

# Impact of Using Soft Exposure Thresholds in Automatic Contact Tracing

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**Abstract**— Current automatic exposure notification apps primarily operate based on hard distance/time threshold guidelines (e.g., 2 m/15 min in the United States) to determine exposures due to close contacts. However, the possibility of virus transmission through inhalation for distances over the specified distance threshold might necessitate consideration of soft distance/time thresholds to accommodate all transmission scenarios. In this paper, using a simplifying approximation on the instantaneous rate of the viral exposure versus distance, we extend the definition of “contact” by proposing a soft distance/time threshold which includes the possibility of getting exposed at any distance (within certain limits) around an infected person. We then analyze the performance of automatic exposure notification with Bluetooth-based proximity detection by comparing the exposure results when soft or hard thresholds are used. This study is done through an agent-based simulation platform that allows for a comprehensive analysis using several system parameters. By tuning the parameters of the proposed soft thresholds, a more accurate determination of possible exposures at any distance would be possible. This would enhance the effectiveness of an automatic contact tracing system. Our results indicate the noticeable impact of using the soft distance/time threshold on the exposure detection accuracy.

**Keywords**- Proximity Detection, Bluetooth, Exposure Determination, COVID-19, Contact Tracing

## I. INTRODUCTION

Contact tracing is a well-established technique used by public health professionals to trace and identify “contacts” of a known infectious person. In the United States, a “close contact” is defined by the Centers for Disease Control and Prevention (CDC) as someone who was within 2 meters of an infected person for at least 15 minutes within a 24-hour period starting from 2 days before appearance of symptoms (or, for asymptomatic cases 2 days prior to positive specimen collection) until the time the person is isolated [1]. The total exposure time needed for determination of this “close contact” is a cumulative total of 15 min or more over a 24-hour period. For example, this total could be acquired through three individual 5-minutes encounters with three separate infected individuals during a 24-hour window. The World Health Organization (WHO) also has a similar definition for close contact through proximity and duration of exposure, except that the proximity threshold is 1 m, instead of the 2 m considered by CDC [2].

During a pandemic efficient implementation of contact tracing is critical to limit an outbreak. However, resource limitations in cases of large outbreaks could create many challenges for executing manual contact tracing. Automatic contact tracing (also known as Automatic Exposure

Notification) is an electronic notification protocol based on a proximity detection mechanism such as Bluetooth ranging. The availability of Bluetooth (or Bluetooth Low Energy (BLE)) technology in today’s smart phones have prompted governments and industry to also consider automatic exposure notification as a tool to complement manual contact tracing in combating the spread of the virus during COVID-19 pandemic [3, 4]. Usage of this technology involves installing an app developed through collaboration between industry and government agencies and published by authorized health authorities.

COVID-19 infections are primarily through exposure to respiratory fluids (i.e., droplets and aerosol particles) carrying the SARS-CoV-2 virus. These respiratory fluids are released during breathing, coughing, sneezing, speaking, etc. Droplets typically clear from the air surrounding the infectious source within seconds to minutes; however, aerosol particles (i.e., very fine droplets) can remain in the air for minutes to hours. Inhalation of the air containing these droplets or aerosols could cause transmission of the virus to an individual who is in proximity of an infected person. The infection risk of the individual inhaling this air depends on the amount of virus to which he is exposed. These infectious particles move outward from the infected person during exhalation or sneezing/coughing. The larger size droplets fall to the ground in the immediate vicinity of the source due to the gravity. However, aerosol particles remain in the air and become diluted with growing volume and mixing with the stream of the air surrounding the source. Since the concentration of these particles is greatest within one to two meters of the infectious source, the risk of virus transmission is highest within that range. Nevertheless, transmission can still occur from inhalation of the virus in the air farther than two meters from the infected person. Increased exhalation (e.g., shouting, singing, exercising), prolonged exposure over 15 minutes and environments such as enclosed spaces have been known to contribute toward infections through inhalation at distances greater than 2 meters from the source [5, 6, 7, 8, 9]. In general, the risk for such infections decreases with increasing distance from the source.

The automatic exposure notification apps primarily operate based on the hard distance/time thresholds outlined by the health organizations (e.g., 2 m/15 min by the CDC or 1 m/ 15 min by the WHO) to determine exposures as a result of close contacts. However, the possibility of virus transmission through inhalation for distances over two meters might necessitate consideration of a soft distance/time threshold to accommodate

all transmission scenarios. In addition, higher transmissibility of the virus variants (such as Omicron) might also require consideration of longer (or shorter) distances/time threshold for exposure determination. Ultimate exposure determination depends on the amount of virus inhaled by the exposed individual; however, there are no simple methodology to ascertain that amount in practice. Accurate mathematical representation of the spatial distribution of the virus density over distance depends on many factors and scenarios. Using a simplifying approximation on the instantaneous rate of the viral exposure versus distance, we can extend the definition of “contact” by proposing a soft distance/time threshold which includes the possibility of getting exposed at any distance (within certain limits) around an infected person.

In this paper, we analyze the performance of automatic exposure notification with BLE-based proximity detection by comparing the exposure results when soft or hard thresholds are used. This study is done through an enhanced agent-based simulation platform which was originally presented in [10]. By tuning the parameters of the proposed soft thresholds, a more accurate determination of possible exposures at any distance would be possible. This flexibility would allow optimization of the soft threshold parameters based on factors such as the surrounding environment (e.g., indoor vs. outdoor), an individual’s health, the severity of the outbreak in the community, etc. The rest of this paper is organized as follows. Section II presents mathematical derivation of the soft thresholds that are needed to calculate exposure. Section III describes the agent-based simulation platform that has been developed to study potential exposures using the BLE-based proximity detection. Simulation results and analysis are provided in Section IV. Finally, conclusions and plans for future work are described in Section V.

## II. USING SOFT THRESHOLDS FOR EXPOSURE DETERMINATION

Consider a healthy individual that is located at distance  $d(t)$  to an infected person at time  $t$ . Assume that  $v(d(t), t)$  is the instantaneous rate of the viral exposure experienced by this individual at time  $t$ . The total viral exposure by the individual during time interval  $[0, T]$  would be:

$$V_T = \int_0^T v(d(t), t) dt \quad (1)$$

To simplify, we assume that  $v(d(t), t)$  is the following function of distance  $d(t)$ :

$$v(d(t), t) = kd(t)^{-\alpha}, \quad (2)$$

where  $\alpha$  represents the decay factor of the viral exposure intensity with distance and  $k > 0$  is a constant coefficient. Clearly the distribution of the viral particles may not be the same (i.e., uniform) in all directions around the infected person and could depend on the source orientation and initial jetting of exhalations. However, to reduce the complexity of our analysis

we do not consider dependency on direction in this paper. Therefore, from (1) and (2), we have:

$$V_T = k \int_0^T d(t)^{-\alpha} dt. \quad (3)$$

The individual receiving this total viral exposure is considered to be “Exposed” at time  $t \geq T$  if  $V_T \geq V^*$ , i.e.,

$$V_T = k \int_0^T d(t)^{-\alpha} dt \geq V^* \quad (4)$$

where  $V^*$  is a constant representing the critical threshold for the total viral exposure. Using the current CDC definition of hard exposure thresholds (i.e.,  $D_H = 2$  m and  $T_H = 15$  min),  $V^*$  can be obtained as follows:

$$V^* = kD_H^{-\alpha}T_H \quad (5)$$

Therefore, the individual at risk is considered exposed if:

$$V_T = k \int_0^T d(t)^{-\alpha} dt \geq kD_H^{-\alpha}T_H \quad (6)$$

Equation (6) defines the soft threshold (i.e., boundary in the distance/time space) for identification of exposure compared to the hard thresholds used for this purpose. This concept is further illustrated in Fig. 1a, and Fig. 1b.

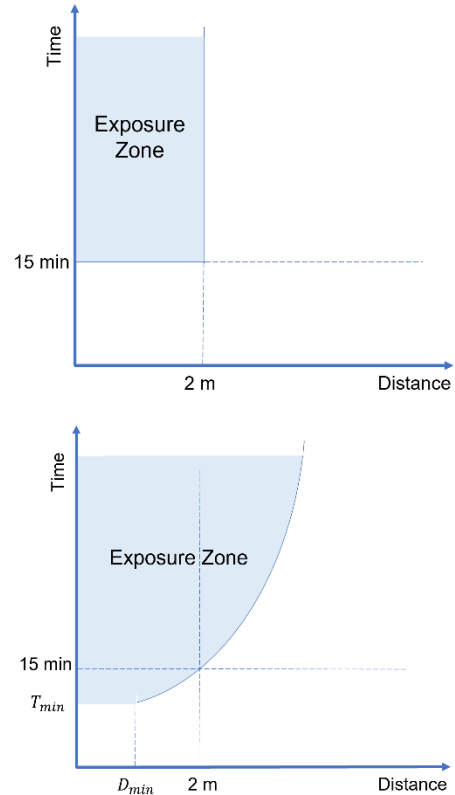


Fig. 1: Exposure zone according to (a) hard thresholds (b) soft thresholds

The boundary function for exposure zone in Fig. 1b can be obtained from equation (6) as:

$$T(D) = 15 \left(\frac{D}{2}\right)^\alpha \quad (7)$$

As indicated earlier in this section,  $\alpha$  is a parameter that specifies the rate at which viral particles decay with distance. In the simplest scenario, assuming a spherical spatial distribution of viral particles in the space surrounding the source,  $\alpha$  may be approximated by 2. However, many environmental characteristics such as obstacles along the exhalation path, indoor vs. outdoor, air flow quality, or even temperature may impact the actual value of  $\alpha$ . For example, in indoor environments with low air circulation, the value of  $\alpha$  may be much lower than outdoor environments. In addition,  $\alpha$  could also be a function of distance itself as droplets may dissipate differently with increasing distance compared to aerosols. The impact of  $\alpha$  on the soft threshold (i.e., boundary of the exposure zone) is shown in Fig. 2. As observed, the size of the exposure zone depends on the values of  $\alpha$ .

Another parameter to consider when using soft thresholds is  $d_{min}$ . It indicates the minimum distance below which the required exposure time for occurrence of a ‘contact’ does not decrease. A value of  $d_{min} < 2$  meters would represent scenarios where less than 15 minutes are sufficient for positive exposure determination. It should be noted that when  $d_{min} = 2$  m, increasing value of  $\alpha$  would cause the soft threshold exposure zone to asymptotically converge to the exposure zone defined by the hard thresholds (i.e., Fig. 1a).

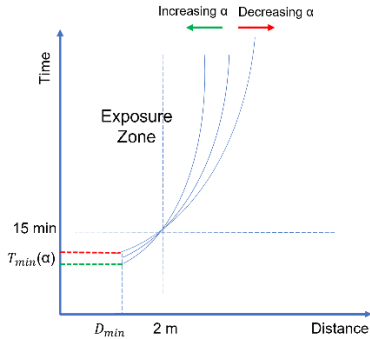


Fig. 2: Impact of  $\alpha$  on the exposure zone

Assuming that the BLE signal measurements are done at a rate of  $1/\Delta$  seconds, a non-stationary individual within the exposure zone will pick up incremental exposures every  $\Delta$  seconds. The values of these incremental exposures are not equal and depend on the distance of the individual from the infected agent at the time of measurement. Consider  $Exp(t, d(t))$  to be the total effective exposure time that an individual has experienced up to time  $t$ .  $d(t)$  is the distance of the agent from an infected individual at time  $t$ . Then, the total effective exposure time at  $t + \Delta$  (i.e.,  $Exp(t + \Delta, d(t + \Delta))$ ) depends on how  $d(t)$  changes from time  $t$  to time  $t + \Delta$ . In general, this can be approximated by the following equation:

$$Exp(t + \Delta, d(t + \Delta)) = Exp(t, d(t)) + \beta \left( \frac{\Delta \times 15}{T(d(t))} \right) + (1 - \beta) \left( \frac{\Delta \times 15}{T(d(t + \Delta))} \right) \quad (8)$$

where  $0 \leq \beta \leq 1$  is a constant parameter indicating the impact of transition from  $d(t)$  to  $d(t + \Delta)$ . If  $\beta$  is close to one, it means that the individual stayed mostly at distance  $d(t)$  to the infected individual during the time interval  $[t, t + \Delta]$ . Likewise, a  $\beta$  value close to zero means that the individual stayed mostly at a distance  $d(t + \Delta)$  to the infected individual during the time interval  $[t + \Delta]$ . Since no information would be available on the transition of the individual from  $d(t)$  to  $d(t + \Delta)$ , then in practice three approaches can be followed to calculate accumulative exposure: A) an aggressive approach where  $\beta$  is considered to be zero, or B) a conservative approach where  $\beta$  is considered to be 1, and C) a medium approach where  $\beta$  is taken to be 0.5.

If the individual is stationary, then  $d(t + \Delta) = d(t)$  and all three approaches would be similar. For a moving individual if  $\Delta$  is small, then the difference between these approaches is again minimal. The three strategies could differ significantly when  $\Delta$  becomes large relative to the speed of the individual. In practice, a small  $\Delta$  means higher measurement and processing rate by the contact tracing application on the mobile phone. This could result in a higher battery consumption and therefore higher frequency of recharging which might not be desirable in practice.

As mentioned earlier, in automatic contact tracing, the Bluetooth signal is used to estimate the distance between two individuals carrying mobile phones. Knowing this proximity and its duration, the ‘close contact’ guidelines using the hard thresholds [1] are typically used to determine the possibility of exposure to the virus. With soft thresholds, the same methodology using the updated exposure zone in Fig. 1b can be used to determine whether any exposure has occurred. Further details of this methodology are discussed in the next section.

### III. SIMULATION PLATFORM

In [10], we proposed an agent-based simulation platform to better understand the impact of Bluetooth proximity estimation error on automatic exposure determination. The platform considered people walking in a plaza, campus area, or neighborhood. The basic dynamics in the agent’s mobility were based on the algorithms in [11]. In order to more accurately reflect agents’ traffic in an open area, we have incorporated two enhancements in our platform. In the basic platform, all agents within the simulation would simply randomly walk towards a given goal. So, the first enhancement is to overcome the possibility of the agents becoming jammed or trapped against one another with no way to proceed during the simulation [12]. Jamming can occur when all the agents in a simulation have similar goals, such as all trying to reach the same area within the simulation field. To avoid jamming in our simulation and thus biasing the results, our platform periodically randomizes the goals of each agent.

The second enhancement is to allow some interaction between agents within the simulation. In other words, we allow an agent to start a ‘conversation’ with another agent, which effectively immobilizes both agents for a random period of

time. If two (or more) agents are within a certain proximity of each other, then, with a given probability, a conversation is started. The length of this conversation is random variable with a truncated Normal distribution. In our simulations, we have chosen the standard deviation of this distribution to be 1 min while the mean is considered as a parameter. This enhancement enables us to better consider the possibility of exposure with face-to-face interactions among the agents.

Using this enhanced platform, we can track the true and estimated distances between any two moving agents at fixed time interval of size  $\Delta$ . The estimated distance is calculated as the summation of the true distance plus an error which is due to the BLE proximity detection mechanism. In the BLE-based proximity detection mechanism, there is an underlying error in the process that converts the signal strength into distance. This error is due to the variation in propagation of the Bluetooth signal. The variation is caused by many factors such as the surrounding environments, phones positions and orientations relative to the individuals carrying them, antenna gain patterns of the phones, etc. Assuming a Lognormal pathloss distribution for the BLE channel with a Gaussian distributed shadowing and fading component with standard deviation  $\sigma$ , the distribution of the error in the estimated distance would be a function of ( $\sigma$ ), pathloss exponent ( $n$ ) and the true distance between the agents. This distribution is shown in equation (9) [10].

$$f_Y(y) = \frac{1}{\sqrt{2n}} \frac{10n}{\sigma \log_e(10)} \frac{1}{(d_0 + y)} e^{-\frac{1}{2} \left( \frac{10n}{\sigma \log_e(10)} \right)^2 \left( \log \left( \frac{y}{d_0} + 1 \right) \right)^2} \quad (9)$$

During a simulation, each healthy agent maintains two parameters: (a) True Effective Exposure Time ( $T\_Exp(t, d(t))$ ), and (b) Estimated Effective Exposure Time ( $E\_Exp(t, d(t))$ ). The true effective exposure parameter keeps track of the total exposure time based on equation (8) using the true distance from infected agent. Likewise, the estimated effective exposure parameter shows the total exposure time when the estimated distance from infected agents is used in equation (8). The true and estimated effective exposure parameters are updated every  $\Delta$  seconds after incorporating the population dynamics in the simulation platform. The parameters are used to make exposure determination for all healthy agents at any time during the length of a simulation. Comparison of the values of these parameters to the soft thresholds discussed in the previous section will lead to 4 possible states for each agent including two types of errors in exposure determination.

A false positive exposure error occurs when the true effective exposure parameter of a healthy agent is outside the exposure zone shown in Fig. 1b while its estimated effective exposure parameter is within the exposure zone. Conversely, a false negative exposure error occurs when the agent's true effective exposure parameter falls inside the exposure zone of Fig. 1b while the estimated effective exposure parameter shows the accumulated exposure time still outside that zone. The diagram shown in Fig. 3 describes possible states for an agent during the

simulation and conditions for transitioning from one state to the other.

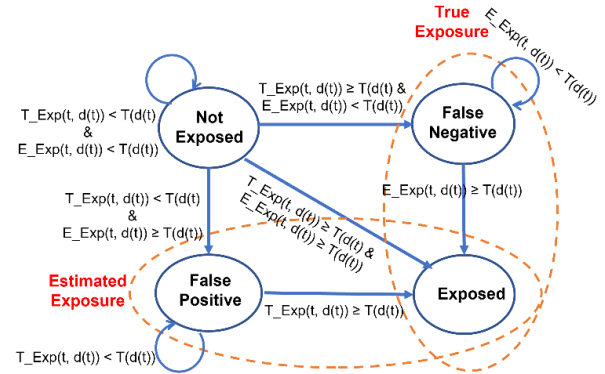


Fig. 3: State diagram of the agents in the simulation platform

To reduce the number of exposure checks at each time interval, a cutoff radius can be considered for the BLE signal measurement around any infected agent in the simulation. The maximum radius of Bluetooth signal in favorable environment (i.e., minimal fading and shadowing) is typically considered to be 10 m. However, this range could vary for harsh environment or equivalently environment with high standard deviation of fading (i.e.,  $\sigma$ ).

#### IV. SIMULATION RESULTS AND ANALYSIS

Assuming a fixed pathloss exponent of  $n = 2$  for the BLE signal propagation, a fading standard deviation of  $\sigma = 4$ , and a distance measurement interval of  $\Delta = 1$  sec, extensive simulations have been done using the platform discussed in the previous section to investigate the impact of  $\alpha$ ,  $d_{min}$  and the average conversation length between agents on the number of exposed individuals. The results presented in this paper consider a population of 135 agents moving within an area of size 162 m  $\times$  35 m for 8 hours (i.e., typical length of a workday). These numbers are chosen based on a standard laboratory building inside the campus area of the National Institute of Standards & Technology where the authors work. The number of infected individuals at the beginning of the simulation is set to 5% of the population. In addition, the probability of two agents starting a conversation once they are within one meter distance of each other is assumed to be 0.01.

Figure 4 shows the number of exposed agents versus time for the soft exposure threshold with parameter  $\alpha = 2, 2.5, 3, 3.5, 4$  as well as the hard thresholds of 2 m/15 min (per CDC guidelines). Here, it is also assumed that  $d_{min} = 1$  m and the average conversation length is set to 3 min. As observed, the number of exposed agents is noticeably more at any given time during the simulation when a soft threshold is used. In addition, this number grows faster for higher values of  $\alpha$ . Although this may be counterintuitive, but the trend versus  $\alpha$  depends on many factors such as  $d_{min}$ , agents mobility pattern, average conversation length, and population density. For  $d_{min} < 2$ , as shown in Fig. 2, the change in the size of the exposure zone



below the 15 min threshold as  $\alpha$  changes will impact all agents that fall within the range  $[d_{min}, 2\text{ m}]$ . When  $\alpha$  increases, the size of that zone will increase as well. This will result in higher number of exposed agents, especially when the mobility pattern of the agents leads to more occurrence of agents within that exposure zone.

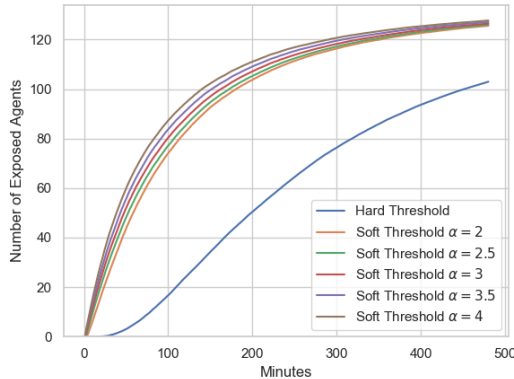


Fig. 4: Impact of  $\alpha$  on the number of exposed agents when  $d_{min} = 1\text{ m}$

On the other hand, if  $d_{min} = 2\text{ m}$ , then the number of exposed agents versus time decreases as  $\alpha$  increases. This trend, shown in Figure 5 for  $\alpha = 2, 2.5, 3, 3.5, 4, 20$ , is simply due to the reduction in the size of the exposure zone above the 15 min threshold. In the limit, for very large values of  $\alpha$ , the soft exposure threshold would converge to the hard thresholds as defined by CDC, and the number of exposed agents would become identical. This can be observed in Fig. 5 when  $\alpha=20$ . The curve corresponding to  $\alpha=20$  is much closer to the results obtained when using the hard thresholds.

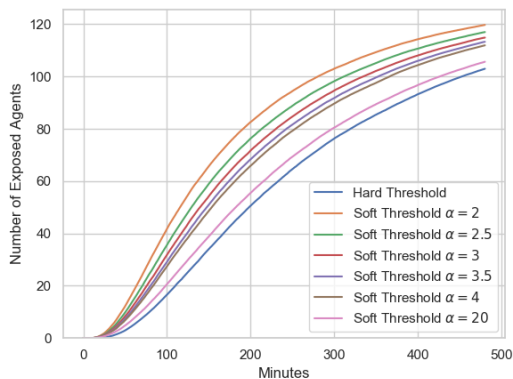


Fig. 5: Impact of  $\alpha$  on the number of exposed agents when  $d_{min} = 2\text{ m}$

In general, for a fixed value of  $\alpha$ , higher values of  $d_{min}$  would decrease the size of the exposure zone; and therefore, the number of exposure opportunities will be reduced. Figure 6 shows the number of exposed agents versus time for the soft exposure threshold with parameter  $d_{min} = (1, 1.5, 2)\text{ m}$  as well as the hard thresholds of  $2\text{ m}/15\text{ min}$  (per CDC guidelines). Here, it has been assumed that  $\alpha = 2$ , the average conversation length =  $3\text{ min}$ , and the probability of initiating a conversation

is again set to 1%. As expected, the total number of exposed agents decreases with increase in the values of  $d_{min}$ .

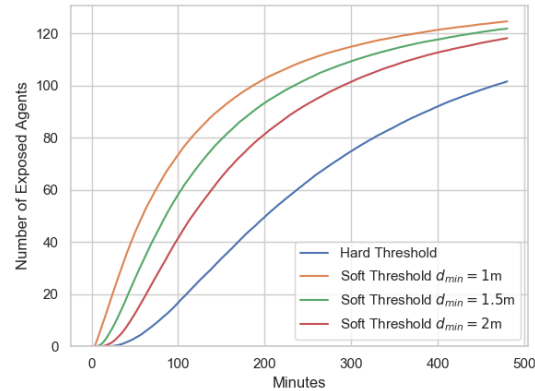


Fig. 6: Impact of  $d_{min}$  on the number of exposed agents

Aside from the soft threshold parameters ( $d_{min}$  and  $\alpha$ ), agent mobility attributes also impact the exposure results. As mentioned in Section 3, the mobility pattern considered for this study allows occasional stops for mobile agents that are within certain distance of each other. This feature represents random conversations that could take place among individuals in their work environment. Figure 7 shows the number of exposed agents over time when the average conversation length is assumed to be  $3\text{ min}$ ,  $6\text{ min}$  or  $10\text{ min}$ . A soft exposure threshold with parameters  $\alpha = 2$  and  $d_{min} = 1\text{ m}$  has been considered for these results. It might be expected that higher average conversation time should increase the number of exposed agents at any time. However, longer conversation between healthy agents reduces their exposure opportunity from other infected agents as they become immobilized during their conversation time. In addition, infected agents who get involved in a longer conversation have less opportunity to expose other healthy agents in the environment. Therefore, as seen in Fig. 7, higher average conversation time leads to lower number of exposed agents over time. It should be noted that this trend may change for different mobility pattern.

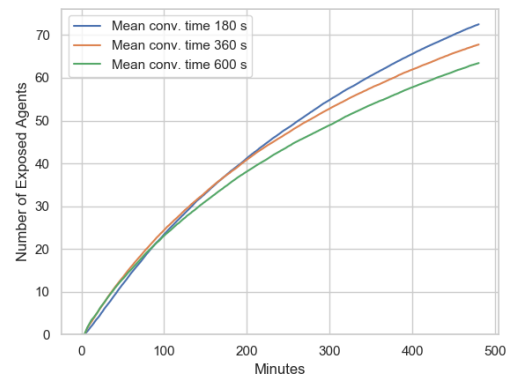


Fig. 7: Impact of the average conversation length on the number of exposed agents

As described by the state diagram in Fig. 3., the number of false exposure determinations can also be studied using our

simulation platform. As the sequence of the error in the estimated distances of agents is most likely correlated in time and not an independent and identically distributed process, a filter (i.e., windowing function) can be used to exploit this correlation, smooth the estimated distances and reduce the occurrence of false exposure determinations. Assuming  $\alpha = 2$ ,  $d_{min} = 2$  and an average conversation time of 3 min, Fig. 8 shows the total average number of false negatives plus positives with and without filtering during the simulation. Here we are showing the impact of using a simple 3-point moving-average window. As shown in Fig. 8, the number of false determinations initially rise and then drop as more transitions to the fully exposed state occurs. However, the total number of false determinations is substantially lower when the 3-point moving average window is used. Further results elaborating the use of filtering has been omitted for brevity.

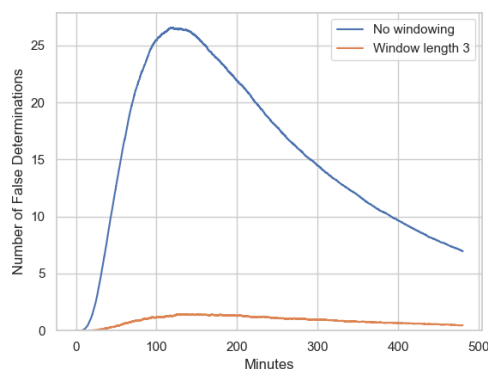


Fig. 9: Number of false determinations with and without a 3-point moving-average window

## V. CONCLUSIONS AND FUTURE WORK

The ultimate goal of contact tracing is to accurately notify the right people (i.e., people who were truly exposed) to quarantine in a timely manner and let other individuals who were not exposed to function in the community as usual. In this way, not only the spread of the virus is better controlled but also the negative economic impacts of general public lockdowns are avoided or minimized.

This paper extends the current hard exposure thresholds to a parameterized soft threshold in order to allow for more general exposure determination scenarios. These scenarios include cases with 1) distances beyond the current hard proximity threshold and 2) exposure time below the current hard time threshold. The impact of the soft thresholds for various system parameters have also been reported using an agent-based simulation platform. The authors plan to further investigate the accuracy of exposure determination by varying the BLE signal measurement frequency as well as the impact of using various filtering on the sequence of the estimated distances. The concept of soft exposure thresholds presented here can be further customized for classes of individuals according to factors such as age, health condition, transmissibility of the virus at a given geographical area, etc. The authors further plan

to investigate identification of such classes and their corresponding soft exposure thresholds.

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