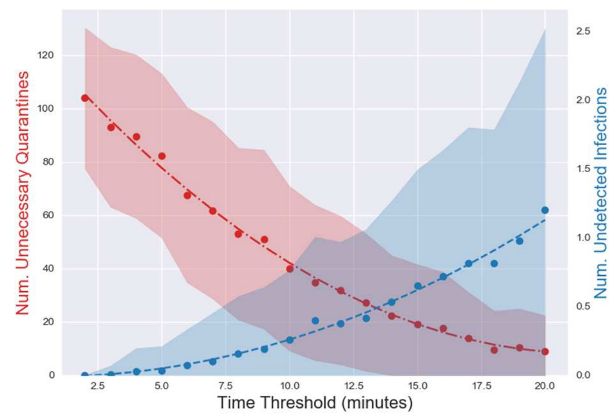

Fig. 7: Number of unnecessary quarantines and undetected infections for various time thresholds (K=5400 s, L=0.9, σ=4)

Similarly, for a given time t and time threshold T_T, $N_{UQ}(t, T_T, D_T)/\ N_{UI}(t, T_T, D_T)$ is an increasing/decreasing function of D_T respectively. This is described qualitatively by noticing that increasing the distance threshold D_T will increase the size of the exposure zone. Therefore, similar argument provided in the previous paragraph illustrates the monotonic increase/decrease of the N_{UQ} and N_{UI} performance metrics. Figures 8 and 9 show the unnecessary quarantines and undetected infections for various distance thresholds $D_T \in \{2, 2.25, 2.5, \ldots, 4\}$ when T_T=15 min and $\sigma = 0$ and 4 respectively. The mobility pattern considered for these results involves constant movements by the agents with occasional stops between two or more agents to emulate conversation. Due to this mobility pattern, agents will not experience sufficient exposure when $D_T < 2$ m, and the monotonic behavior of the undetected infection metric would not exist for small distance thresholds. The authors plan to further investigate the impact of mobility on the performance metrics described here.

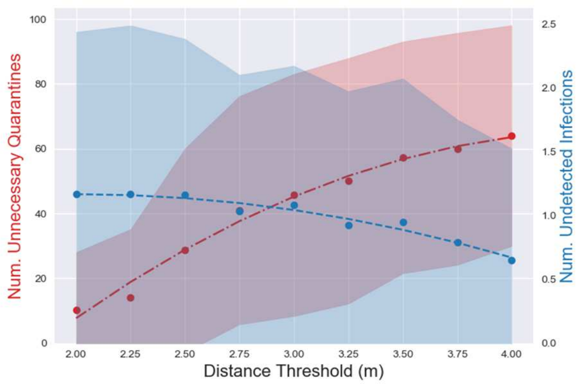

Fig. 8: Number of unnecessary quarantines and undetected infections for various distance thresholds (K=5400 s, L=0.9, σ=0)

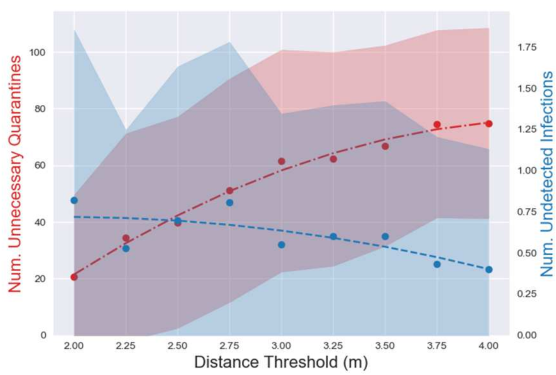

Fig. 9: Number of unnecessary quarantines and undetected infections for various distance thresholds (K=5400 s, L=0.9, σ=4)

V. Conclusions and Future Work

The objective of contact tracing is to accurately notify the right people (i.e., people who were truly exposed) to quarantine in a timely manner and let other individuals who were not exposed to function in the community as usual. In this way, not only the spread of the virus is better controlled but also the negative economic impacts of general public lockdowns are avoided or minimized. In this paper, we have highlighted the impact of time and distance thresholds on the performance of a Bluetooth-based contact tracing system using unnecessary quarantines and undetected infections as metrics. The results show an inherent trade-off between these two metrics. Using proper cost models, it would be possible to optimize the exposure notification system based on a variety of parameters such as environment, transmissibility of the virus, status of the pandemic, and general health or immunity level of the individual using the system. The authors plan to continue this research and further investigate this trade-off. In addition, using soft distance/time thresholds as proposed in [15] would more accurately models the exposure process and possible infections. The authors also plan to extend the current results with soft distance/time thresholds.

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