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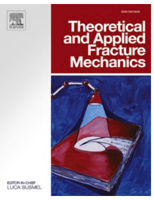

Assessment of macroscopically inhomogeneous fracture toughness data sets using the simplified and multimodal master curve methods

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ABSTRACT

A number of large, historical fracture toughness data sets have been analyzed according to the provisions of ASTM E1921-22a, in order to compare two Master Curve methods for assessing potential macroscopic material inhomogeneity: simplified method and multimodal approach. Analyses conducted on 51 data sets, 20 of which inhomogeneous, demonstrated substantial equivalence between the two approaches, even below the current applicability limit of the multimodal method (20 data points). A revised (lower) limit might be considered for future revisions of the E1921 standard. Overall, the simplified method provides slightly more conservative assessments, while the multimodal approach is more accurate. Freeware tools are currently available for the application of both methodologies, and of ASTM E1921 in general, for the analysis of fracture toughness data obtained in the ductile-to-brittle transition regime.

1. Introduction

The Master Curve (MC) methodology was developed in the 1980 s [1] to statistically analyze fracture toughness test results obtained in the ductile-to-brittle transition regime, where varying amounts of stable and unstable crack propagation can occur. The outcome consists in the determination of a reference temperature, T_0, which characterizes the fracture toughness of ferritic steels experiencing elastic or elastic–plastic instabilities due to cleavage cracking.

The weakest-link theory [2], applied to a three-parameter Weibull distribution of fracture toughness values, K_{Jc}, is used to characterize the statistical effects of specimen size on fracture toughness in the transition regime, while enforcing a limit on K_{Jc} values to ensure high constraint along the crack front when fracture occurs.

Once T_0 is established, the median fracture toughness of the material for a standard specimen of 1 in. or 25.4 mm thickness is represented by a fixed-shaped curve (the so-called Master Curve), such that at T_0 the median fracture toughness $K_{Jc,med} = 100\ \text{MPa}\sqrt{\text{m}}$. From a structural integrity and safety perspective, tolerance bounds can be established corresponding to low fracture probabilities, such as 5 % or 2 %. The standard deviation of the data distribution is a function of the Weibull slope and $K_{Jc,med}$.

The MC procedure was first standardized by the American Society for Testing and Materials (ASTM International) in 1997, and has undergone multiple revisions up to present (the current version, at the time of writing, is ASTM E1921-22a [3]).

Until the mid-2010 s, the procedure standardized by ASTM E1921 was based on the assumption that the material could be considered macroscopically homogeneous, in that its tensile and toughness properties could be classified as homogeneous. If a macroscopically inhomogeneous material was analyzed by the standard MC procedure, a nonconservative and often inaccurate estimate of T_0 could be obtained and the confidence bounds could also become nonconservative. Examples of inhomogeneous materials are multi-pass weldments, thick-section steels that exhibit a gradient of mechanical properties through their thickness, or data sets obtained by combining test results from different heats of the same steel.

Investigations aimed at modifying the MC procedure to account for macroscopic inhomogeneities were conducted since the beginning of the 21st century [4], and led to incorporating provisions in ASTM E1921 for identifying (screening criterion) and assessing (simplified method, bimodal approach, multimodal approach) macroscopically inhomogeneous materials. The first edition of E1921 that included such provisions was ASTM E1921-19, where Appendix A5 (“Treatment of Potentially Inhomogeneous Data Sets”) was added to provide the user with statistical methods that can be applied when data sets fail the screening criterion contained in the main body of the standard (section 10.6.3).

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Nomenclature

ASTM	American Society for Testing and Materials
β	sample size uncertainty factor
$K_{CENS,i}$	toughness value used in the screening criterion of ASTM E1921 section 10.6.3, $MPa\sqrt{m}$
K_{Jc}	fracture toughness at cleavage in terms of stress intensity factor, $MPa\sqrt{m}$
$K_{Jc,i}$	fracture toughness measured in the i-th test, $MPa\sqrt{m}$
$K_{Jc,med}$	median fracture toughness, $MPa\sqrt{m}$
$K_{Jc(0.02)}$	fracture toughness corresponding to the 2 % lower confidence bound, $MPa\sqrt{m}$
MC	Master Curve
MLNH	parameter used to quantify the likelihood that the data set is inhomogeneous, according to the multimodal method
MM	Multimodal Method
N	number of data points within $T_0 - 50\,^\circ C \le T \le T_0 + 50\,^\circ C$ in a data set, to be used in homogeneity assessments
N_{tests}	overall number of data points in a data set, independent of the test temperature
PCCv	fatigue precracked Charpy-type specimen
r	number of uncensored data points in a data set
S	cumulative survival probability in the multimodal method
SCI	Screening Criterion Index
SM	Simplified Method
σ_{exp}	experimental uncertainty of the reference temperature (assumed equal to 4 °C)
σ_{Tm}	uncertainty of T_m, °C
T_i	test temperature for the i-th specimen tested, °C
T_m	multimodal reference temperature for an inhomogeneous data set, °C
T_m*	margin-adjusted multimodal reference temperature for an inhomogeneous data set, °C
T_0	Master Curve reference temperature, °C (corresponding to $K_{Jc,med} = 100\ MPa\sqrt{m}$ for specimens of 1 in. = 25.4 mm thickness)
$T_{0,hom}$	reference temperature obtained from the standard Master Curve analysis for a homogeneous material, °C
$T_{0,i}$	single-data T_0 estimate, used in the simplified method for $N < 10$, °C
T_{0IN}	reference temperature for an inhomogeneous data set, calculated with the simplified method, °C
$T_{0,\max}$	maximum value of $T_{0,i}$, used in the simplified method for $N < 10$, °C
$T_{0,scrn}$	screening reference temperature value obtained from the screening procedure, °C
$T_{0,stepn}$	reference temperature obtained from the n-th step of the screening procedure, °C
XTC(T)	Compact Tension specimen with X in. thickness
XTSE(B)	Single-Edge Bend specimen with X in. thickness

2. Treatment of inhomogeneous data sets according to ASTM E1921

2.1. Screening criterion

To assess the possibility of a data set being macroscopically inhomogeneous, the SINTAP approach [5] is recommended, as summarized below.

1. For every test result in the data set, the value $K_{CENS,i}$ is calculated as:

$$K_{CENS,i} = 30 + 70exp\left[0.019\left(T_i - T_{0,hom}\right)\right] \tag{1}$$

where T_i is the test temperature of the i-th test, and $T_{0,hom}$ is the reference temperature for the data set obtained under the assumption of macroscopic homogeneity (standard MC analysis). Temperatures are in °C and fracture toughness values are in $MPa\sqrt{m}$.

2. After size correction and censoring for constraint loss and excessive ductile crack extension, $K_{Jc,i}$ and $K_{CENS,i}$ values are compared, and $K_{JC,i}$ is replaced by $K_{CENS,i}$ if $K_{JC,i} \ge K_{CENS,i}$.

3. The revised data set is analyzed using the homogeneous approach, and a new value of reference temperature, $T_{0,step2}$, is obtained.

4. If $T_{0,step2} \ge T_{0,hom} + 0.5$ °C, steps 1–3 above are repeated, after replacing $T_{0,hom}$ in Eq. (1) with $T_{0,step2}$.

5. This iterative process continues until the difference between T_0 values calculated from two successive iterations becomes smaller than 0.5 °C. When this happens, the highest calculated $T_{0,stepn}$ is defined as $T_{0,scrn}$.

According to the E1921 screening criterion, the data set is considered macroscopically homogeneous if the following inequality holds:

$$T_{0,scrn} - T_{0,hom} \le 1.44\sqrt{\frac{\beta^2}{r}} \tag{2}$$

where β is a sample size uncertainty factor that depends on $K_{Jc,med}$ and r is the number of uncensored data in the data set.

If the inequality in Eq. (2) is violated, the data set may be representative of a macroscopically inhomogeneous material, and the E1921 user is encouraged to use the statistical procedures in Appendix X5 to more accurately (and conservatively) characterize the material.

2.2. Simplified method (SM) to characterize Material's inhomogeneity

If the data set fails the screening criterion in Eq. (2) and includes<20 data points ($N < 20$), a simplified method [5,6] is proposed to determine a generally conservative estimate of the material's reference temperature, T_{0IN}, which can be used in lieu of $T_{0,hom}$.

Within the range $N < 20$, a further distinction between $N < 10$ and $N \ge 10$ is made, as detailed below.

If $N < 10$, a single-data T_0 estimate, $T_{0,i}$, is calculated for every non-censored data point as:

$$T_{0,i} = T_i - \frac{1}{0.019} ln\left[\frac{\left(K_{Jc,i} - 20\right)N^{0.25} - 10}{70}\right] \tag{3}$$

The maximum value of $T_{0,i}$ is defined as $T_{0,\max}$, and must be compared with $T_{0,scrn}$ calculated during the screening phase. If:

$$T_{0,max} - T_{0,scrn} > 8^{\circ}C \tag{4}$$

then $T_{0,\max}$ is the conservative estimate of the reference temperature for the inhomogeneous data set, T_{0IN}. Otherwise, $T_{0IN} = T_{0,scrn}$.

If $10 \le N < 20$, it is assumed that $T_{0IN} = T_{0,scrn}$.

When using the simplified method, confidence bounds are obtained by replacing $T_{0,hom}$ with T_{0IN} in the corresponding formulae.

2.3. Advanced statistical methods to characterize potentially inhomogeneous large data sets

For data sets including at least 20 test results, two more advanced statistical approaches [4] are proposed in Appendix X5 of ASTM E1921:

- The *Bimodal Method*, which applies to data sets containing two toughness distributions, such as heat-affected zones.

- The *Multimodal* (MM) *Method*, which applies to data sets containing randomly distributed toughness populations, such as heterogeneous ferritic steels.

This study did not consider the bimodal method at all, and only focused on the multimodal approach. It was therefore assumed that the inhomogeneity of several examined data sets was always of multimodal (random) nature.

Analytical details of the MM approach will not be provided here, as they can be found in Appendix X5 of ASTM E1921. The most relevant features of this method are the following.

(a) The MM distribution is fully defined by the mean reference temperature of all toughness populations, T_m, and its standard deviation around the mean, σ_{Tm}.
(b) Both T_m and σ_{Tm} are determined by maximizing the logarithm of a likelihood function.
(c) A parameter is defined, *MLNH*, to quantify the likelihood that the data set is potentially inhomogeneous:

$$MNLH = \frac{\sigma_{Tm}}{\sqrt{\frac{\beta^2}{r} + \sigma^2_{exp}}} \tag{5}$$

Where β and r have been defined before, and $\sigma_{exp} = 4$ °C is the experimental uncertainty of the reference temperature. A data set is considered potentially homogeneous when $MNLH > 2$.

If $\sigma_{Tm} < 10$ °C, the confidence of *MLNH* to correctly identify a data set as inhomogeneous is poor.

(d) The tolerance bounds of the MM distribution do not have an exact analytical expression, but must be established using numerical methods to evaluate an infinite integral.

3. Data sets investigated and analyses performed

Ten large publicly available data sets, each including between 55 and 734 fracture toughness test results and corresponding to eight different ferritic steels, were analyzed in this study. These are the same data sets for which similar MC analyses were performed in a recent publication [7], to which this author collaborated. From each data set, several sub-sets were extracted, corresponding to different specimen configurations and/or sizes, or specific participants in the case of round-robins. The total number of data sets and sub-sets considered in this study is 51 (Table 1). Details of each data set are provided in the references listed in the last column of the table.

In Table 1, the number of fracture toughness results available in each data set is indicated by N_{tests}, while N corresponds to the number of data points with temperatures within the range $T_0 - 50\ °C \leq T \leq T_0 + 50\ °C$, which must be included in the analyses according to ASTM E1921. Tests conducted at temperatures below $T_0 - 50$ °C are assumed to represent lower shelf fracture behavior, where the weakest-link assumption is invalid and the Weibull distribution does not apply. Conversely, tests above $T_0 + 50$ °C are associated to substantially upper shelf behavior.

Many of the data sets listed in Table 1 were generated before the MC methodology was even formulated. Specifically, these data sets were obtained with no particular effort to target a temperature range close to the middle of the transition region. It is not surprising, therefore, that N (to be used in the selection of the inhomogeneous analysis approach to be used) is often significantly lower than N_{tests}. For the 10 data sets combined, $N_{tests} = 1996$ and $N = 977$ (48.9 %). The overall number of uncensored data, r, is 851, or 87.1 % of N.

Incidentally, three different and inconsistent definitions of N can be found in the current version of ASTM E1921 [3]: "*number of specimens tested*" (10.2.4, X5.3.2.3, and X5.3.3.3), "*total number of censored and uncensored data*" (10.4), and "*total number of K_{Jc} values in the data set*" (X5.2.1.1). Based on the discussion above, only the second definition appears accurate, while the remaining two are misleading and should be corrected in the next revision of the standard. In this study, N is defined as the number of specimens tested within ± 50 °C of the calculated reference temperature (homogeneous or inhomogeneous).

Table 1
Fracture toughness data sets analyzed in this study.

Data set	Sub-set	N_{tests}	N	r	References
72 W unirradiated	All tests1TC	77	44	44	[8]
	(T)2TC	34	23	23	
	(T)4TC	20	15	15	
	(T)	16	6	6	
72 W irradiated	All tests1TC	56	16	16	
	(T)2TC	29	9	9	
	(T)	19	6	6	
73 W unirradiated	All tests1TC	80	55	54	
	(T)2TC	38	30	29	
	(T)	20	15	15	
	4TC(T)-8TC(T)	22	10	10	
73 W irradiated	All tests1CT	55	19	19	
	(T)2TC	29	20	16	
	(T)4TC	18	5	5	
	(T)	8	5	5	
Ingham et. al	All tests1TSE	216	52	36	[9]
	(B)	70	21	21	
	, 1st set1TSE	57	8	7	
	(B)	61	5	5	
	, 2nd set2TSE	30	6	6	
	(B)4TSE	172	24	24	
	(B)	44	28	10	
	All exc. PCCv				
	PCCv				
EURO	All tests	734	278	265	[10]
	All exc. SX9	692	314	287	
	SX9 block	42	37	32	
JSPS Round-Robin	All tests	116	85	85	[11]
Midland 1 Weld irradiated	All testsMC	111	63	40	[12–16]
	(T)	51	51	31	
	PCCv	18	18	16	
	0.5TC(T)-1TC	42	25	22	
	(T)	13	13	8	
	Lab A	13	11	5	
	Lab B	12	12	8	
	Lab C	13	13	10	
	Lab D				
Plate 13A	All tests1TSE	340	154	124	[17]
	(B)2TSE	70	21	21	
	(B)4TSE	61	61	28	
	(B)	30	6	6	
	PCCv0.5TC	44	28	12	
	(T)1TC	38	20	20	
	(T)2TC	54	54	50	
	(T)4TC	26	12	12	
	(T)	6	6	6	
CRIEPI Round-Robin	All tests	211	211	168	[18]
	Lab 1	58	58	41	
	Lab 2	20	20	17	
	Lab 3	32	32	28	
	Lab 4	22	22	18	
	Lab 6	10	10	9	
	Lab 7	24	24	15	
	Lab 8	29	29	25	

On each data set listed in Table 1, the following analyses were performed, irrespective of the homogeneous or inhomogeneous outcome of the screening criterion:

(a) Standard MC analysis ($T_{0,hom}$).
(b) Application of the screening criterion, Eq. (2).
(c) Calculation of T_{0IN} according to the simplified method.
(d) Multimodal inhomogeneity analysis (T_m, σ_{Tm}, *MLNH*).

The analysis results are summarized in Table 2. Note that, while T_{0IN} is a conservative estimate of the reference temperature for an inhomogeneous data set, T_m is the mean reference temperature for the multiple

Table 2

*Results of the analyses conducted on the data sets/sub-sets listed in Table 1. Data sets with $N \geq 20$ are in **bold**.*

Data set	Sub-set	N	$T_{0,hom}$ (°C)	Screening criterion	T_{0IN} (°C)	T_m (°C)	σ_{Tm} (°C)	T_m^*(°C)	*MLNH*
72 W unirradiated	**All tests**	**44**	**−58.8**	**HOM**	**−55.8**	**−59.0**	**1.18**	**−59.0**	**0.244**
	1TC(T)2TC	**23**	**−56.6**	**INHOM**	**−50.5**	**−54.9**	**8.21**	**−54.6**	**1.497**
	(T)4TC	15	−61.5	HOM	−58.8	−61.7	1.17	−61.7	0.191
	(T)	6	−59.5	HOM	−46.5	−59.7	0.10	−59.7	0.012
72 W irradiated	All tests1TC	16	10.6	INHOM	21.0	17.4	14.0	22.9	2.325
	(T)2TC	9	4.0	INHOM	20.6	14.4	19.3	26.0	2.676
	(T)	6	26.6	HOM	26.6	26.4	0.1	26.4	0.012
73 W unirradiated	**All tests**	**55**	**−60.4**	**HOM**	**−59.6**	**−59.8**	**5.4**	**−59.6**	**1.150**
	1TC(T)2TC	**30**	**−59.2**	**HOM**	**−59.2**	**−59.4**	**0.8**	**−59.4**	**0.153**
	(T)	15	−61.8	HOM	−57.2	−60.9	5.8	−60.7	0.938
	4TC(T)-8TC(T)	10	−61.4	HOM	−58.8	−66.4	1.2	−66.4	0.170
73 W irradiated	All tests	19	33.3	HOM	34.3	34.0	5.4	34.2	0.939
	1TC(T)2TC	**20**	**38.3**	**HOM**	**40.0**	**39.5**	**0.1**	**39.5**	**0.017**
	(T)4TC	5	20.4	HOM	24.2	20.2	0.1	20.2	0.011
	(T)	5	29.9	HOM	29.9	29.6	0.1	29.6	0.011
Ingham et. al	**All tests**	**52**	**−104.9**	**INHOM**	**−96.2**	**−104.3**	**7.3**	**−104.1**	**1.460**
	1TSE(B),1st 1TSE(B),2nd	**21**	**−104.8**	**HOM**	**−102.8**	**−104.9**	**1.2**	**−104.9**	**0.205**
	2TSE(B)	8	−105.0	HOM	−105.0	−105.2	0.1	−105.2	0.013
	4TSE(B)	5	−101.0	HOM	−101.0	N/A	N/A	N/A	N/A
	All exc. PCCv	6	−91.6	HOM	−91.6	−91.8	0.03	−91.8	0.004
	PCCv	**24**	**−103.9**	**HOM**	**−102.0**	**−102.7**	**5.4**	**−102.5**	**0.994**
		28	**−108.2**	**INHOM**	**−90.2**	**−118.0**	**35.1**	**−88.4**	**5.045**
EURO	**All tests**	**278**	**−91.3**	**INHOM**	**−86.2**	**−87.2**	**10.7**	**−85.9**	**2.578**
	All exc. SX9	**314**	**−88.4**	**INHOM**	**−82.6**	**−85.5**	**8.7**	**−85.2**	**2.102**
	SX9 block	**37**	**−106.0**	**HOM**	**−102.8**	**−103.9**	**9.4**	**−103.6**	**1.837**
JSPS Round-Robin	**All tests**	**85**	**−106.2**	**HOM**	**−106.2**	**−106.2**	**0.1**	**−106.2**	**0.022**
Midland 1 Weld irradiated	**All tests**	**63**	**15.2**	**INHOM**	**36.4**	**25.3**	**33.8**	**30.9**	**6.885**
	MC(T)	**51**	**15.3**	**INHOM**	**36.3**	**20.2**	**41.9**	**37.0**	**8.004**
	PCCv	18	30.6	INHOM	42.7	37.7	19.1	49.1	3.095
	0.5 T-1TC(T)	**25**	29.0	**INHOM**	**35.0**	**31.5**	**12.5**	**35.3**	**2.255**
	Lab A	13	34.9	HOM	30.3	34.7	0.1	34.7	0.012
	Lab B	11	35.0	HOM	45.0	N/A	N/A	N/A	N/A
	Lab C	12	14.7	INHOM	27.7	18.7	34.9	25.8	4.280
	Lab D	13	34.8	INHOM	52.7	45.7	37.0	55.7	4.927
Plate 13A	**All tests**	**154**	**−95.8**	**INHOM**	**−85.9**	**−92.7**	**12.4**	**−89.0**	**2.874**
	1TSE(B)	**21**	**−104.8**	**HOM**	**−103.2**	**−104.9**	**1.2**	**−104.9**	**0.214**
	2TSE(B)4TSE	**61**	**−101.0**	**HOM**	**−101.0**	**N/A**	**N/A**	**N/A**	**N/A**
	(B)	6	−91.6	HOM	−91.6	−91.8	0.03	−91.8	0.004
	PCCv	**28**	**−107.4**	**INHOM**	**−91.6**	**−111.9**	**26.9**	**−108.9**	**4.102**
	0.5TC(T)	**20**	**−70.1**	**INHOM**	**−58.9**	**−78.7**	**5.8**	**−78.5**	**1.022**
	1TC(T)2TC	**54**	**−82.2**	**INHOM**	**−74.5**	**−64.2**	**17.3**	**−60.7**	**3.649**
	(T)4TC	12	−86.6	HOM	−84.3	−85.4	0.1	−85.4	0.015
	(T)	6	−84.6	HOM	−80.2	−84.8	0.1	−84.8	0.012
CRIEPI Round-Robin	**All tests**	**211**	**−102.9**	**INHOM**	**−99.9**	**−102.0**	**12.7**	**−97.9**	**3.008**
	Lab 1	**58**	**−105.9**	**INHOM**	**−99.3**	**−104.3**	**19.4**	**−97.8**	**3.916**
	Lab 2	**20**	**−99.3**	**INHOM**	**−92.1**	**−97.0**	**15.1**	**−96.5**	**2.547**
	Lab 3	**32**	**−102.9**	**HOM**	**−102.9**	**−103.1**	**0.1**	**−103.1**	**0.019**
	Lab 4	**22**	**−104.0**	**HOM**	**−104.0**	**−104.7**	**0.1**	**−104.7**	**0.017**
	Lab 6	10	−94.1	HOM	−88.9	−97.8	0.1	−97.8	0.013
	Lab 7	**24**	**−111.0**	**HOM**	**−110.7**	**−111.4**	**11.2**	**−109.4**	**1.778**
	Lab 8	**29**	**−99.2**	**HOM**	**−99.2**	**−99.0**	**5.7**	**−98.8**	**1.047**

toughness populations represented in the data set. For T_m to be directly comparable to T_{0IN} (and $T_{0,hom}$), a margin adjustment must be added, in accordance with section X5.3.3.7 of ASTM E1921. This temperature shift is a function of σ_{Tm}, and is negligible when $\sigma_{Tm} \leq 10$ °C. This margin-adjusted multimodal reference temperature is designated as T_m^* in Table 2.

Analyses were performed using a spreadsheet-based software suite developed at NIST by the author [19], except for the MM calculations, which were conducted by means of the open code T0TEM (T_0 Test Evaluation Module – Vers. 1.5), developed by NASA [20] and explicitly mentioned in the current version of E1921. T0TEM, unlike the NIST software, accommodates an unlimited number of data points, and allows running the MM analysis even on a homogeneous data set, by selecting the option "Mesh Calculation", which solves the multimodal equations using an iterative mesh. Note that MM calculations failed for three data sets.

In the following sections, the outcomes of the homogeneous, simplified, and multimodal analyses will be compared for both homogeneous and inhomogeneous data sets.

4. Results of analyses performed

4.1. Detection of potential inhomogeneity

According to ASTM E1921, a data set of any size can be examined for possible potential inhomogeneity by verifying the screening criterion in Eq. (2). Another inhomogeneity criterion, based on the MM approach and only applicable if $N \geq 20$, is $MNLH > 2$.

It is possible to define a parameter similar to *MNLH*, which we will call *SCI* (Screening Criterion Index), given by:

$$SCI = \frac{T_{0,scrn} - T_{0,hom}}{1.44\sqrt{\frac{\beta^2}{r}}} \tag{6}$$

Based on Eq. (2), a data set can be considered macroscopically inhomogeneous if $SCI \geq 1$.

Thirty-one of the 51 data sets investigated (60.8 %) screened homogeneous according to Eq. (2), or $SCI < 1$. Excluding the three data sets for which MM results were not available, the remaining 28 data sets were also classified as homogeneous based on the MM approach ($MLNH \leq 2$).

Of the remaining 20 data sets for which $SCI \geq 1$, 17 (85 %) were also recognized inhomogeneous based on the MM approach ($MLNH > 2$). The remaining three [Ingham 2TSE(B), Midland 1 Weld Lab B, and plate 13A 2TC(T)] screened inhomogeneous according to Eq. (2), but yielded $MLNH < 2$. All had enough data points for the MM approach to be legitimately used ($N \geq 20$). There were no cases where a data set screened homogeneous based on *SCI* and inhomogeneous according to *MLNH*. All this suggests that *SCI* is slightly more restrictive, or conservative, than *MLNH*.

As could be expected, a strong proportional correlation (Pearson's coefficient = 0.81) was found between *SCI* and *MLNH*, as shown in Fig. 1. The considerable agreement between the two screening indices does not appear to depend on the size of the data set.

4.2. Assessment of homogeneous data sets

As mentioned above, 31 of the 51 investigated data sets screened homogeneous according to Eq. (2) or $SCI < 1$. For a homogeneous data set, it is expected that all the reference temperatures that can be calculated ($T_{0,hom}$, T_{0IN}, T^*_m) should be within a few °C, at least for large data sets ($N \geq 20$). For smaller data sets ($N < 20$), one could envisage a few T^*_m values displaying larger deviations, due to the uncertainties of the MM calculations applied to small data sets. However, this is not what the data illustrated in Fig. 2 show: all T_m values were found to be within ± 5 °C of $T_{0,hom}$, irrespective of *N*. Conversely, differences>10 °C were obtained for a couple of T_{0IN} values, with the simplified method providing higher (i.e., more conservative) values than the homogeneous analysis. In the case of data sets including at least 20 data points, all calculated reference temperature values fall within a narrow ± 5 °C band.

The relationship between homogeneous and MM analyses is also depicted in Fig. 3, which shows the difference between T^*_m and $T_{0,hom}$ as a function of the number of data points in the set. Positive differences correspond to $T^*_m > T_{0,hom}$ (MM conservative), negative difference to $T^*_m < T_{0,hom}$ (MM non–conservative). The two extreme negative values correspond to $N = 10$. For $N > 10$, the largest recorded difference is 2.4 °C (MM conservative).

4.3. Assessment of inhomogeneous data sets

For the 20 macroscopically inhomogeneous data sets, the main expectation is that both T_{0IN} and T^*_m are higher (more conservative) than $T_{0,hom}$. As far as a direct comparison between T_{0IN} and T^*_m goes, the former is expected to be generally higher, particularly for $N < 20$, where the MM approach is not considered to be accurate.

The margins of conservatism for the two inhomogeneity methodologies are plotted in Fig. 4 as a function of data set size. We observed that:

- For the simplified method, T_{0IN} is always more conservative (higher) than $T_{0,hom}$, irrespective of *N*. Margins range between 1.6 °C and 21.2 °C. Although a slight tendency for the margin to decrease as the size of the data set increases can be observed, due to pronounced scatter the slopes of the fitting lines in Fig. 4 are not statistically different from zero, as shown by statistical *t*-tests at the 95 % confidence level.
- For the MM approach, only one negative (non-conservative) difference was recorded, −8.4 °C, for a data set with $N = 20$. This corresponds to one of the three data sets for which *SCI* and *MLNH* provided conflicting results. All the T^*_m values for small data sets were found to be higher (more conservative) than $T_{0,hom}$.
- On average, SM is slightly conservative with respect to MM, and differences tend to vanish for larger sample sizes.

In Fig. 4, black filled symbols indicate the three data sets screened as inhomogeneous by *SCI* and homogeneous by *MLNH*.

4.4. Comparison of lower bound confidence bounds

Lower bound curves, corresponding to low failure probabilities (for example, 2 %), are extremely important from a structural integrity

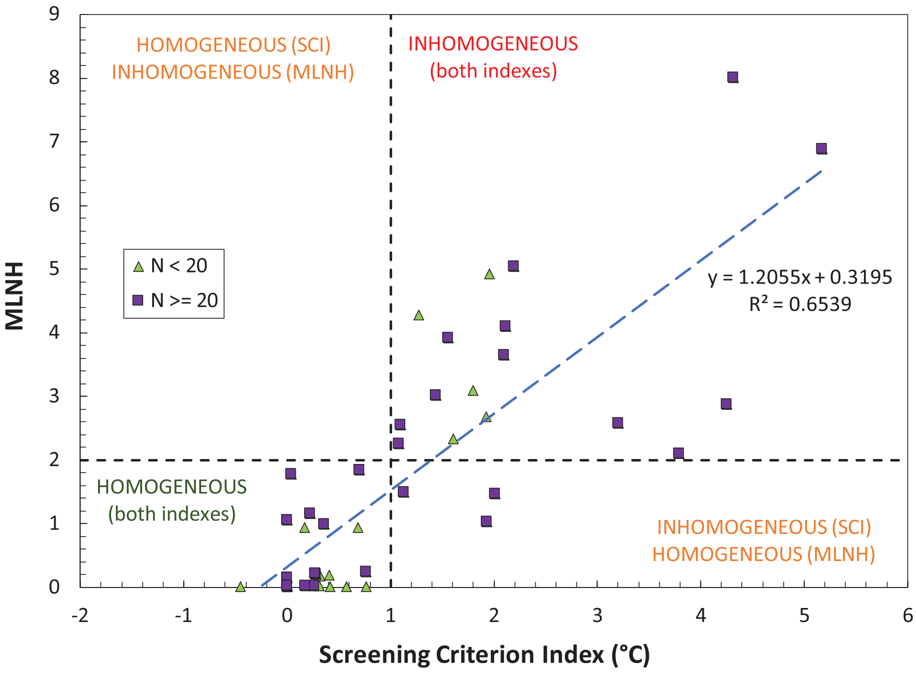

Fig. 1. Relationship between SCI and MLHN for the data sets investigated in the study. The dashed line is a fit of MLNH vs. SCI, and it can be observed that is slope is relatively close to unity.

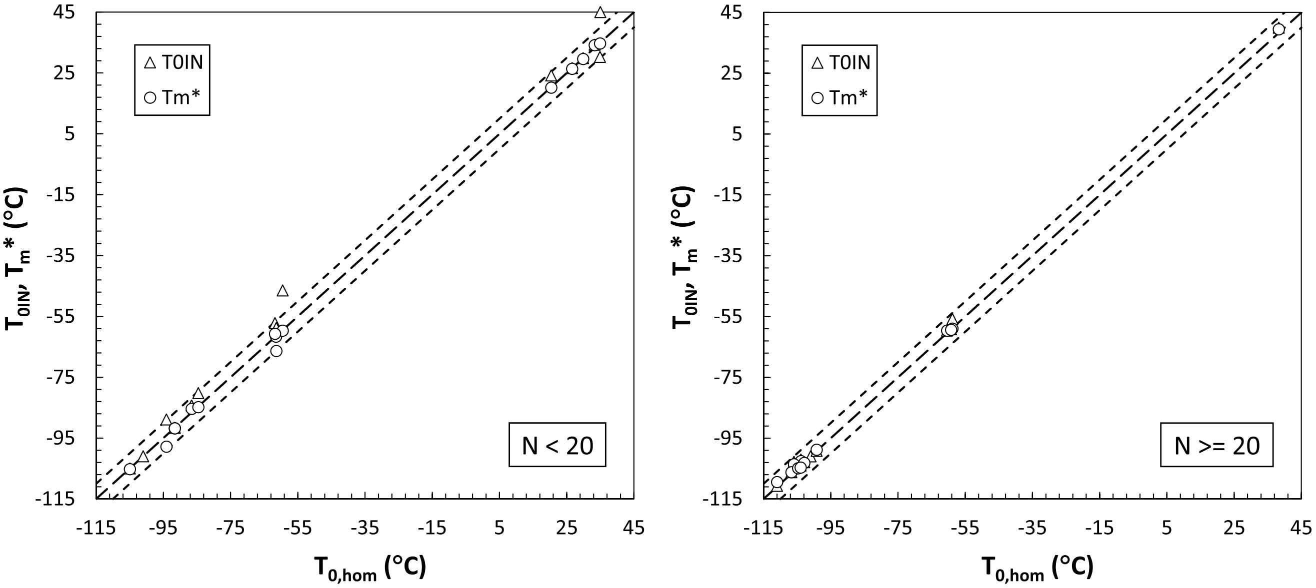

Fig. 2. Comparison between $T_{0,hom}$, T_{0IN}, and T_m^* for homogeneous data sets. Left side: $N < 20$, right side: $N \geq 20$. Dotted lines correspond to ± 5 °C with respect to the 1:1 equality line.

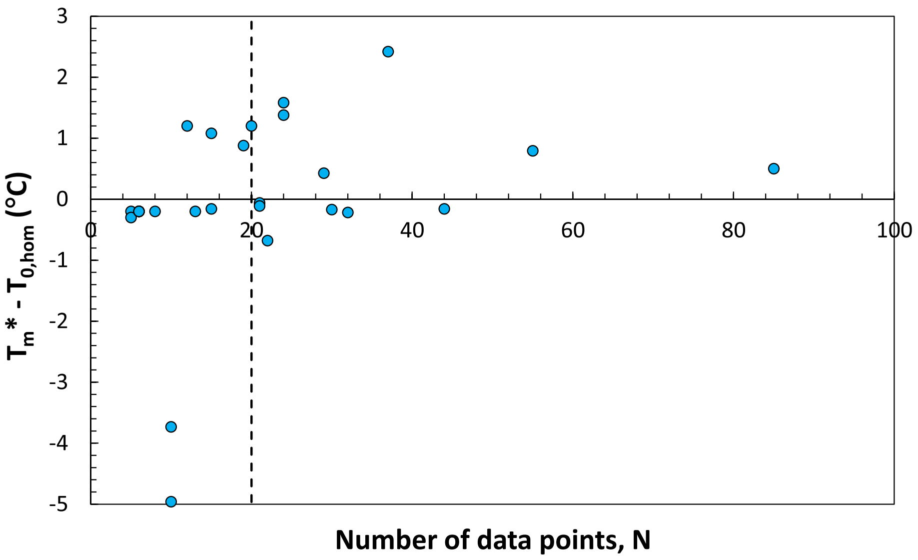

Fig. 3. Differences between multimodal (margin-adjusted) and homogeneous reference temperatures for homogeneous data sets as a function of data set size.

perspective. Their reliability can be assessed by comparing the theoretical failure probability (2 % in this study) with the percentage of experimental data points falling below the specific lower bound curves.

For the standard (homogeneous) analysis, the 2 % confidence bound is given by:

$$K_{Jc(0.02)} = 20 + \left[ln\left(\frac{1}{1-0.02}\right)\right]^{0.25}\left[11 + 77 \bullet e^{0.019\left(T - T_{0,hom}\right)}\right] \quad (7)$$

If a different lower bound is desired (for example, 5 %), it's sufficient to replace 0.02 with 0.05 in Eq. (7) above.

In the case of an inhomogeneous data set, the 2 % lower confidence bound for the simplified method can be obtained from Eq. (7) by simply replacing $T_{0,hom}$ with T_{0IN}. As for the MM method, as already mentioned, there is no analytical expression for the confidence bounds. These are established by solving the following equation:

$$S = 1 - 0.\text{xx} \quad (8)$$

where S is the cumulative survival probability at a level of 0.xx (in this study, 0.xx = 0.02). Providing the analytical expression of S is unnecessary here, but the reader is referred to Eq. (X5.23) of ASTM E1921–22a [3].

For every one of the 20 data sets screened inhomogeneous in this investigation, we compared the actual 1 T-normalized K_{Jc} test results with 2 % lower confidence bounds for the homogeneous analysis, the simplified method, and the MM approach. An example is shown in Fig. 5 for one of the largest data sets investigated here (Plate 13A, all tests, $N = 154$).

The percentage of data points falling below the 2 % confidence bounds (dotted lines in Fig. 5) is shown in Table 3 for each of the 20 inhomogeneous data sets. In the table, percentages larger than 2% are marked in **bold**, while the lowest percentage for each data set is presented in *italic*. Percentages are also shown in Fig. 6 as a function of the number of data points in the data set, N. Note that, for the two largest data sets (EURO and EURO without SX9 block), 2 % confidence bounds

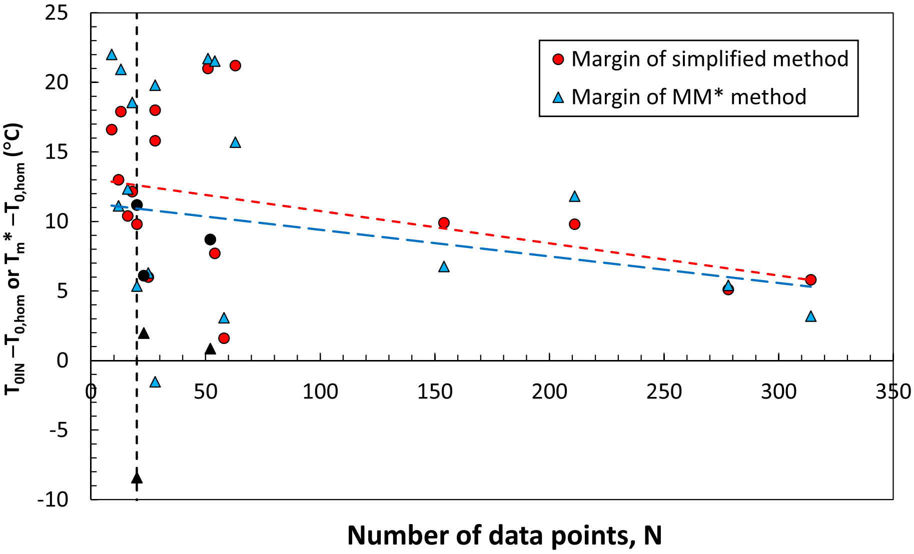

Fig. 4. Margin of conservatism of the simplified and MM methods as a function of the number of data points for inhomogeneous data sets. Linear fits are just a guide for the eye.

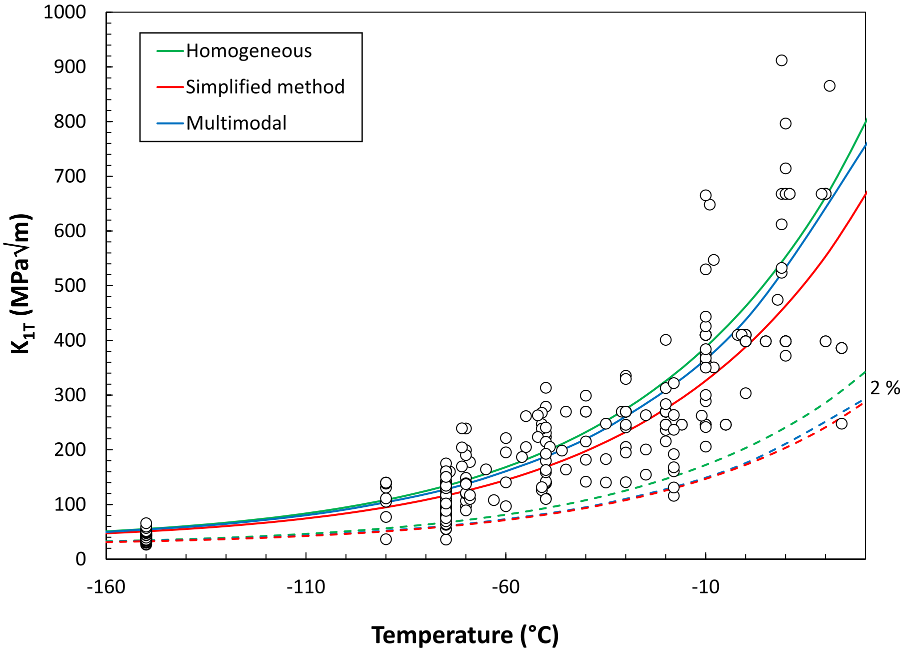

Fig. 5. Comparison between data points, homogeneous, inhomogeneous Master Curves, and corresponding 2 % confidence bounds (dotted lines) for Plate 13A (all tests).

could not be established, as T0TEM only provides the following choices: 15 %, 10 %, 5 %, and 1 %. For these two data sets, the 5 % confidence bound was used instead.

The overall impression that emerges for the examination of Table 3 and Fig. 6 is that the conservatism of the simplified and MM methods is substantially equivalent, as compared with the distribution of the experimental data points. For $N < 20$ (5 data sets, upper section of Table 3), the performance of the two methods is identical, with no data points falling below the 2 % curves. In all cases, the MM curve is the lowest of the three lower bounds. For the 15 larger data sets ($N \geq 20$, lower section of Table 3), the lowest percentage corresponds to SM in 4 cases and to MM in 4 cases, while for the remaining 11 data sets percentages are identical between SM and MM. In most cases (13 out of 20, or 65 %), the lowest 2% confidence bound was provided by MM, while SM provided the lowest curve in four cases, and for the remaining three data sets SM and MM are substantially overlapping.

Not considering the two EURO data sets for which 5 % confidence bounds were used, the average percentages of experimental data points falling below the 2 % confidence limit are 1.7 % for the simplified method and 2.0 % for the multimodal approach (as compared to 7.6 % for the standard homogeneous MC analysis).

5. Conclusions

The Master Curve analyses performed in accordance with ASTM E1921 on 10 large, “historical” data sets and a number of corresponding sub-sets (for a total of 51 data sets analyzed) allowed comparing two different inhomogeneity assessment methods: the simplified method

Table 3

Percentage of data points falling below the 2 % lower confidence bounds for the homogeneous, simplified, and multimodal analyses. (*) Data compared to the 5 % lower confidence bound. The thick black line separates data sets with $N < 20$ and $N \geq 20$.

Data set	**Sub set**	***N***	**Below 2 % confidence bound**			**Lowest curve**
			HOM	**SM**	**MM**	
72 W irr	1TC(T)	9	**30.4%**	*0.0%*	*0.0%*	**MM**
Midland irr	Lab C	12	**11.1%**	*0.0%*	*0.0%*	**MM**
Midland irr	Lab D	13	**10.0%**	*0.0%*	*0.0%*	**MM**
72 W irr	All tests	16	**6.3%**	*0.0%*	*0.0%*	**MM**
Midland irr	PCCv	18	*0.0%*	*0.0%*	*0.0%*	**MM**
Plate 13A	0.5TC(T)	20	**10.5%**	***5.3%***	**13.2%**	**SM**
CRIEPI R-R	Lab 2	20	*0.0%*	*0.0%*	*0.0%*	**MM**
72 W unirr	1TC(T)	23	**6.3%**	*0.0%*	**3.1%**	**SM**
Midland irr	0.5TC(T)-1TC (T)	25	**5.4%**	***2.7%***	***2.7%***	SM/ MM
Ingham	PCCv	28	**4.5%**	***2.3%***	***2.3%***	SM/ MM
Plate 13A	PCCv	28	***2.3%***	***2.3%***	***2.3%***	**MM**
Midland irr	MC(T)	51	**16.1%**	**3.2%**	*0.0%*	**MM**
Ingham	All tests	52	1.0%	1.0%	*0.5%*	**SM**
Plate 13A	1TC(T)	54	**7.4%**	*1.9%*	*1.9%*	**MM**
CRIEPI R-R	Lab 1	58	**7.5%**	**5.0%**	***2.5%***	**MM**
Midland irr	All tests	111	**11.8%**	*1.3%*	*1.3%*	**MM**
Plate 13A	All tests	154	**5.0%**	***3.5%***	**3.8%**	**SM**
CRIEPI R-R	All tests	211	*1.9%*	*1.9%*	*1.9%*	**MM**
EURO	All tests (*)	278	**15.8%**	**12.3%**	***11.3%***	**MM**
EURO	W/O SX9 (*)	314	**14.3%**	***11.4%***	**11.6%**	SM/ MM

and the multimodal method. Note that both approaches were used on all data sets, regardless of their size (number of data points, *N*), as one of the main objectives of this study was to assess the applicability and reliability of the multimodal approach for small data sets ($N \leq 20$).

The two approaches were comparatively evaluated in terms of inhomogeneity screening capability and performance in evaluating macroscopically homogeneous and inhomogeneous data sets. As mentioned above, the importance of the number of K_{Jc} values in the data set, *N*, was also assessed, as the multimodal method is currently restricted to $N \geq 20$.

The main conclusion of this study is that the two methods deliver substantially equivalent results, even below $N = 20$. More specifically:

- The screening criterion of ASTM E1921, section 10.6.3, appears slightly more restrictive than the criterion based on the *MLNH* parameter of the multimodal method. Out of 20 data sets that were screened inhomogeneous according to section 10.6.3, only three yielded $MLNH < 2$ (i.e., homogeneous according to the multimodal approach). The remaining 31 data sets were found to be homogeneous using both approaches. By introducing a parameter similar to *MLNH*, the Screening Criterion Index (*SCI*), the condition for inhomogeneity according to 10.6.3 can be expressed as
- $SCI \geq 1$.
- For homogeneous data sets, reference temperatures calculated according to the standard Master Curve methodology, $T_{0,hom}$, and the multimodal approach (after margin adjustment), T^*_m, were found in excellent agreement (within ± 5 °C), irrespective of the data set size (*N*).
- In the case of inhomogeneous data sets, the reference temperature estimate provided by the simplified method, T_{0IN}, was found to be generally conservative, sometimes significantly, as expected. As for the multimodal approach, only one margin-adjusted T^*_m value was found to be lower than $T_{0,hom}$, corresponding to a data set with $N = 20$. Generally speaking, the multimodal method appears slightly less conservative than the simplified method.
- The available experimental data points were compared to 2 % lower confidence bounds provided by the standard Master Curve analyses, the simplified method, and the multimodal approach. Once again, the performance of the two latter methods appears fully equivalent.

It can be concluded the simplified method can be used without excessive conservatism even for large data sets, while the multimodal approach provides reliable results even below $N = 20$. A lower limit for its applicability (10? 15?) probably exists, but it should be established through Monte Carlo analyses of a large number of simulated fracture toughness data sets.

The simplified method, which originates from the SINTAP project conducted in the late 1990 s [21], is easy to apply and provides conservative assessments regardless of the size of the data set. On the other hand, the multimodal approach is much more complex and requires specific calculations tools, which are nowadays however freely available in the form of software developed at NIST [19] and NASA [20].

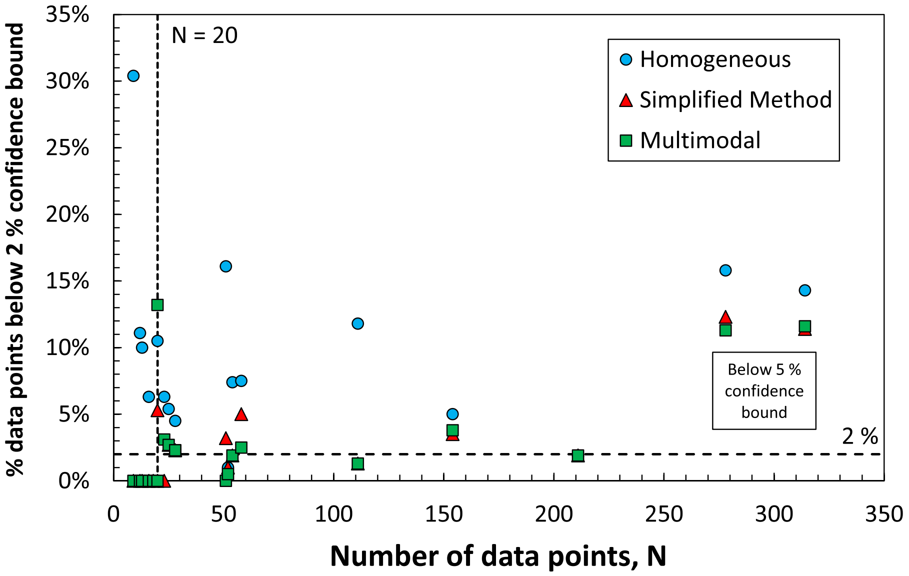

Fig. 6. Percentage of data points below 2 % confidence bounds for inhomogeneous data sets, as a function of N. For the two largest data sets, percentages refer to the 5 % lower bound.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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