DIMENSIONAL METROLOGY IN DETERMINATION OF *G* WITH BIPM'S TORSION BALANCE

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INTRODUCTION

In general terms, three inputs are required to calculate the value of *G* (a.k.a. the Newtonian Constant of Gravitation): force, mass, and dimension. To generate values of *G,* scientists have conceived and built various devices. One such device is called a torsion balance and researchers at the BIPM (Bureau International des Poids et Mesures) have designed and constructed one and used it to realize and study *G* for over two decades [1-3]. In 2016, NIST (National Institute of Standards and Technology) was provided the opportunity to obtain this device to repeat the *G* experiments performed by the BIPM.

The BIPM torsion balance is comprised of two sets of cylinders, which need to be measured for their relative positions in order to calculate the value of *G*. As shown in FIGURE 1, these two sets of cylinders are arranged in a circular pattern with one set nested inside the other. The outer cylinders (also known as the source masses) reside on a carousel near the outer boundary of the device, and a set of four inner cylinders (also known as the test masses) reside inboard of the outer cylinders and within a vacuum chamber.

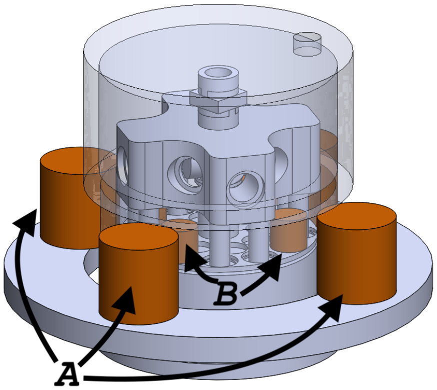

Figure 1. The BIPM Torsion Balance, "A" are the Outer Cylinders (a.k.a. Source Masses) and "B" are the inner cylinders (a.k.a. test masses) a vacuum chamber houses the inner masses [2]

The inner cylinders on the Torsion Balance are arranged on a platter (called the Torsion Disk) such that they are equally spaced in a circular pattern with a pitch circle diameter (PCD) of approximately 240 mm.

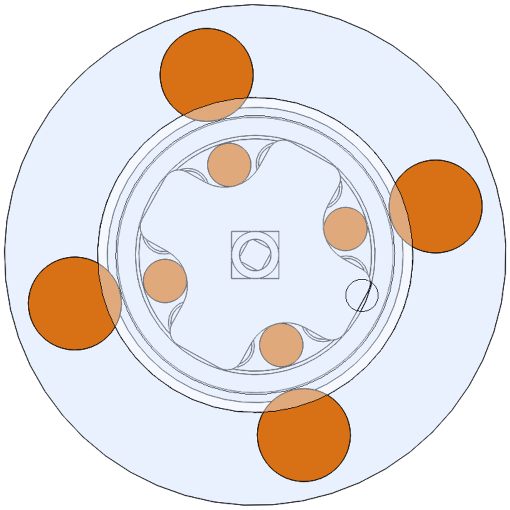

Figure 2. Top view of torsion balance shown in Figure 1

These cylinders work in relationship with the outer cylinders, which have a nominal PCD of 428 mm, to generate a gravitational torque that is precisely measured in the gravitational measurement experiment. For the Cavendish mode, the equation for calculating *G* is as follows [6]:

$$G = \Delta\phi \frac{16\pi}{T_0^2}\frac{R^5}{70Mr^2}\left(1+\frac{I_{\text{disk}}}{4mr^2}\right) \qquad \text{Eq. (1)}$$

For the dimensional metrology portion of this experiment, the elements from the above equation that are of interest are *R* and *r,* which are the pitch circle radii of the outer and inner masses, respectively. To enable the measurements of these parameters during the experiment, the torsion balance was constructed to be contained within the measurement volume of a Cartesian coordinate measuring machine (CMM).

The CMM used is a Zeiss Xenos having a measuring volume of 900 mm x 1200 mm x 700 mm and utilizing a tactile probe to measure points on the surface of the cylinders. Least-squares fits are applied to the points measured on the tops and sides of the masses to construct planes, and cylinders, respectively. The geometric center of the cylinder is calculated by projecting the planes down the axis of the cylinder to a distance that is half the cylinder height and calculating that intersection point. The four intersection points obtained from each set of masses are used to calculate a circle using least-squares method. The diameters of the two circles are the desired PCDs.

What may sound like a straightforward measurement is complicated by physical obstacles inherent to the design of the device which block access by the CMM probe. As a result, only portions of the cylinders are available for probing by the CMM to infer their locations. Relying only on partial data for these cylinders, combined with imperfect form of the cylinders (ranging from 10 µm to 20 µm), would likely result in a higher-than-desired measurement uncertainty [4].

With the aid of a new CMM (described above) and the facilities at NIST Advance Measurement Laboratory, we hope to improve upon the measurement results obtained by previous researchers using the BIPM torsion balance.

INNER AND OUTER MASS MEASUREMENT CHALLENGES AND POTENTIAL ERRORS

The masses originally designed for this torsion balance were constructed out of tellurium copper. At the micrometer level these masses have a tri-lobed shape, and with the outer masses even being slightly tapered. If the CMM could fully access and measure the entire surface of the mass, this form error would be less of an issue in determining its location. However, the CMM can only access about 135º of the cylinder surface on the inner masses and about 190º to 210º of the outer masses. Measuring a portion of a tri-lobed shaped cylinder to infer its actual diameter and location results in an error in determining both, as shown in Figure 3 [4].

Fortunately, for the small inner masses (ø=35 mm x h=35 mm), new ones were constructed via diamond turning with a form error of 0.250 µm or better. With the improved form, the difference in the resultant PCD from a partial coverage measurement and all-around measurement of the inner masses is around 300 nm. This means that even with partial measurement coverage a more accurate PCD can be calculated for the inner masses.

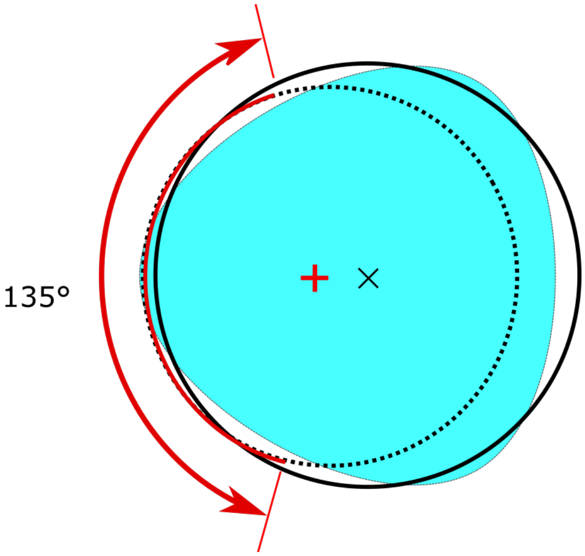

Figure 3: Example of cylinder position difference between partial arc measurement ("+" sign), and full circumference measurement ("X" sign) of cylinder

The outer masses have better CMM measurement access than the inner ones when the vacuum chamber is removed. This allows for different measurement strategies to be used to observe how increases in measurement coverage, relative to what can be accessed with the vacuum chamber on, affect the calculated outer mass PCD. In addition, the outer masses can be removed from the device to be further evaluated to see how corrections can be made to account for low CMM probing coverage. Eventually, these results can be used to make the necessary corrections to provide the best estimate of the outer mass PCD for the *G* calculation. The inner masses don't provide this flexibility, which influenced the decision to produce new cylinders using diamond turning.

In addition to the imperfect shape (i.e. form) of the masses, the measurements are also influenced by the CMM and the point probing strategy. The CMM used, when specified and evaluated using the ISO 10360-2 documentary standard, measures with a maximum permissible error of (0.3+0.001*L*) [µm], where *L* is in units of mm. However, the ISO standard is defined using point-to-point bidirectional measurements (using, for example, gauge block end standards) to evaluate measurement error. In the case of the of the torsion balance, the PCD is the feature of interest, as such a task specific measurement uncertainty needs to be evaluated. This evaluation will have to convolve the number of points measured on the surface of each mass.

In the ideal scenario a dense set of points would be measured on the full surfaces of the inner and outer masses to get the most accurate representation of their locations. But due to geometric and practical constraints this is not possible.

MEASUREMENT CORRECTION STRATEGIES

When the torsion balance device is in normal operation, the outer masses toggle to two different positions to generate a gravity signal (more details about how this device operates is in [1-3]). In these two different positions, the PCD for the outer masses will differ by about 1 µm to 2 µm. What is of extreme interest is the actual PCD of these masses. As mentioned above, when the torsion balance is in operation, only a partial arc of cylinder coverage is available to be measured and evaluated for an apparent PCD. To obtain a more accurate representation for PCD a correction factor needs to be determined.

To do so multiple measurement experiments were performed. These had two objectives: (1) to determine the uncertainty attributed to the CMM, and (2) to determine the uncertainty contributed by the imperfect form of the cylinders. Both of these effects are amplified by the fact that sampling is limited to only part of the cylinder, as mentioned above.

To determine CMM measurement errors a calibrated step gauge [5] is measured in the CMM work volume where the torsion balance will sit. The step gauge used was calibrated on the NIST M48 CMM to an expanded uncertainty (*k*=2) of 0.11+0.2*L* [µm] (where *L* is in units of meters). Measurement errors from this experiment will be used to correct measurement results from measuring the masses in the torsion balance.

During the torsion balance experiment, the CMM measures all accessible portions of the masses, which includes their side walls, and tops. To calculate the geometric center of each cylinder, first the side wall measurement points are used to calculate a cylinder axis through a least-squares fit to determine its location and orientation. Next, a planar least-squares fit is applied to the measurement points representing the cylinder tops to determine its location and orientations. Finally, to calculate the geometric center of each mass, its top plane is projected halfway down along its axis. Where the projected plane and axis intersect is considered the geometric center of each mass.

The PCD for the inner and outer mass clusters are calculated by performing a circle fit to the geometric centers of the four masses for each cluster. For the inner, and outer masses the nominal PCDs are 240.064 mm and 427.9 mm respectively. However, corrections need to be applied to the measured values to obtain a better estimate of the true PCD values.

To obtain correction values for the SMs, each individual cylinder was evaluated to determine how the geometric center shifts based on how much of its surface is used in a least-squares fit, when compared to using its entire surface. Each SM was placed in a kinematic fixture that supports it using the same points as when it is mounted on the torsion balance.

Three probing patterns were used during this evaluation. The first pattern is the same used when each SM is mounted in the torsion balance. This type of pattern only partially measures the surface of the cylinder. Keep in mind that there are two toggle positions for the torsion balance's carousel, the counterclockwise (Position 1) and the clockwise (Position 2). For each of the SMs the probing coverages are different for each of these positions. The second pattern is one that uses that same point spacing used in the first pattern except that it is expanded to cover the entire cylinder. While this pattern covers the entire cylinder, it has a relatively coarse spacing and provides some indication of what a low measurement point density does to resolving its location. The third pattern consists of 9 rings of 37 points distributed along the length of the cylinder and covers it entirely. This pattern will be used to represent the entire cylinder. The figure below shows an example of the measured points, with their deviations from the least squares fit magnified 2000 times, Figure 4.

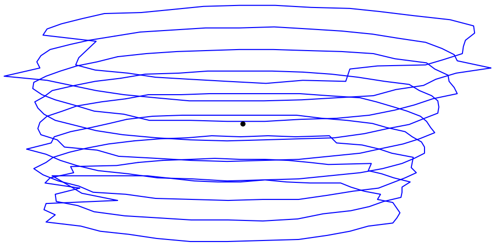

Figure 4: Magnified form error of points measured from a Source Mass (SM). Form error is about 20 μm (2,000 times magnified)

From the experiments mentioned a precise estimate of how much of a correction needs to be made for each SM to account for the reduced measurement coverage is calculated. The shift for each SM towards the center is outlined in Table 1.

Table 1: Estimated shift for each SM center coordinate to correct for reduced measurement coverage

	Position 1		Position 2	
Mass No.	X Shift (μm)	Y Shift (μm)	X Shift (μm)	Y Shift (μm)
1	1.336	-0.022	0.151	-1.287
2	0.000	-0.563	-1.061	-0.433
3	0.653	1.010	-1.723	0.097
4	-1.513	1.395	-1.221	1.321

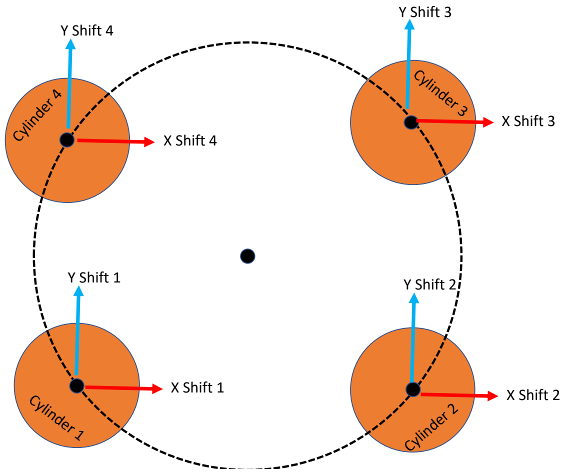

Figure 5: Shift of SM centers when applying corrections from Table 1

RESULTANT OUTER PCD MEASUREMENT UNCERTAINTY

Without the corrections the PCD for the SMs are 427.9314 mm and 427.9303 mm for one specific measurement set, for the two toggle positions. With the corrections, the measurements are 427.9342 mm and 427.9327 mm.

The following table outlines the preliminary estimates for the most significant sources for measurement uncertainty for the outer PCD of the masses.

Table 2: Uncertainty evaluation for outer PCD

Uncertainty Source	**Value (*μm*)**
Uncorrected (?) CMM error (including repeatability)	0.3
Cylinder Imperfect Form (after correction)	0.2
Effect of imperfectly known temperature after correction	0.25
Effect of imperfectly known CTE after correction	0.1
Standard combined uncertainty	0.45

CONCLUSION

The BIPM torsion balance presented unique dimensional metrology challenges. Through the work outlined above, the resultant measurement uncertainty for the outer PCDs is sufficiently low to lead to one of the best estimates of the Newtonian gravitational constant ever performed. Even with pleasing results, any future effort could benefit from a few modifications. One suggestion would be to use spherical masses rather than cylinders. One reason is that a sphere can be manufactured with very low form error, on the order of hundreds of nanometers, using dedicated manufacturing equipment; usually for a lower cost than diamond turning. The next design suggestion would be to reduce the number of obstructions around the masses to allow for better access by the CMM probe. Another step would be to use (for certain components that affect the PCD) low-expansion material to minimize dimensional changes when the temperature varies. NIST has been honored to partake in this experiment that has been challenging scientists for over two centuries. It is our hope that NIST's contributions in this field can help inspire and inform further research.

Disclaiimer:
Any mention of commercial products within this document is for information only; it does not imply recommendation or endorsement by NIST.

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