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NIST Alternatives for Resilient Communities (NIST ARC) Software Tool

Mathematical Programming Model

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Abstract

The National Institute of Standards and Technology (NIST) Alternatives for Resilient Communities (NIST ARC) software is an interactive tool for developing alternative sets of actions that meet community resilience and cost goals, given hazard and interdependency information and socio-economic data. Community resilience planning is challenging as it involves several large-scale systems and public sector decision-making with numerous stakeholders. The goal of NIST ARC is to decrease a community's burden in developing viable alternatives for stakeholder consideration. This technical note details NIST ARC's mathematical programming model, which is the leading technical contribution of NIST ARC. The model variables, parameters (data), and equations of the two-stage stochastic mixed integer linear programming model are described, with the full model given in the Appendix. Results for a realistic example designed to test the suitability of the mathematical programming model for supporting interactive community resilience planning are presented. Finally, the NIST ARC decision support application that enables the use and application of the model, the plans for its further development and testing, and its role within the broader set of NIST-funded tools and guidance for the Community Resilience Program are briefly described.

Keywords

Community resilience, disaster resilience, mathematical programming, linear programming, stochastic programming, hazards, simulation, optimization.

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1. Introduction

Systematic decision-making tools are needed to achieve community resilience [1–3]. This document describes the modeling behind the initial version of a new decision-making tool for community resilience planning developed by the Community Resilience Program at the National Institute of Standards and Technology (NIST). The NIST Alternatives for Resilient Communities model, or NIST ARC, is designed as an interactive tool to support community resilience planning. This technical note, the first of a series, focuses on the mathematical programming model that underpins NIST ARC. The equations of the mathematical programming model capture many aspects of community resilience and model infrastructure systems and population dislocation and reoccupation. Results from testing the mathematical programming model suggest that it will support the interactive identification of viable alternatives. After describing the mathematical programming model and its testing, the tool's software architecture, implementation, and plans for further development are briefly outlined. Subsequent reports in the series will focus on the application of NIST ARC through demonstration cases, its use in conjunction with other NIST tools, and extensions incorporating additional systems impacting community resilience.

1.1. Community Resilience

Community resilience refers to preparing for anticipated hazards, adapting to changing conditions, and recovering rapidly from disruptions [4–6]. The NIST Community Resilience Planning Guide (CRPG) draws attention to the importance of the functioning of the built environment on the social functions they support, including, among others, government, economics, health, education, community services, religion, culture, and media. Buildings serve as the interface between infrastructure systems such as energy, communication, water and wastewater, transportation, and a community's human social system [7, 8]. NIST ARC is a product of the NIST project entitled *Development of a First-Generation Community-Resilience Systems Model*, which aims to incorporate physical, social, and economic systems models at the community scale into a decision support tool that can prescribe alternative solutions for improving a community's resilience. While government leaders are typically restricted to making public decisions regarding public properties, such as water utilities and road networks, desired solutions can be implemented through a range of policies, including grants, tax credits, and low interest loans [9–13], that impact private entities.

The model has several potential users, but the primary target user of NIST ARC is an analyst who would work with a collaborative community resilience planning team. The NIST CPRG recommends forming a collaborative team with a good representation of all stakeholders. The analyst initially would advise the community on gathering the required inputs to NIST ARC, which consists of hazard and interdependency information and socio-economic data. However, the analyst's primary role would be to facilitate the planning team's use of NIST ARC to find alternatives. The creative aspect of NIST ARC, which the analyst would facilitate, is incorporating the team's feedback into new runs of the model. Other users would likely include policy analysts and researchers seeking to improve the several governmental programs providing community resilience funding. The form of the NIST ARC model is conducive to evaluating such policies.

NIST ARC applies optimization methods to deal with the daunting number of alternative actions or policies that a community can adopt to improve its resilience. The unmanageably (absent optimization) large number of resilience-improving alternatives stems from the combinatorial nature of the actions that could be taken within and across several large-scale physical, economic, and social systems. Reliance on a manual trial and error process to identify solutions, even with access to an easy-to-use simulation model linking decisions to resilience and cost metrics, is unlikely to yield satisfactory alternatives. It is difficult to know, for example, what specific changes to make to a set of decisions under consideration that will result in greater resilience while meeting budget and other constraints.

1.2. Modeling for Decision Support

There are several approaches to optimization modeling from which to choose, each striking a different balance of model tractability, convenience, and model validity. NIST ARC takes a *mathematical programming* approach, sharing some design elements of interdependent network design and restoration models that have been proposed [14–19]. Approximations are made to make the mathematical programming model amenable to solution by efficient, well-established methods. The design goal is to make the minimal degree of approximation necessary to permit its use in real-time, interactive sessions with the collaborative planning team. The resultant model described in this report is a *risk-averse two-stage stochastic mixed integer linear program* that captures mitigation and recovery and is consistent with other simulation models [20–23] in the stochastic modeling of hazards and building failure.

The novelty of the mathematical programming model lies in its ability to capture many essential aspects of community resilience:

- Varied options for improving resilience:
    - Increasing the resistance of the built environment to various hazard scenarios
    - Adding redundancy to systems and networks to avoid system failure
    - Adding backup storage to survive temporary losses of inputs needed for system operation (e.g., fuel, water)
- Recovery time modeling, here modeled with discrete event simulation (DES).
- Protections that are offered by "lines of defense" in which one system component may protect another system or component (e.g., a levee offering protection to a building or group of buildings).
- Connection of the built environment to socially important objectives, here minimizing permanent population dislocation and the extent and duration of temporary population dislocation.
- Building functionality, i.e., the availability of a building for its intended purpose, which depends on its level of damage and access to essential services [24], the loss of which can result in population dislocation and other social costs (e.g., school closures and business interruptions) [25].

The result of a run of the model is a set of optimal changes to the built environment that minimizes the extent and duration of temporary population dislocation and the extent of permanent dislocation while considering mitigation and recovery costs.

The mathematical programming model incorporates empirical relationships from, or is otherwise informed by, a range of modeling approaches developed in engineering and the social sciences:

- Population dislocation models [26] that estimate household responses to damages on an aggregated level; this approach is applied in hazard risk estimation tools, such as HAZUS [27] and MAEViz [28]. Dislocation models at an individual building-level [29–32]. Factors influencing dislocation go beyond building damage, including social characteristics, such as racial, demographic, and tenure status.
- Some models evaluate mitigation for buildings considering that building functionality is solely dependent on building damage [33–35], while others take into account, as is the case with NIST ARC, both damage and access to essential services in defining building functionality [7, 8, 36].
- Quantitative approaches for the recovery of buildings that include probabilistic models for recovery time evaluation and cost estimation [37–39]. Infrastructure system recovery models have limited their scope to the restoration of infrastructure components with limited or no consideration given to societal costs.

In addition, the mathematical programming model is designed to address other aspects of community resilience planning that require consideration and methodological approaches:

- Decision-making for low-probability, high-consequence events. The mathematical programming model includes a method for decision-making under uncertainty that allows for examining tradeoffs between cost and risk. A risk-averse decision approach is selected, where conditional-value-at-risk (CVaR) is defined as the risk measure.
- The folding of resilience into community plans [5, 6]. The design of the mathematical programming model supports this by including the ability to identify *alternative solutions or policies*, the “A” in NIST ARC. The mathematical programming model incorporates the mathematical programming technique *Modeling to Generate Alternatives (MGA)*[40–42] to explore the community's flexibility in meeting resilience and cost objectives.

1.3. Other Computational Tools

NIST ARC fits within a spectrum of available computational tools for community resilience planning. At one end, there are tools whose central aim is to make users aware of system interdependencies through the geospatial and interactive display of networks, such as All-Hazards Analysis (AHA) [43], Geospatial Risk and Resilience Assessment Platform (GRRASP) [44], and the Regional Resiliency Assessment Program (RRAP) Dependency Analysis Framework [45]. While such tools are valuable for gaining a community’s understanding of the importance of system interdependencies, they typically focus on connectivity alone. At the other end of the spectrum is the NIST-funded Center for Risk-Based Community Resilience Planning model and computational platform IN-CORE [46], which evaluates alternatives with resilience metrics through a spatial and temporal simulation model resolution chain and includes optimization methods. NIST ARC sits between these modeling approaches, making approximations to gain model tractability while capturing essential features of community resilience planning. All such tools together represent a comprehensive approach to community resilience planning, with graphical network tools playing a key role in initially scoping out and communicating interdependencies, NIST ARC as a screening tool to identify promising alternatives, and IN-CORE for higher resolution resilience analysis.

1.4. Report Scope

This report focuses on the mathematical programming model central to the NIST ARC. In addition to describing its equations, the model's ability to generate timely, meaningful solutions is tested here. The testing is conducted using data from a flood-impacted community, the City of Lumberton, North Carolina. Only a brief description of the decision support application built around the mathematical programming model is provided here as they are the subjects of subsequent reports.

1.5. Report Organization

The organization is as follows. **Section 2** describes NIST ARC’s mathematical programming model, including its scientific basis. Only the critical elements of the model required for a general understanding of the model are described; the entire model is given in **Appendix A.** Following the model's description, in **Section 3**, the results of the testing of the model are presented and discussed. This is followed by, in **Section 4**, a brief description of the decision support application built around the mathematical programming model. **Section 5** presents the plans for the tool’s further development and testing and concluding remarks.

2. Mathematical Programming Model

The goal of the NIST ARC mathematical programming model is to represent the interactions of the built environment and human social systems that impact a community’s resilience and to help identify optimal decisions to achieve resilience goals. The decisions here refer to recommended mitigation and recovery actions, which appear as variables in the model. Model parameters and restrictions present user-defined data, and physical modeling of the systems is captured with model constraints. The model objectives relate to the community’s resilience metrics and costs. When a model is solved, a solution is returned, corresponding to the variables' values, i.e., decisions regarding what mitigation and recovery actions should be taken that are feasible and optimal. Model constraints impose feasibility conditions, whereas the resilience goals, i.e., the objective function, are used to check optimality. The scope of the current model version is limited to a community with a built environment that includes critical infrastructure components and residential buildings and a human social system that is made up of people living in these buildings. Here, residential buildings are considered the interface between the built environment and the human social system. **Fig. 1** presents various components of a community's built environment, their connectivity and interdependencies, and their relationship with the human social systems. With the help of this figure, the decision-making problem and the approach for translating it into a mathematical programming model are discussed.

In **Fig. 1**, significant elements of the built environment and human social systems are labeled numerically. Three vertical panes visually represent a community in three stages: the mitigation stage before a hazard event, immediately after a hazard event, recovery stage after a hazard event. The rectangle with the dashed boundary represents a community in each pane, and its five layers show distinct community entity sets. The network structure represents the connectivity and interdependency among these entities. The network comprises distinct classes of nodes, vertices, and links, or arcs, connecting two nodes. Each network node represents a physical structure (e.g., power substation) or an abstract entity (e.g., neighborhoods). Directed arcs represent connections between two nodes, which are shown as arrows. Each directed arc has a starting node that can send a product or service and an end node receiving it.

The left pane represents the situation before a hazard event, where the built environment is presented by the top four layers and the human social system by the bottom layer. Layer "1" represents the infrastructure components that can protect against a hazard, i.e., protector components. An example (label "7") of such a component is a levee or a berm if the considered hazard is flooding. Arcs along this layer indicate that two protector components can be connected, with one receiving protection from another; for example, a levee can protect a berm, itself a flood protection measure, if the height of the levee surpasses that of the berm and flood elevation.

Other community elements can also be protected and will be discussed once they are introduced. Layers "2" and "3" represent utility service networks. Layer "2" represents, for example, an electric power network, and layer "3" is a potable water network. Arrows between two layers (i.e., networks) signify the dependencies among the nodes of the two networks. An example is the power requirement at a water treatment plant in a potable water network. Without receiving power from the electric power network, the water treatment plant is unable to process and supply potable water. If the nodes in the utility network components remain functional, i.e., their structural integrity remains intact, and required products are available for operation, service will remain uninterrupted at the terminal demand nodes. The term "service area" describes these terminal demand nodes, which are represented by gray rectangular blocks. Each service area comprises several neighborhoods. Examples of a neighborhood are a US Census block or an area with the same Zip Code. In layer "4" of the left pane, one service area is expanded to reveal three neighborhoods, each bounded by a small rectangle. Each neighborhood is characterized by demographic and socio-economic features, e.g., racial composition, average income, and the proportion of renters. A neighborhood contains residential buildings defined by different structural attributes, such as archetypes and hazard-resistant features. The buildings within a neighborhood serve as interfaces between the built environment and human social system, i.e., population, shown in layer "5". It is mentioned earlier that a protector component can provide protection to some of the other community elements. The arcs connecting a node with a protector component are termed “lines of defense.” Arc labeled “6a” represents the line of defense via which a protector component can protect a node in the electric power network, whereas the arc labeled “6b” shows the line of defense along which a neighborhood can receive protection. More on this topic is discussed in **Section 2.5**.

Community components have their initial ability to withstand hazards. However, mitigation actions can be taken to reinforce their abilities and thereby increase a community's resilience. Some of these mitigation decisions are alluded to in **Fig. 1** by green checkmarks (“✓”) on the left pane. For example, the checkmark above the protector node labeled “7” indicates that a mitigation action is taken to increase its hazard resistance. As such, it can protect the nodes that are connected to it via a line of defense (e.g., the node in layer “2” via arc “6a”). Mitigation actions can also be taken for utility nodes, including increasing resistance and product storage. An increase in resistance of a node in the electric power network node is shown by the checkmark above “8a”, whereas the checkmark below “8b” represents an increase in power storage (e.g., generator) at a node in the potable water network. Mitigation actions can also be taken to retrofit residential buildings to withstand hazards of higher intensity. Green checkmarks above buildings in layer 4 represent such actions, one of which is labeled “9”.

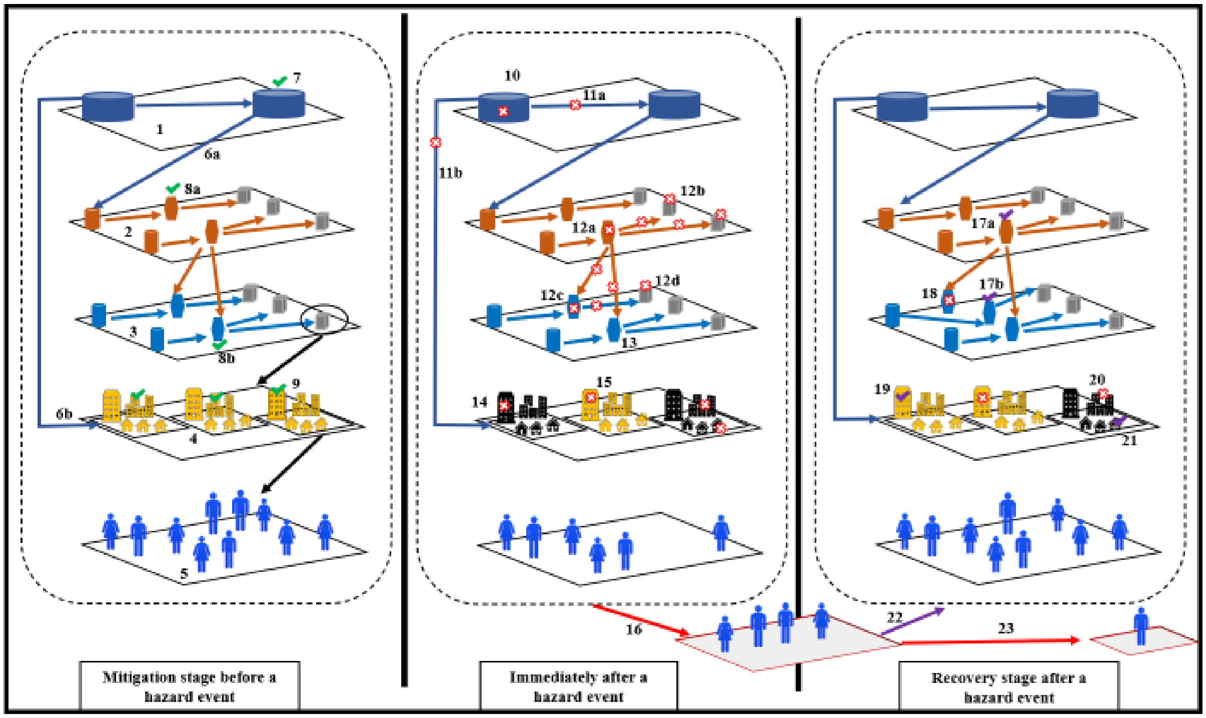

Fig. 1. Illustration of the built environment and human social system of a community

On the center pane of **Fig. 1**, the immediate aftermath of a hazard is presented. The top three layers represent the same set of built environment components as in the left pane, but the fourth layer shows neighborhoods from different service areas, and the bottom layer shows the effect of functionality losses in the built environment on the community population. In this pane, a red cross mark (“×”) placed on a node or arc represent the loss of functionality of the entity. The failure of a protector node is indicated by “10”, whereas “11a” indicates the failure of the line of defense connecting this node to the second protector node. Since resistance was added to the second node as a mitigation action (shown by “7”), it still functions as intended. Failure of the line of defense connecting the failed protector node with a neighborhood is denoted by “11b”. This results in damages in the corresponding neighborhood and buildings unless the buildings were retrofitted to withstand the hazard. For utility network nodes, loss of functionality can result from damage, unavailability of products, or both. Loss of functionality of a node in the electric power network due to damage is labeled “12a”. The arcs coming out of this node also lose functionality, as does the end node of these arcs. The loss of functionality at a service area due to the unavailability of electric power is indicated by “12b”. The loss of functionality of a water network node that is both damaged structurally and fails to receive power from the electric power network is labeled “12c”, which results in functionality loss in a service area due to the unavailability of potable water, indicated by “12d”. Label “13” points to the water network node that remains functional even though the flow of power from the electric power network is interrupted since mitigation action of increasing power storage was taken (shown by “8b”). The consequences of functionality loss in utility network nodes propagate as service outages to service areas, neighborhoods, and residential buildings. Buildings themselves are vulnerable and experience damage after a hazard event. The neighborhood labeled “14” experience service outages (indicated by shaded building facades) as well as building damages (red cross marks), whereas the neighborhood labeled “15” only suffered building damages without service interruption. Building damages and service outages result in societal consequences, such as population dislocation, which is indicated by label “16”. It shows people being dislocated from the community.

The right pane of the figure shows a community’s post-hazard recovery stage. The layers represent the same set of elements as in the middle pane. Purple check marks on this pane indicate recovery decisions. Repair and restoration of a node in the electric power network are labeled “17a”. On the other hand, a recovery decision for the water network entails the activation of an initially dormant node labeled “17b”. The node in the water network that lost functionality immediately after the hazard event due to damage and power unavailability (denoted by “12c”) remains damaged and non-functional, indicated by label “18”. Recovery of utility network nodes results in service restoration in the service areas, neighborhoods, and residential buildings. Service restoration in a neighborhood is denoted by labels “19” and yellow facades of buildings. Label “19” also indicates that the building damaged by the hazard (label “14”) is repaired so that its displaced occupants can return. Labels “20” and “21” indicates the neighborhood where services have not been restored so that the evacuated building cannot be reoccupied. Label “20” indicates that the buildings have not been repaired to allow reoccupation, whereas the label “21” denotes a repaired building; however, it cannot be reoccupied since services have not been restored. The result of building repair and service restoration is indicated by label “22”, which shows dislocated people's return to reoccupying their residential buildings. Some dislocated people, however, need to wait until services are restored in their neighborhoods, and their buildings are repaired sufficiently. A fraction of the population dislocates permanently due to failure to repair their residential buildings, as indicated by “23”.

In summary, some or all the components of the built environment experience damage after a hazard event. Depending on a component's resistance, dependence on other entities, and hazard load, it may lose functionality and result in unwanted consequences in the social system, such as population dislocation. For assessing community resilience, two metrics are considered in the presented model: population dislocation and recovery delay following a hazard. The purpose of the model is to help decision-makers to achieve resilience goals, i.e., minimize the costs associated with these metrics by recommending: (i) mitigation actions to minimize the number of people dislocating from the community and (ii) recovery actions to maximize the number of people returning to the community following dislocation with minimum delay. An optimization modeling approach is adopted with necessary assumptions and approximations to conform the model to a form amenable to efficient, well-established solution methods. A variety of data are needed to represent the attributes of the community entities, which are introduced as model parameters. Resilience planning decision levers, i.e., recommended actions for the community entities, are introduced as model variables, whereas restrictions and limitations imposed on these entities are introduced as model constraints. The objective function weighs various functions, i.e., mathematical expressions constructed with model parameters and variables, to achieve resilience goals. A set-theoretic approach is used to construct mathematical expressions for model variables, parameters, constraints, and objective functions.

With set theory, large mathematical programs can be expressed very concisely. A set is a collection of unique objects or object identifiers, each of which is called a set element. A set is described with its elements separated by commas and within a pair of braces. The binary relationship of whether an object is a member of a set is the fundamental aspect of set theory. If object a belongs to set A, the membership can be expressed as $a \in A$, whereas the relationship between an object d and set A, can be expressed as $d \notin A$, if it is not a member of A. Another useful concept of set theory is subsets. Set B is a proper subset of A, i.e., $B \subset A$, if all the elements of set B are also in set A, but not all elements of set A are in set B (i.e., sets A and B are not equal). Set operations, such as the union of sets, the intersection of sets, and set difference, are among the most used and therefore discussed here:

- The union of two sets A and C is denoted $A \cup C$, which results in a set of all elements that are a member of either set A, or set C, or a member of both sets. Mathematically, it is expressed as: $A \cup C = \{x : x \in A \; or \; x \in C\}$.
- The intersection of two sets A and C is denoted $A \cap C$, which results in a set of elements that are in members of both sets A and C. Mathematically, it is expressed as: $A \cap C = \{x : x \in A \; and \; x \in C\}$.
- The set difference of two sets A and C is denoted $A \backslash C$ and is the set of elements that are in set A but not in set C. Mathematically, it is expressed as: $A \backslash C = \{x : x \in A \; and \; x \notin C\}$.

A set can contain elements that are numbers, text, tuples etc. Set elements can be used as identifiers of objects. For example, if a set is defined so that its elements are the names of community components, each element is an identifier of the corresponding component. Tuples are used as identifiers of objects when more than one attribute is needed to represent it. A tuple is an ordered list of elements and is represented by its elements separated by commas within parentheses. The number of elements in a tuple is used to describe it; for example, a tuple with three elements is called a 3-tuple, and a tuple with n elements is an n-tuple. For example, two attributes are required to describe a utility network arc in the presented model: the arc's starting node and the arc's ending node. In a set defined to contain the arcs of a utility network, each element of the set is a 2-tuple.

Various sets are defined to represent the community and the decision-making problem. Set elements are used as identifiers of physical and abstract entities required to formulate the mathematical programming model. An index is used to represent an arbitrary element of a set, and indexed expressions are mapped to set elements. Indexed expressions include model parameters, variables, and constraints. Uppercase letters are used as symbols for sets and parameters. However, a single bar on top of the symbol is used to distinguish it from set notations for parameters. Variables are represented by lowercase letters. Using conventional notation practice, 'V' is used as a symbol to designate node sets, and 'A' is used to denote arc sets. Subscripts are used to represent the sets over which the variables, parameters, or sets are defined. Superscripts are used to provide information about the corresponding model components as well as distinguishing notation features.

Node set V is defined as the set containing all relevant community nodes. Three subsets of V are defined for distinct community components. These are set $V^{\mathrm{PR}+}$ that contains protector nodes, set V^{UT} containing all nodes in utility service networks, and set V^{N} containing nodes representing neighborhoods. Index i is generally used to denote elements of node sets. A line of defense, or arc connecting a node $j \in V$ with a protector node $i \in V^{\mathrm{PR}+}$, is contained in set A^{PR}, i.e., each element of A^{PR} is a 2-tuple representing the start and end node of a line of defense. Index (i, j) is used to denote the elements of this set. A fourth node subset $V^{\mathrm{PR}-}$ is defined as the set containing nodes that are connected to protector nodes and (potentially) receive protection along the lines of defense, i.e., $V^{\mathrm{PR}-} = \{j \in V\colon (i, j) \in A^{\mathrm{PR}}\}$. It should be noted that a node contained in V^{UT}, or V^{N}, or even $V^{\mathrm{PR}+}$may also be a member of the set $V^{\mathrm{PR}-}$ if it is connected to a protector node along a line of defense arc. More on this topic is discussed in **Section 2.5**. The set of all utility products is denoted by K, and is generally indexed by k. Notations for sets of nodes and arcs that are present in the network for utility product $k \in K$ are themselves subscripted by k. For example, set V_k^{UT} contains nodes that are present in the network for utility product k. Index i is used to represent elements of this set. Set A_k^{UT}, on the other hand, contains directed arcs in the network for utility product k. Elements of this set are 2-tuples, with each tuple containing identifiers of an arc's starting node, its ending node, and utility product, respectively, as its elements. To represent the elements of the set A_k^{UT}, index (i, j) is used. Two subsets of set V_k^{UT} are defined based on the nature of their function in the network. Utility network nodes $i \in V_k^{\mathrm{UT}}$ that have arcs originating from them are contained in the "output connection" node set $V_k^{\mathrm{UT}+}$, whereas the nodes $i \in V_k^{\mathrm{UT}}$ with incoming arcs are contained in "input connection" node set $V_k^{\mathrm{UT}-}$. Due to the functional interdependencies (discussed in **Section 2.9**) among the utility networks, an "output connection" node $i \in V_k^{\mathrm{UT}+}$ in network k can be an "input connection" node $i \in V_l^{\mathrm{UT}-}$ in network $l \in K$. Several "input connection" nodes $i \in V_k^{\mathrm{UT}-}$ in network for utility product k represent terminal demand nodes, i.e., service areas containing several neighborhoods. Each neighborhood $i \in V^{\mathrm{N}}$ consists of multiple residential buildings. Three sets associated with residential buildings are defined, set B contains all building archetypes, set and S contains all retrofit strategies, and set F for all damage states after an event. These sets are indexed by b, s, and f, respectively. Finally, set E is defined that contains all hazard events with index e used to represent an element of set E in mathematical expressions.

With the sets defined above, now the modeling approach can be described. The full description of model components, in a conventional format, is given in **Appendix A**, in which all variables, constraints, and the model's objective function are described in detail.

2.1. Resistance and Product Storage at Network Nodes

Initial hazard resistance of a utility network node $i \in V^{\mathrm{UT}}$ is represented by the parameter $\bar{R}_i^0$. Variable r_i is introduced as a mitigation decision. Variable r_i represents the *increase in resistance* of node $i \in V^{\mathrm{UT}}$. The upper bound on this variable is imposed by the parameter $\bar{R}_i^{\mathrm{U}}$, which denotes the maximum resistance that can be added. For a network node $i \in V_k^{\mathrm{UT}}$ in utility service $k \in K$, the initial storage of product k at the node is represented by the parameter $\bar{S}_{ik}^0$. This parameter indicates the maximum length of time that the node can remain functional without product k being received from an outside source. Variable s_{ik} is introduced, which represents the *increase in storage* of product k at node $i \in V_k^{\mathrm{UT}}$. The upper bound on the variable s_{ik} is imposed by the parameter $\bar{S}_{ik}^{\mathrm{U}}$.

2.2. Protector Nodes

As discussed in the above section, set $V^{\mathrm{PR}+}$ consists of protector nodes that can protect one or more protected nodes contained in $V^{\mathrm{PR}-}$. Lines of defense, or arcs connecting a protected node with a protector node, are contained in set A^{PR}. A protector node can be a levee for flood protection or a wildland-urban interface (WUI) buffer for wildfire protection. They offer protection to other nodes, but only if they are installed and only up to a certain level of protection.

The initial status of protector node i is denoted by the binary parameter $\bar{Z}_i^0$ ($Z_i^0 = 1$: already installed; $\bar{Z}_i^0 = 0$: otherwise), and the initial hazard resistance of an installed node is denoted by the parameter $\bar{R}_i^0$. The mitigation decisions for protector nodes are *whether to install* a protector node i, represented by the binary variable z_i^{N} (i.e., $z_i^{\mathrm{N}} = 1$: to be installed; $z_i^{\mathrm{N}} = 0$: otherwise) and *the increase in resistance,* r_i, at an installed protector node i.

2.3. Hazard Resistance and Storage Limits on Nodes

The limits on increase in resistance r_i are given by the following constraints:

$$r_i \leq \bar{R}_i^{\mathrm{U}}(\bar{Z}_i^0 + z_i^{\mathrm{N}}), \qquad \forall i \in V^{\mathrm{PR}+} \tag{1}$$

$$r_i \leq \bar{R}_i^{\mathrm{U}}, \qquad \forall i \in V \backslash V^{\mathrm{PR}+} \tag{2}$$

Constraint (1) indicates that resistance can be added to a protector node only if it is installed either initially ($\bar{Z}_i^0 = 1$) or it is newly installed ($z_i^{\mathrm{N}} = 1$); at most, one of these conditions can be met (i.e., $\bar{Z}_i^0 + z_i^{\mathrm{N}} \leq 1$). The notation $V \backslash V^{\mathrm{PR+}}$ in constraint (2) means the nodes in V that are not in $V^{\mathrm{PR+}}$.

For an increase in storage s_{ik}, the following constraint imposes the limit:

$$s_{ik} \leq \bar{S}_{ik}^{\mathrm{U}}, \qquad \forall k \in K, i \in V_k^{\mathrm{UT}} \tag{3}$$

2.4. Post-mitigation Hazard Resistance and Storage

Post-mitigation hazard resistance r_i^{PM} and storage s_{ik}^{PM} are defined for convenience to simplify the mathematical expressions of the later constraints.

$$r_i^{\mathrm{PM}} \leq \bar{R}_i^0 (\bar{Z}_i^0 + z_i^{\mathrm{N}}) + r_i, \qquad \forall i \in V^{\mathrm{PR+}} \tag{4}$$

$$r_i^{PM} \leq \bar{R}_i^0 + r_i, \qquad \forall i \in V \backslash V^{\mathrm{PR+}} \tag{5}$$

and storage s_{ik}^{PM}:

$$s_{ik}^{\mathrm{PM}} = \bar{S}_{ik}^0 + s_{ik}, \qquad \forall k \in K, i \in V_k^{\mathrm{UT}} \tag{6}$$

It should be noted here that in constraints (5), $\bar{R}_i^0$ represents the average hazard resistance of neighborhood node $i \in V^{\mathrm{N}}$ (for flooding, average first-floor elevation (FFE) of the buildings in neighborhood i).

2.5. Lines of Defense

Whether a protector node offers protection is dependent on both its hazard resistance and that of the (potentially) protected node. For example, if a building located just behind a levee has been elevated above the height of the levee, the levee offers no protection; the building would then be considered self-protected, represented by the binary variable z_i^{SP} (i.e., $z_i^{\mathrm{SP}} = 1$: self-protected; $z_i^{\mathrm{SP}} = 0$: parent-protected). The hazard resistance of the "child" node, in this example, the building, is the maximum afforded by its "parent," the levee, and itself. The parent node $i \in V^{\mathrm{PR+}}$ potentially, depending upon its post-mitigation resistance protects child node $j \in V^{\mathrm{PR-}}$. The set A^{PR} contains all such pairings (i, j); all children node j within A^{PR} are contained in the set $V^{\mathrm{PR-}}$. The hazard resistance of a utility node $j \in V^{\mathrm{PR-}}$ *after* considering parent protections is r_j^{EFF}. The following disjunctive constraints ensure this desired behavior:

$$r_i^{\mathrm{EFF}} \leq r_i^{\mathrm{PM}} + \bar{M}\left(1 - z_i^{\mathrm{SP}}\right), \qquad \forall i \in V^{\mathrm{PR-}} \tag{7}$$

$$r_j^{\mathrm{EFF}} \leq r_i^{\mathrm{PM}} + \bar{M}\, z_j^{\mathrm{SP}}, \qquad \forall (i,j) \in A^{\mathrm{PR}} \tag{8}$$

$\bar{M}$, a parameter with a large positive number as its value, enforces the condition that only one of these restrictions, i.e., one of (7) and (8), will have an effect.

The above constraints also can accommodate a hierarchy of protection, where the parent node $i \in V^{\mathrm{PR}+}$ of $(i,j) \in A^{\mathrm{PR}}$ itself may be a child of component $l \in V^{\mathrm{PR}+}$ via $(l,i) \in A^{\mathrm{PR}}$. For example, in Lumberton, NC [46, 47], while a levee offered a certain level of protection to the water treatment plant, a berm around the water treatment plant was constructed with a height designed to exceed that of the adjacent levee. Here, the levee is the parent of the berm, and the berm is the parent of the treatment plant.

For nodes that are not members of the set $V^{\mathrm{PR}-}$, which are not connected to any protector node, the following constraint gives their effective resistances:

$$r_i^{\mathrm{EFF}} = r_i^{\mathrm{PM}}, \qquad \forall i \in V \backslash V^{\mathrm{PR}-} \tag{9}$$

The representation of buildings and people in neighborhoods is described in later sections. It is important to note here, however, that neighborhoods can also be protected via lines of defense.

2.6. Mitigation Constraints for Residential Buildings

Each building in the community, regardless of its location and archetype, follows a retrofit strategy before mitigation actions are taken. The hazard resistance of a building depends on the (initial or newly) adopted retrofit strategy, its archetype, and the average hazard resistance of the neighborhood in which the building is located according to the following constraint:

$$\bar{R}_{ibs}^{\mathrm{BUILD}} = r_i^{\mathrm{EFF}} + \bar{\Delta}_{bs}^{\mathrm{RESISTANCE}}, \qquad \forall i \in V^{\mathrm{N}}, b \in B, s \in S \tag{10}$$

Here, $\bar{R}_{ibs}^{\mathrm{BUILD}}$ denotes the hazard resistance of the buildings of archetype b with retrofit strategy s in neighborhood i; $\bar{\Delta}_{bs}^{\mathrm{RESISTANCE}}$ is the improvement in hazard resistance if retrofit strategy s is applied to buildings of archetype b. The mitigation decision variable $n_{ibss'}^{\mathrm{RFIT}}$, i.e., how many buildings of archetype b and retrofit strategy s in neighborhood i should be upgraded with a better strategy s', is restricted by the following constraint.

$$n_{ibs}^{\mathrm{PM}} = \bar{N}_{ibs}^{0} + \sum_{s' \in S} n_{ibs's}^{\mathrm{RFIT}} - \sum_{s' \in S} n_{ibss'}^{\mathrm{RFIT}}, \qquad \forall i \in V^{\mathrm{N}}, b \in B, s \in S \tag{11}$$

Here, $\bar{N}_{ibs}^{0}$ and n_{ibs}^{PM} are the number of buildings with the retrofit strategy before and after mitigation actions are implemented.

2.7. Recovery Constraints for Infrastructure Components

Recovery decisions are made in the second stage of the model. At this stage, mitigation actions have already been implemented, and the hazard loadings have become known. The second-stage variables are defined over each event $e \in E$. As such, the variables and constraints in this stage are indexed by the events.

2.8. Post-event Integrity, Recovery, and Activation of Utility Network Nodes

The binary variable y_{ie}^{S} represents the post-event structural integrity of a utility node ($y_{ie}^{\mathrm{S}} =$ 1: survival; $y_{ie}^{\mathrm{S}} = 0$: failure). A node survives if its effective resistance exceeds the hazard loading L_{ie}. The following constraint enforces this restriction:

$$r_i^{\mathrm{EFF}} - L_{ie} \geq \bar{M}\left(y_{ie}^{\mathrm{S}} - 1\right), \qquad \forall i \in V, e \in E \tag{12}$$

Based on a node's post-event survival, two types of decisions can be made- whether a failed node should be recovered ($y_{ie}^{\mathrm{R}} = 1$: recovered; $y_{ie}^{\mathrm{R}} = 0$: not recovered), and whether a survived node that was dormant before the event should be activated ($y_{ie}^{\mathrm{A}} = 1$: activated; $y_{ie}^{\mathrm{A}} = 0$: not activated). The following constraints enforce the restrictions on these decisions:

$$y_{ie}^{\mathrm{R}} \leq 1 - y_{ie}^{\mathrm{S}}, \qquad \forall i \in V, e \in E \tag{13}$$

$$y_{ie}^{\mathrm{A}} \leq y_{ie}^{\mathrm{S}}(1 - \bar{Y}_i^0), \qquad \forall i \in V, e \in E \tag{14}$$

Here, parameter $\bar{Y}_i^0$ denotes the initial status of node i ($\bar{Y}_i^0 = 1$: in use; $\bar{Y}_i^0 = 0$: dormant).

2.9. Functional Dependencies

For each product $k \in K$, a network arc is represented by a tuple $(i, j) \in A_k^{\mathrm{UT}}$ that represents the connectedness between nodes $j \in V_k^{\mathrm{UT}}$ and $i \in V_k^{\mathrm{UT}}$. Recovery decisions include *whether arc* $(i, j) \in A_k^{\mathrm{UT}}$ *will be used* to send product $k \in K$, which is represented by the binary variable y_{ijke}^{FLOW} ($y_{ijke}^{\mathrm{FLOW}} = 1$: is in use; $y_{ijke}^{\mathrm{FLOW}} = 0$: not in use). The usability of an arc depends on the condition of the end nodes. Arc $(i, j) \in A_k^{\mathrm{UT}}$ can only be used if both end nodes have their integrity intact, recovered, or activated from dormant states.

$$y_{ijke}^{\mathrm{FLOW}} \leq y_{ie}^{\mathrm{S}} \bar{Y}_i^0 + y_{ie}^{\mathrm{R}} + y_{ie}^{\mathrm{A}}, \qquad \forall k \in K, (i, j) \in A_k^{\mathrm{UT}}, e \in E \tag{15}$$

The amount of flow of product k along an arc $(i, j) \in A_k^{\mathrm{UT}}$ is restricted by the capacity of the arc $\bar{Q}_{ijk}^{\mathrm{U}}$ and usability of the arc y_{ijke}^{FLOW}.

$$q_{ijke}^{\text{FLOW}} \leq \bar{Q}_{ijk}^{\text{U}} y_{ijke}^{\text{FLOW}}, \qquad \forall k \in K, (i,j) \in A_k^{\text{UT}}, e \in E \tag{16}$$

Connectivity, dependency, and redundancy among nodes of the networks are modeled with the help of the network architecture. To capture these concepts, the sets "input connections," or $V_k^{\text{UT}-}$, and "output connections" or $V_k^{\text{UT}+}$ are used to represent utility network nodes of product k. An illustrative example is given in **Fig. 2**, where node i is the receiving node of arcs in networks of products a and b, i.e., $i \in V_a^{\text{UT}-}$ and $i \in V_b^{\text{UT}-}$. Node i depends on predecessor nodes for receiving products a and b as inputs. Two predecessor nodes (nodes 1 and 2) supply product a, and one predecessor node (node 3) supplies product b.These inputs go towards meeting node i's own demand and to generate product c to send on to successor nodes along the arcs for product c. Therefore, node i is a source node of an arc for sending product c, i.e., $i \in V_c^{\text{UT}+}$. Node i produces product c and supplies it to two successor nodes (nodes 4 and 5). If, for example, node i represented a water treatment plant, a may be the product 'power' and b the product 'raw untreated water,' and c the product 'treated water.'

Note in the illustrative example, too, that redundancy is captured. There is redundancy in the connectivity supplying a, whereas there is no such redundancy in the supply of b.

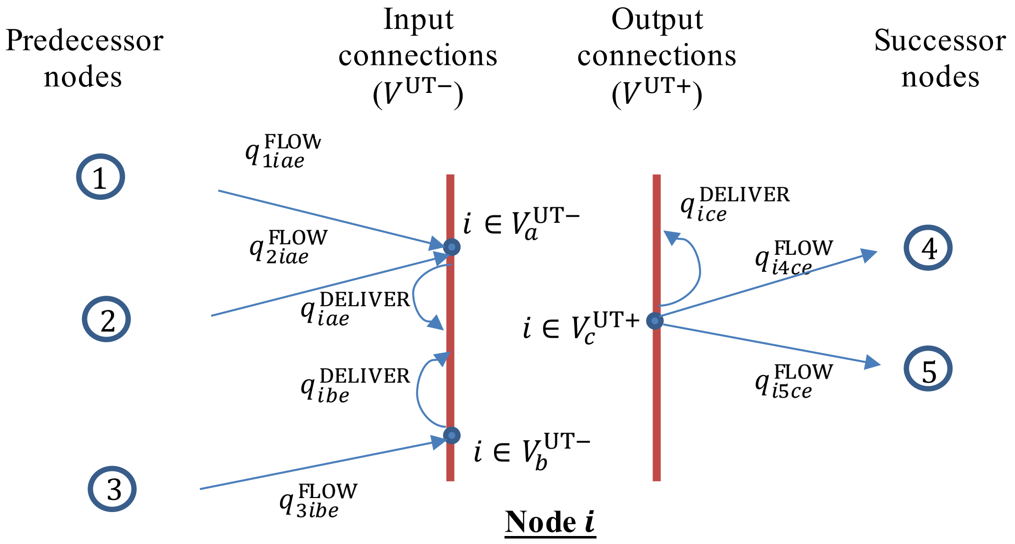

Fig. 2. Illustration of the modeling of one network node, node i.

A key assumption of the model is that there is a linear input-output relationship at utility network nodes. In the water treatment example, generating one unit of treated water at the treatment plant requires one unit of raw water and a certain amount of power. The parameter $\overline{\text{H}}_{ikl}$ represents the functional dependency at node $i \in V_k^{\text{UT}+}$, which is the minimum requirement for product l to produce a unit amount of product k at node i. The relationship between the amount of input and output of products at node $i \in V_k^{\text{UT}+}$ is dictated by the following constraint.

$$q_{ile}^{\mathrm{SUPPLY}} + \sum_{(j,i)\,\in A_l^{\mathrm{UT}}} q_{jile}^{\mathrm{FLOW}} \geq \overline{\mathrm{H}}_{ikl} \left(q_{ike}^{\mathrm{DELIVER}} + \sum_{(i,j)\,\in A_k^{\mathrm{UT}}} q_{ijke}^{\mathrm{FLOW}} \right) \qquad \forall k \in K, l \in K, i \in V_k^{\mathrm{UT+}}, e \in E \tag{17}$$

For "output connection" nodes $i \in V_k^{\mathrm{UT+}}$ without functional dependencies, the total outgoing flow of the product is capped at $q_{ike}^{\mathrm{SUPPLY}}$, the amount of product supply that can be added at the node. Similarly, the total delivered amount $q_{ike}^{\mathrm{DELIVER}}$ at "input connection" node $i \in V_k^{\mathrm{UT-}}$ is capped at total incoming flow. The bounds on the amount of product k supplied from and delivered to utility network nodes are enforced by the following constraints respectively:

$$q_{ike}^{\mathrm{SUPPLY}} \leq \bar{Q}_{ik}^{\mathrm{S}}, \qquad \forall k \in K, i \in V_k^{\mathrm{UT+}}, e \in E \tag{18}$$

$$q_{ike}^{\mathrm{DELIVER}} \geq \bar{Q}_{ik}^{\mathrm{D}}, \qquad \forall k \in K, i \in V_k^{\mathrm{UT-}}, e \in E \tag{19}$$

2.10. Recovery and Service Restoration Times of Utility Network Components

Critical measures of resilience include recovery times ($t_{ie}^{\mathrm{RECOVERY}}$) of utility network nodes and service restoration times ($t_{ike}^{\mathrm{RESTORE}}$). The recovery time of a node is the latest of the elapsed times required for the restoration of the services. The service restoration time of a component is the latest of: (i) the elapsed time for integrity recovery and production startup time and (ii) the recovery time of the component(s) from which it receives supply.

In the case where a component remains functional before and after a hazard, i.e., $y_{ike}^{\mathrm{F}} = 1$, there is no delay in recovering that component. The determination of whether a component remains functional throughout, i.e., the determination of y_{ike}^{F}, is constrained by:

$$y_{ike}^{\mathrm{F}} \leq y_{ie}^{\mathrm{S}} \bar{Y}_i^0, \qquad \forall k \in K, i \in V_k^{\mathrm{UT}}, e \in E \tag{20}$$

$$s_{ik}^{\mathrm{PM}} - t_{ike}^{\mathrm{RESTORE}} \leq \bar{M}(y_{ike}^{\mathrm{F}} - 1), \qquad \forall k \in K, i \in V_k^{\mathrm{UT}}, e \in E \tag{21}$$

Note that the availability of storage, s_{ik}^{PM}, figures into whether the component remains functional. The component will remain functional if sufficient storage covers the time before inputs are restored.

2.11. Recovery Decisions for Residential Buildings

Two different methodologies are integrated into the model to make recovery decisions regarding residential buildings and determine the effect of these decisions on societal costs and benefits. The first is an engineering approach for characterizing the uncertainty in residential building damages conditioned on hazard loading. The second approach quantifies the societal impact of residential building damage and subsequent recovery decisions. These two approaches are described in brief in the following sub-sections.

2.11.1. Damage States of Residential Buildings

A probabilistic model that utilizes the fragility functions for residential buildings is used to characterize the uncertainty in residential building damage following a hazard. In structural engineering studies, a common approach is to categorize post-hazard building damages into several states. An ordered set $F = \{f_1, f_2, \dots, f_{n_D}\}$ is defined that containing all damage states ordered by the increasing degree of damage, where n_D denotes the total number of damage states. For a hazard event, $e \in \mathrm{E}$, a building of archetype $b \in B$ with post-mitigation retrofit strategy $s \in S$ in neighborhood $i \in V^{\mathrm{N}}$ may fall into each damage state $f \in F$ with specific probabilities depending on the hazard loading ($\bar{L}_{ie}$) and the building's resistance ($\bar{R}_{ibs}^{\mathrm{BUILD}}$). Fragility functions for the buildings are used to calculate the exceedance probabilities of damage states resulting from hazard loading. Log-normal distributions are the most suitable for characterizing these distributions. The probabilities of buildings falling in a damage state that is equal to or worse than f conditioned on the uncertain deficit of resistance $X = \left(\bar{L}_{ie} - \bar{R}_{ibs}^{\mathrm{BUILD}}\right)$ are as follows:

$$P\left[DS \geq f \middle| X = \left(\bar{L}_{ie} - \bar{R}_{ibs}^{\mathrm{BUILD}}\right)\right] = \Phi\left(\frac{\ln\left(\bar{L}_{ie} - \bar{R}_{ibs}^{\mathrm{BUILD}}\right) - \mu_{bf}}{\xi_{bf}}\right) \tag{22}$$

In (22), 'DS' is a random variable representing the building damage state, μ_{bf} and ξ_{bf} are the median capacity and the standard deviation of the natural log of resistance capacity to resist damage state f by the buildings of archetype b, respectively. The probabilities of these buildings to be in each damage state $f \in F$, where $F = \left\{f_1, f_2, \dots, f_{n_D}\right\}$, can be expressed as the following:

$$\begin{aligned} &P\left[DS = f_k \middle| X = \left(\bar{L}_{ie} - \bar{R}_{ibs}^{\mathrm{BUILD}}\right)\right] \\ &= \begin{cases} 1 - P\left[DS \geq f_k \middle| X = \left(\bar{L}_{ie} - \bar{R}_{ibs}^{\mathrm{BUILD}}\right)\right], & k = 1 \\ P\left[DS \geq f_k \middle| X = \left(\bar{L}_{ie} - \bar{R}_{ibs}^{\mathrm{BUILD}}\right)\right] - P\left[DS \geq f_{k+1} \middle| X = \left(\bar{L}_{ie} - \bar{R}_{ibs}^{\mathrm{BUILD}}\right)\right], & 2 \leq k \leq (n_D - 1) \\ P\left[DS \geq f_k \middle| X = \left(\bar{L}_{ie} - \bar{R}_{ibs}^{\mathrm{BUILD}}\right)\right], & k = n_D \end{cases} \end{aligned} \tag{23}$$

The probabilistic model estimates the probabilities of a building falling into various damage states conditional upon the hazard load and building resistance. The state $f \in F$ with the maximum probability is set as the post-hazard event damage state of the building. The parameter $\bar{W}_{ibsfe}^{\mathrm{DS}}$ is set to 1 for the appropriate combination of archetype b, retrofit strategy s, neighborhood $i \in V^{\mathrm{N}}$, damage state f, and hazard event $e \in E$.

Using parameter $\bar{W}_{ibsfe}^{\mathrm{DS}}$ and post-mitigation retrofit strategies for buildings n_{ibs}^{PM}, the number of buildings falling into each damage under each event is determined by the model as follows:

$$n_{ibfe}^{\mathrm{DS}} = \sum_{s \in S} n_{ibs}^{\mathrm{PM}} \, \bar{W}_{ibsfe}^{\mathrm{DS}}, \qquad \forall i \in V^{\mathrm{N}}, b \in B, f \in F, e \in E \tag{24}$$

2.11.2. Population Dislocation due to Residential Building Damage

A population dislocation model is used to quantify a hazard's societal impact, i.e., dislocation due to building damage, subsequent repair, and reoccupation. This model, adopted from the social science field, utilizes demographic characteristics of the hazard-affected region and building damage states to determine the thresholds for population dislocation and reoccupation. The probability that occupants will dislocate from buildings is calculated using one of several variants of the population dislocation algorithm. A commonly used version of the algorithm [26] based on the logistic regression approach is:

$$\bar{P}_{ibf}^{\mathrm{DISLOCATION}} = \frac{1}{1 + e^{-\left(\beta_0 + \beta_1 \bar{P}_{bf}^{\mathrm{LOSS}} + \beta_2 \bar{P}_i^{\mathrm{RENT}} + \beta_3 \bar{P}_i^{\mathrm{AMI}} + \beta_4 \bar{P}_i^{\mathrm{HISP}}\right)}} \tag{25}$$

Here, $\bar{P}_{bf}^{\mathrm{LOSS}}$ represents the proportion of property value loss if a building of archetype b falls into damage state f, $\bar{P}_i^{\mathrm{RENT}}$ denotes the proportion of renters in neighborhood i, $\bar{P}_i^{\mathrm{AMI}}$ and $\bar{P}_i^{\mathrm{HISP}}$ denote the proportion of Native American and Hispanic populations in neighborhood i, respectively. Parameters $\beta_1, \beta_2, \beta_3, \beta_4$ are the logistic regression coefficients.

A binary parameter $\bar{W}_{ibf}^{\mathrm{DMDL}}$ is introduced to indicate whether buildings of archetype b and damage state f in neighborhood i reach a dislocation threshold, $\bar{P}^{\mathrm{THRESHOLD}}$. The threshold parameter is set after considering the hazard type and affected region. Values for parameter $\bar{W}_{ibf}^{\mathrm{DMDL}}$ are determined as follows:

$$\bar{W}_{ibf}^{\mathrm{DMDL}} = \begin{cases} 1, & if \quad \bar{P}_{ibf}^{\mathrm{DISLOCATION}} \geq \bar{P}^{\mathrm{THRESHOLD}} \\ 0, & \quad otherwise \end{cases}, \tag{26}$$

$$\forall i \in V^{\mathrm{N}}, b \in B, f \in F$$

Using parameter $\bar{W}_{ibf}^{\mathrm{DMDL}}$ and the number of buildings in damage state f, the following constraint enforces the lower bound on the number of buildings of archetype b in neighborhood i that reach the dislocation threshold due to damage considering the neighborhood survival status:

$$n_{ibe}^{\mathrm{DMDL}} \geq \sum_{f \in F} n_{ibfe}^{\mathrm{DS}} \bar{W}_{ibf}^{\mathrm{DMDL}} - \bar{M} y_{ie}^{\mathrm{S}}, \qquad \forall i \in V^{\mathrm{N}}, b \in B, e \in E \tag{27}$$

2.11.3. Repair and Reoccupation of Residential Buildings

From the dislocation algorithm, the reoccupation threshold, i.e., the worst damage state that allows reoccupation, is also identified. Binary parameter $\overline{W}_{ibf}^{\mathrm{DMRO}}$ is introduced to indicate whether occupants dislocated from a building of archetype b in neighborhood i can reoccupy if the building is in the damage state f after repair. Values for parameter $\overline{W}_{ibf}^{\mathrm{DMRO}}$ are determined as follows:

$$\overline{W}_{ibf}^{\mathrm{DMRO}} = \begin{cases} 1, & if \quad f \leq \operatorname*{argmax}_{f' \in F} \left\{ \overline{P}_{ibf'}^{\mathrm{DISLOCATION}} : \overline{P}_{ibf'}^{\mathrm{DISLOCATION}} < \overline{P}^{\mathrm{THRESHOLD}} \right\} \\ 0, & otherwise \end{cases}, \tag{28}$$

$$\forall i \in V^{\mathrm{N}}, b \in B, f \in F$$

The number of repaired buildings $n_{ibff'e}^{\mathrm{REPAIR}}$ and post-repair number of buildings in each damage state n_{ibfe}^{PR} are restricted by the following constraint:

$$n_{ibfe}^{\mathrm{PR}} = n_{ibfe}^{\mathrm{DS}} + \sum_{f' \in F : f' > f} n_{ibf'fe}^{\mathrm{REPAIR}} - \sum_{f' \in F : f' < f} n_{ibff'e}^{\mathrm{REPAIR}}, \tag{29}$$

$$\forall i \in V^{\mathrm{N}}, b \in B, f \in F, e \in E$$

The upper bound on the number of post-repair reoccupied buildings n_{ibe}^{DMRO} is imposed by the following constraint:

$$n_{ibe}^{\mathrm{DMRO}} \leq \sum_{f \in F} \sum_{f' \in F : f' < f} n_{ibff'e}^{\mathrm{REPAIR}} \overline{W}_{ibf'}^{\mathrm{DMRO}}, \tag{30}$$

$$\forall i \in V^{\mathrm{N}}, b \in B, e \in E$$

Here n_{ibe}^{DMRO} is the number of buildings of archetype b in neighborhood i that only experience temporary dislocation; occupants dislocating from these buildings eventually reoccupy after repair completion. The quantity ($n_{ibe}^{\mathrm{DMDL}} - n_{ibe}^{\mathrm{DMRO}}$) represents the number of buildings of archetype b in neighborhood i whose occupants dislocate permanently.

2.11.4. Population Dislocation due to Service Outage

Apart from building damage, product, i.e., utility service outages, can also cause population dislocation. Those that do dislocate because of a service outage only reoccupy their buildings after services are restored. A terminal demand node or service area and all neighborhoods therein share the same service restoration time. The binary variable $x_{ie}^{\mathrm{OT}}, i \in V^{\mathrm{N}}$ is introduced to identify whether a neighborhood experiences a service outage, which occurs if the service restoration time t_{ike}^{RESTORE} at the corresponding service area $j \in V_k^{\text{UT}-}: \bar{I}_{ijk}^{\text{M}} = 1$ exceeds the neighborhood's tolerance period $\overline{\text{T}}_i^{\text{TOL}}$, i.e., the length of time it can sustain without a utility service. It is assumed that a building may reach the dislocation threshold due to a service outage only if it has not been retrofitted and does not reach the damage-induced dislocation threshold. The lower bound on the number of buildings that reach the dislocation threshold in a neighborhood due to service outage is imposed by the following constraint:

$$n_{ie}^{\text{OTDL}} \geq \sum_{b \in B}\left(\sum_{s \in S \cap S'} n_{ibs}^{\text{PM}} - n_{ibe}^{\text{DMDL}}\right) - \bar{M}\left(1 - x_{ie}^{\text{OT}}\right), \tag{31}$$

$$\forall i \in V^{\text{N}}, e \in E$$

Occupants of buildings dislocate only temporarily due to service outages and return after services are restored at the corresponding service area.

2.11.5. Recovery Times of Residential Neighborhoods

Building repair times and service restoration times dictate the recovery times of neighborhoods. The repair completion time of buildings of archetype b with damage state f in neighborhood i are determined according to the following constraints:

$$t_{ibfe}^{\text{REPAIR}} \geq \left(\overline{\text{T}}_i^{\text{DELAY}} + \overline{\text{T}}_{bff'}^{\text{REPAIR}}\right) x_{ibff'e}^{\text{REPAIR}}, \tag{32}$$

$$\forall i \in V^{\text{N}}, b \in B, f \in F, f' \in F, e \in E: f' < f$$

Here, $\overline{\text{T}}_i^{\text{DELAY}}$ denotes the average time to begin repair in the neighborhood, $\overline{\text{T}}_{bff'}^{\text{REPAIR}}$ denotes the average time of repair to improve the damage state from f to f', and binary variable $x_{ibff'e}^{\text{REPAIR}}$ indicates whether any building has undergone such repair. The recovery time of a neighborhood is the latest of the building repair completion time and the service restoration time in that neighborhood.

2.12. Objective Function and the Optimization Model

The objective of the mathematical programming model is presented as a mean-risk function containing two components, the expected total costs, and conditional value-at-risk ($CVaR_\alpha$). Expected total costs include event-agnostic mitigation costs and event-dependent expected recovery costs. The $CVaR_\alpha$ represents the risk measure associated with the variability of the expected total cost. The model allows the decision maker to set a risk preference by introducing two parameters, α and γ. Parameter γ represents the exchange rate of the expected total cost for risk, whereas parameter α denotes the confidence level. In the presence of uncertainty, the expected total cost distribution is a random variable, and the value-at-risk (VaR_α) is the α-quantile of the distribution of this random variable, i.e., the total expected cost exceeds VaR_α with a probability of $(1-\alpha)$. The resulting optimization problem can be expressed as follows:

$$\min_{x\in\mathcal{X}}\{\mathbb{E}[f(x,e)]+\gamma CVaR_\alpha(f(x,e))\} \tag{33}$$

where $\mathbb{E}[f(x,e)]$ is the expected total cost and $CVaR_\alpha(f(x,e))$ denotes the conditional expected total cost value exceeding the value-at-risk (VaR_α) at confidence level α. A reformulation of the optimization problem (31) can be expressed following [48, 49]:

$$Minimize\left\{(1+\gamma)\bar{\mathbf{c}}^{\mathrm{T}}\boldsymbol{x}+\sum_{e\in E}\lambda_e\bar{\mathbf{q}}_e^T\boldsymbol{y}_e+\gamma\left(\eta+\frac{1}{1-\alpha}\sum_{e\in E}\lambda_e v_e\right)\right\} \tag{34}$$

subject to:

$$W_e\boldsymbol{y}_e=h_e-T_e\boldsymbol{x},\qquad \forall e\in E \tag{35}$$

$$\boldsymbol{x}\in\mathcal{X} \tag{36}$$

$$\boldsymbol{y}_e\in\mathcal{Y},\qquad \forall e\in E \tag{37}$$

$$v_e\geq\bar{\mathbf{q}}_e^T\boldsymbol{y}_e-\eta,\qquad \forall e\in E \tag{38}$$

$$\eta\in\mathbb{R},\ v_e\geq 0,\qquad \forall e\in E \tag{39}$$

The mean-risk objective function of the model is presented in Eq. (34). Event-independent first-stage mitigation cost functions are presented by $\bar{\mathbf{c}}^{\mathrm{T}}\boldsymbol{x}$, where $\bar{\mathbf{c}}^{\mathrm{T}}$ is the cost vector, and $\boldsymbol{x}$ is the mitigation decision vector. The second stage expected recovery cost function, is presented by $\sum_{e\in E}\lambda_e\bar{\mathbf{q}}_e^T\boldsymbol{y}_e$, where $\bar{\mathbf{q}}_e^T$ and $\boldsymbol{y}_e$ are the recovery cost and decision vectors for event e, respectively and λ_e is the probability of the event e. Conditional value-at-risk $CVaR_\alpha$ is presented by $\left(\eta+\frac{1}{1-\alpha}\sum_{e\in E}\lambda_e v_e\right)$, where η represents the α-quantile of the recourse cost distribution and v_e is the amount by which the recourse cost exceeds η under the event e. Constraint set that enforces the relationship between mitigation decision $\boldsymbol{x}$ and recovery decisions $\boldsymbol{y}_e$ are presented in Eq. (35), where W_e, T_e are matrices and h_e is a vector of appropriate dimensions. Set $\mathcal{X}$ in Eq. (36) contains constraints that restrict mitigation decisions $\boldsymbol{x}$, whereas $\mathcal{Y}$ in Eq. (37) represents the constraint set that enforces restrictions on recovery decisions $\boldsymbol{y}_e$. Constraint set in Eq. (38) provides lower bounds for event-specific recourse costs exceeding α-quantile, and Eq. (39) represents variable bounds for η and v_e.

To consider hazard events occurring randomly within a planning horizon and the equivalent present values of the costs and risks associated with these events, an infinite planning horizon and a fixed discount rate ρ are considered. Moreover, it is assumed that each hazard event $e \in \mathrm{E}$ is a Poisson event with an annual occurrence probability $\lambda_{\boldsymbol{e}}$. As such, the objective function's expected recourse cost and conditional value-at-risk components presented in (34) are converted to present values using appropriate multipliers. The mitigation cost components are assumed to be given in their present value terms considering the operations and maintenance costs over their useful lives. With these assumptions, the mean-risk objective function of the model is constructed as follows:

$$Minimize \quad \left[(1+\gamma)Mitigation\_Cost + \left(\frac{1}{\rho}\right)\sum_{e \in E} \lambda_e (Recourse\_Cost)_{\boldsymbol{e}} + \left(\frac{\gamma}{\rho}\right)\left(\eta + \frac{1}{1-\alpha}\sum_{e \in E} \lambda_e v_e\right)\right] \tag{40}$$

In Eq. (40), $Mitigation\_Cost$ represent the cost of mitigation actions, i.e., increasing resistance and product storage at utility network nodes and improving retrofit strategies of residential buildings. Costs of taking recovery actions and penalty costs resulting from dislocation and recovery delays in event e are represented by $(Recourse\_Cost)_{\boldsymbol{e}}$. The entire model is presented in Appendix A, and Appendix B presents the justification for the adjustment made in (39) to obtain equivalent present values for the expected recourse cost and the risk measure.

2.13. Modeling to Generate Alternatives (MGA)

MGA presumes that alternatives composed of maximally different actions will vary with respect to the other important community objectives that are not addressed directly by the mathematical programming model. The Modeling to Generate Alternatives (MGA) approach [40–42, 50, 51] can be used to obtain alternatives that may be preferable when also considering unmodeled objectives. A brief outline of the MGA approach is described here. After the mathematical programming model, as in (33)-(38), is solved and the optimal solution obtained, the following model is constructed.

$$Minimize \quad \sum_{i \in I} \mathrm{x}_i \tag{41}$$

Subject to:

$$g(\mathbf{x}) \leq T \tag{42}$$

$$\mathbf{x} \in \mathcal{X} \tag{43}$$

Here, $\mathbf{x}$ represents the vector of decision variables, and I contains the indices of variables with nonzero values in the optimal solution of the original model with $g(\mathbf{x})$ as its modeled objective function. The parameter T is obtained by adding a slack amount (typically 5%-10% of the objective function value) to the optimal objective function value of the original model. The MGA approach now solves the model presented in Eq. (41)-(43), which has the same set of original model constraints and the one additional constraint shown in Eq. (41); this new constraint imposes an upper bound on the modeled objective function. The objective function of the MGA model ensures that the MGA solutions are maximally different in decision space, while constraint in Eq. (42) ensures that the solutions explored are near in objective space (in terms of the modeled objective function), and Eq. (43) represents the constraints that define the feasible region for $\mathbf{x}$. The solution to the original model, and the resultant solution to the MGA model, the approach presumes, may well perform quite differently against other important community objectives.

2.14. Mathematical Programming Model Extension Plans

In the current version of NIST ARC, only a part of the community is considered. Planned improvements include the incorporation of other entities, e.g., business centers, schools, and transportation systems. Following a hazard event, these community elements, too, are disrupted, impacting people in the community in direct and indirect ways. Other planned future extensions include the consideration of additional hazards. In the next year, FY23, the model will be tested for an earthquake case study, leading to some changes to the mathematical programming model. In FY24, uncertainty associated with the currently considered deterministic parameters will be introduced into the model.

3. The example used for testing

The mathematical programming model has been tested using data for a riverine flood hazard-impacted community, the City of Lumberton, North Carolina, which has been the focus of a longitudinal co-led by NIST and the NIST-funded Center for Risk-Based Community Resilience Planning [47, 52–54]. The testing here is limited to demonstrating the ability to generate solutions for a realistic problem in an amount of time that is supportive of interactive decision-making. The dataset is taken from a case study that forms the basis of a subsequent report detailing the application and use of the NIST ARC tool. It goes into much greater detail on the community than the description that follows.

The input dataset was prepared with data from the study of the hazard-affected community that Hurricane Matthew impacted in October 2016 and Hurricane Florence in September 2018. Two essential services are considered in the test case: electric power and potable water. Abridged versions of these networks are considered to represent the connectivity and interdependencies among various components. Data associated with building archetypes, average monetary values of buildings, damage states, fragility functions, and neighborhood characteristics are obtained from IN-CORE [55] and available literature [23, 37]. Parameters associated with the population dislocation algorithm and restoration and startup times of infrastructure components are obtained from technical investigation reports [47, 52]. US Census tracts are considered as the 'service areas', i.e., the terminal demand nodes of the utility networks. Each census tract contains a collection of census blocks corresponding to the neighborhood set considered in the model. The case study considers 693 US Census blocks grouped in 11 service areas. Socio-demographic characteristics, e.g., the average proportion of the Black and American Indian population and the average renter proportion of these neighborhoods, are used in the population dislocation algorithm. The study area consists of a total of 7,254 residential buildings. Each residential building belongs to one of four archetypes: (i) One-story residential building on a crawlspace foundation, (ii) One-story residential building on a slab-on-grade foundation, (iii) Two-story residential building on a crawlspace foundation, and (iv) Two-story residential building on a slab-on-grade foundation. Four retrofitting strategies are considered: baseline strategy or strategy-0, strategy-1, strategy-2, and strategy-3. Five post-event damage states for residential buildings are considered. As such, the ordered set F in the model contains five damage states, starting with no damage and ending with complete damage, and with slight, moderate, and high damage states in between. The data for tolerance periods of neighborhoods in withstanding service outages randomly within the range of [3,10] days. To guide the model to prioritize more vulnerable neighborhoods, the social vulnerability indices of the neighborhoods (e.g., SoVI) [56] are used to inflate the cost parameters associated with dislocation and reoccupation delays. A variant of the SoVI, e.g., SoVI-Lite [57] parameters, are calculated using U.S. Census data and normalized in the range of [1,2], and then used as multipliers of the average cost values to obtain neighborhood-specific cost parameters. The numerical experiments are run for a budget of $200 million. Neighborhood tolerance periods in withstanding service outages are generated randomly within the 4-7 days range. Social vulnerability indices (SoVI index) [56] of the neighborhoods and average property values of different building archetypes are used to generate cost parameters associated with population dislocation and recovery delays experienced at the neighborhood level.

Nine flood hazard scenarios (**Table 1**) are generated based on flood flow frequency analysis [58]. Data at two stations are utilized: the 23-year discharge and gage height record of a United States Geological Survey (USGS) station 02134170 on the Lumber River at Lumberton, NC [59] and the 95-year historic crest record of National Weather Service (NWS) station LBRN7 at a point just upstream of Lumberton, NC [60]. Given the strength of correspondence (96.2% correlation) in gage height for the historic crests observed at these two stations, and given the much longer LBRN7 record, a form of record extension based on Appendix 8 of [58] is conducted: the LBRN7 data are applied to compute (linear regression: R^2=0.92) the gage height at 02134170 and then mapped to discharge using the USGS station's rating-curve [61]. The now augmented 95-yr discharge record of 02134170 is then input to the USGS peakFQ software [62] to generate discharge estimates corresponding to flood events with different exceedance probabilities corresponding to the commonly referenced return periods (e.g, 100-yr return period) listed in **Table 1**. The peakFQ-estimated discharge estimates are then converted to gage heights using the station rating-curve for 02134170. Applying the gage height linear relationship between the stations, and accounting for differences in their respective datums, a weighted average flood elevation for the city is computed, with weights (75% LBRN7, 25% 02134170) selected based on observed inundation from Hurricanes Matthew and Florence. The exceedance curve was then discretized to arrive at the probabilities assigned to each event in **Table 1**.

Table 1. The flood scenario set for the case study

Flood scenario		Flood elevation (m)	Annual occurrence probability
Annual exceedance probability	Return period (years)		
50%	2	39.77	0.572
20%	5	39.13	0.229
10%	10	38.35	0.100
4%	25	37.70	0.060
2%	50	36.96	0.020
1%	100	39.19	0.010
0.5%	200	35.06	0.005
0.2%	500	34.02	0.003
0.1%	1000	32.61	0.001

Three solutions are presented in **Table 2**, along with their solution times. The presented mathematical programming model is solved with the commercial solver FICO XPRESS [63] on a 16-core 2.60GHz CPU running on a 64-bit Windows 10 system. The model was run for three different risk preference settings, with a 100 million US Dollars budget for taking mitigation and repair actions. Model solution times for the presented solutions vary from 8 to 17 seconds, which supports an interactive decision-making tool.

The results in **Table 2** demonstrate the ability of the model to investigate solutions with varying risk preferences. Referring to Sec. 2.12, the mean-risk model allows a decision maker to set risk preference by tuning parameters α and γ. The risk coefficient γ is a trade-off parameter representing the exchange rate of expected total cost for risk. When the value of γ is set as zero (the first solution), the resulting model becomes a risk-neutral two-stage stochastic programming model, where the objective is only to minimize the expected cost. As a result, low-probability hazard events, even though their impact is high, are not given as much weight as events with high probabilities but low impacts. Increasing the value of γ indicates that higher weight is given on risk, which leads to more conservative, i.e., risk-averse, decision-making (the second and third solutions). Parameter α denotes the confidence level. Higher values for α (the third solution) result in more weight given to the more extreme scenarios and, therefore, more conservative, or risk-averse, and typically more costly decisions.

Table 2. Mitigation decisions for three risk preference settings

	Mitigation Decisions		Mitigation Cost (millions $)	Model Solution Time (seconds)
	For Critical Infrastructure Components	For Residential Buildings		
Risk-Neutral Approach γ=0	1. A Floodgate is installed with an elevation of 39.32 m 2. A Floodwall is installed for Electric Substation-2 with a 39.77 m elevation 3. Surface water pump #2 is elevated by 2.55 m to 39.13 m 4. Surface water pump #3 is elevated by 2.55 m to 39.13 m 5. Groundwater well #6 is elevated by 0.72 m to 39.13 m 6. Groundwater well #8 is elevated by 2.25 m ft to 39.13 m	77 buildings upgraded from strategy-0 to strategy-3	184.18	38.70
Risk-Averse Approach α=0.85, γ=1	1. A Floodgate is installed with an elevation of 39.32 m 2. A Floodwall is installed for Electric Substation-2 with a 39.77 m elevation 3. Electric Substation-3 is elevated by 0.14 m to 39.77 m 4. Surface water pump #2 is elevated by 3.19 m to 39.77 m 5. Surface water pump #3 is elevated by 3.19 m to 39.77 m 6. Groundwater well #3 is elevated by 3.49 m to 39.77 m 7. Groundwater well #6 is elevated by 2.88 m to 39.77 m 8. Groundwater well #7 is elevated by 2.88 m to 39.77 m 96 m	218 buildings upgraded from strategy-0 to strategy-3	222.03	9.07
Risk-Averse Approach α=0.95, γ=1	1. A Floodgate is installed with an elevation of 39.32 m 2. A Floodwall is installed for Electric Substation-2 with a 39.77 m elevation 3. Electric Substation-3 is elevated by 0.14 m to 39.77 m 4. Surface water pump #2 is elevated by 3.19 m to 39.77 m 5. Surface water pump #3 is elevated by 3.19 m to 39.77 m 6. Groundwater well #5 is elevated by 3.49 m to 39.77 m 7. Groundwater well #7 is elevated by 2.88 m to 39.77 m	282 buildings upgraded from strategy-0 to strategy-3	253.36	9.64

Whereas **Table 2** focuses on the mitigation decision of the three solutions, **Table 3** focuses on recovery and associated social costs. In the presented mathematical programming model, population dislocation and recovery time are considered important societal consequences of hazard events or costs. They are generally considered the result of taking inadequate mitigation and recovery actions. In **Table 3**, the societal costs associated with one of the events, which is similar to a 200-year return period flood (similar in magnitude to the flood that occurred following Hurricane Florence in 2018), are presented as it is likely of great interest to decision-makers in a community. The table shows the societal consequences when no mitigation actions are taken and the outcomes from the solutions for three risk preference settings. The solutions vary considerably regarding the extent of temporary and permanent dislocation and the average length of time of reoccupation.

Table 3. Comparison of societal costs for a 200-year return period event for different risk preference

	Temporary Dislocation due to Damage (No. of Households)	Temporary Dislocation Due to Service Outage (No. of Households)	Average Reoccupation Time (days)	Permanent Dislocation due to Damage (No. of Households)
No-Mitigation Approach	1477	2471	34.43	1192
Risk-Neutral Approach (γ=0)	1130	0	40.79	218
Risk-Averse Approach (α=0.85, γ=1)	737	0	33.88	202
Risk-Averse Approach (α=0.95, γ=1)	548	0	29.78	199

The case study results suggest that the mathematical programming model is viable for use in a decision support application that permits exploring alternative solutions for community resilience planning.

4. Decision Support Application

The presented mathematical programming model is being made accessible to the user through the NIST ARC tool. This section describes the decision support application side of NIST ARC. The initial version of NIST ARC rested on the predecessor mathematical programming model [64] that was limited to infrastructure nodes and did not include population dislocation. Its design was motivated based on a prior Decision Support Application utilizing mathematical programming to evaluate alternative strategies [51, 65]. As depicted in **Fig. 3**, in the current version of NIST ARC, the Decision Support Application retrieves needed data from the user and connects with the mathematical modeling language platform AMPL[1] through its Application Programming Interface (API). The task of optimization model generation and interaction with the solver is performed by AMPL as directed through the Decision Support Application. The AMPL system is a software modeling tool that supports mathematical programming models' development, testing, and deployment [66]. Due to its high-level representation of decision problems, it allows fast prototyping and efficiently communicates with popular programming languages, e.g., MATLAB, Python, and C++, via AMPL APIs.

Any of the many solvers with which AMPL can interface can be used to solve the mathematical programming model. The NIST testing for the case study was conducted with the commercial solver FICO XPRESS [63] on a workstation with a 16-core 2.60GHz CPU running on a 64-bit Windows 10 system. XPRESS is used extensively for solving linear

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[1] AMPL, distributed by AMPL Optimization Inc, is a commercial software available for purchase at https://ampl.com/

programming (LP), mixed integer linear programming (MILP), and convex quadratic programming (QP) optimization problems and their variants. Since the decision problem underlying the NIST ARC results in a mixed integer linear programming model, the problem solver applies a combination of the primal and dual simplex method, the branch and bound method, and the cutting-plane method. The AMPL model interacts with XPRESS to solve the model, and solutions are transferred to the NIST ARC decision support application via the AMPL API. XPRESS can be replaced by any of the many commercial or open-source mixed integer linear programming solvers with which AMPL interfaces.

The NIST ARC Decision Support Application is constructed in Python programming language. Python is an objected-oriented, general-purpose, and open-source programming language. Python interacts with the modeling environment of AMPL through its API, creating Python objects representing model entities (objectives, constraints, objective functions), sending data and commands to set up the model, and retrieving solution values once the model is solved. With its vast collection of libraries, Python enables the creation of various tools for the graphical display of solutions and for post-processing analyses of the solutions.

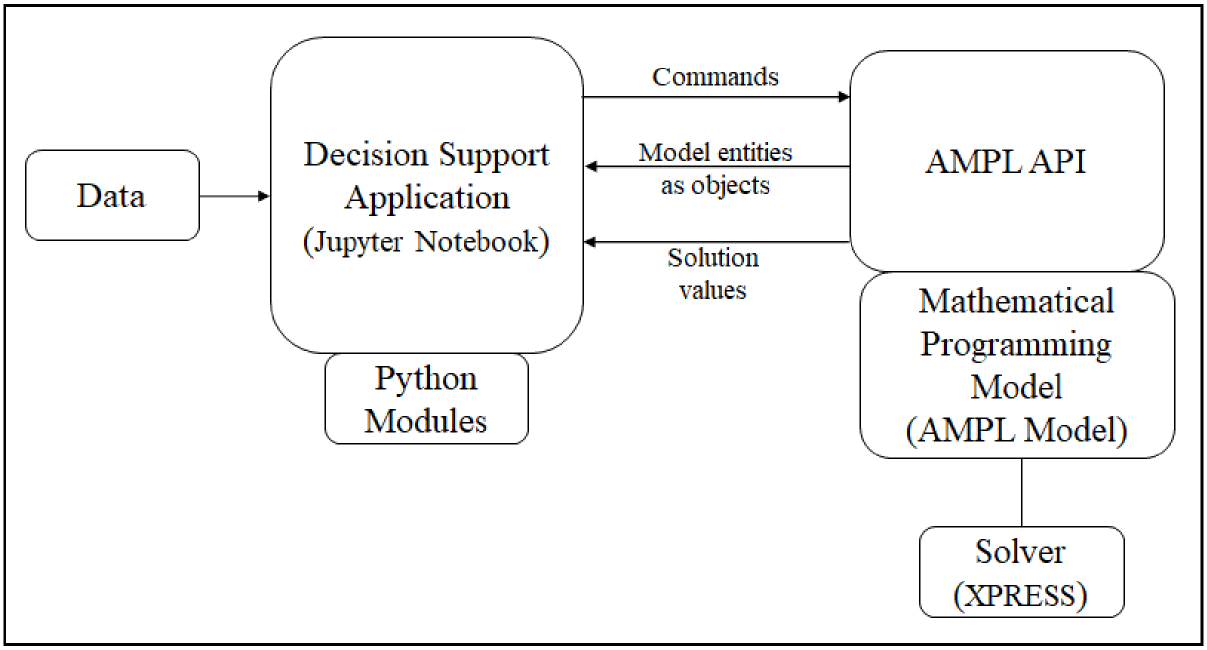

Fig. 3. Software architecture of the decision support application

The Decision Support Application in the current version of NIST ARC takes the form of a Jupyter Notebook. Jupyter is an open-source web browser-based interactive computational environment for creating and sharing documents containing text, code, visualization etc. It also enables users to develop open-source software and services for interactive computing across different programming languages. In NIST ARC, Jupyter “widgets” allow for basic form controls—sliders, checkboxes, buttons—to give a graphical user interface (GUI) experience for Jupyter Notebook users. More powerfully, among the widgets available are advanced controls like interactive maps, 2d and 3d visualizations, and data grids. Data for model parameters are stored in spreadsheets or databases that the notebook user uploads, and this information is then passed to AMPL via its API. Various sets of commands from the notebook can be invoked to direct AMPL to update, change, and solve the model. Once solved, solution values for model entities are retrieved from AMPL back into the notebook for visualization and post-processing analysis. With these capabilities, the model user can interactively change, update, and solve an optimization model, fine-tune solver parameters, and perform post-optimality analysis. Further, a user can visualize the data and solutions tabularly and graphically via maps and plots and interact (e.g., hover information) using advanced functionality on many "dashboard" websites. Detailed instructions for installing and running the NIST ARC Decision Support Application are given in Appendix C.

5. Future Extension Plans

This document has focused on the mathematical programming model and its testing. Future extension plans for the NIST ARC tool can be classified into three groups: improving the mathematical programming model with new features, making an open-source version of the tool, and introducing a web-based interface for users.

5.1. Testing with Open-Source Resources

The current version of NIST ARC utilizes two commercial pieces of software: the AMPL modeling language platform and the FICO XPRESS solver. In FY23, Pyomo [67], a Python-based open-source modeling language, will be tested as a replacement for AMPL. This, in addition to the testing performed in FY22 of open-source and free-for-non-commercial-use solvers, will allow NIST ARC to run with 100% open-source code. Open-source solvers, such as CBC [68], GLPK [69], and SCIP [70], will continue to be tested to identify the most suitable of the open-source and free-for-non-commercial-use solvers, as the comparative performance of solvers is problem-dependent.

5.2. Web-Based Tool

Planned future extensions for the NIST ARC include the development of an easy-to-navigate graphical user interface (GUI) for the Jupyter notebook-based application. Referring to **Fig. 3**, the new architecture will add a layer atop the Jupyter notebook environment. This architecture will facilitate a more streamlined and intuitive use of the decision support application. The Jupyter notebooks of the current and future versions of NIST ARC will serve the important purpose of defining and communicating the web-based tool's use cases and other software requirements.

5.3. Soliciting Stakeholder Feedback

The NIST ARC tool has been designed with the intent that community stakeholders will use it to support risk-informed decisions for community resilience planning. City or county officials and managers of utility services are some examples of the intended user groups. After the tool's deployment, evaluation and feedback will be solicited from users and will be used to update, change, and improve the tool's decision model and architecture. Moreover, feedback will be obtained from researchers from the NIST-funded center to develop plans for integrating NIST ARC with the IN-CORE models.

5.4. Coordination with Other NIST Tools/Software/Guidance

In the development of the NIST ARC, insights were drawn from other NIST resilience planning guides and tools. The long-term success of the NIST ARC decision tool will depend to a large extent on the coordination with the higher-resolution systems models and other planning tools developed by NIST, e.g., the EDGe$ tool for economic analysis for a set of alternatives. Moreover, the NIST ARC tool requires appropriate resilience metrics and modeling based on engineering, economics, and social sciences studies. To this end, close collaboration will remain with the NIST Community Assessment Methodology project.

6. Conclusion

The NIST Alternatives for Resilient Communities (NIST ARC) model is an interactive tool for developing alternative sets of actions for achieving community resilience goals. The purpose of this tool is to help community stakeholders to identify viable alternatives for increasing resilience. To that end, a mean-risk two-stage stochastic programming model was developed that considers expected total costs, including mitigation and event-related costs, and decision-maker risk preferences. The NIST ARC yields solutions that perform well in a typical stochastic environment and in the case of high-impact, low-probability events typically considered in disaster preparedness planning. The outputs from the tool provide insights into a community’s vulnerability against a hazard event, capability to absorb impacts, and ability to recover. Community leaders can use the NIST ARC tool to develop pre-and-post-hazard action plans to achieve community resilience goals while constrained by budget, physical and social constraints, and limits on risk exposure.

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Appendix A. Math Programming Model

The complete mathematical programming model of NIST ARC decision support tool is presented here, starting with the sets' definition and indices.

Sets

E	Set of events (scenarios)
K	Set of products/services/utilities (e.g., power, water etc.)
V	Set of all nodes
$V^{\mathrm{PR+}}$	Set of protective nodes
$V^{\mathrm{PR-}}$	Set of protected nodes
V^{UT}	Set of utility service network nodes
$V_k^{\mathrm{UT+}}$	Set of output connection nodes in utility service network for product $k \in K$
$V_k^{\mathrm{UT-}}$	Set of input connection nodes in utility service network for product $k \in K$
V^{N}	Set of neighborhood nodes

A^{PR}	Set of protection arcs
A_k^{UT}	Set of arcs in utility service network for product $k \in K$
B	Set of building archetypes
S	Set of mitigation retrofitting strategies for residential buildings
F	Ordered set of post-event damage states of residential buildings

Parameters are defined over set elements to provide the model with data to construct constraints and objective functions. The parameters, their notations, and definitions are provided here.

Parameters

$\bar{\Phi}_e$	The probability of event $e \in E$
$\bar{R}_i^0$	The initial resistance to the hazard of node $i \in V$
$\bar{R}_i^{\mathrm{U}}$	The upper bound on the increase in resistance for node $i \in V$
$\bar{L}_{ie}$	The hazard loading at node $i \in V$ under event $e \in E$
$\bar{D}_i^{\mathrm{R}}$	The time requirement for restoring integrity of component $i \in V$
$\bar{D}_i^{\mathrm{A}}$	The time requirement for activating node $i \in V$ from dormant status
Z_i^0	Binary parameter: = 1, if node $i \in V^{\mathrm{PR+}}$ was installed initially = 0, otherwise

$\bar{Y}_i^0$	Binary parameter: = 1, if $i \in V \backslash V^{\text{PR}+}$ was in use initially = 0, otherwise
$\bar{S}_{ik}^0$	The initial backup storage of product $k \in K$ at node $i \in V_k^{\text{UT}}$
$\bar{S}_{ik}^{\text{U}}$	The upper bound on change in backup storage of product $k \in K$ at utility network node $i \in V_k^{\text{UT}}$
$\bar{Q}_{ik}^{S}$	The available supply of product $k \in K$ at node $i \in V_k^{\text{UT}}$
$\bar{Q}_{ik}^{D}$	The demand for product $k \in K$ at node $i \in V_k^{\text{UT}}$
$\bar{Q}_{ijk}^{\text{U}}$	The flow capacity of product $k \in K$ along network arc $(i, j) \in A_k^{\text{UT}}$
$\bar{\text{H}}_{ikl}$	The minimum input of product $l \in K$ required at node $i \in V_k^{\text{UT}+}$ for per unit output of product $k \in K$
$\bar{I}_{ij}^{\text{M}}$	Binary parameter: = 1, if neighborhood $i \in V^{\text{N}}$ is in the service area of node $i \in V_k^{\text{UT}-}, k \in K$ = 0, otherwise
$\bar{\text{T}}_{ik}^{\text{TOL}}$	The maximum restoration delay allowed for utility service $k \in K$ until people in the neighborhood $i \in V^{\text{N}}$ perceive the service to be unavailable
$\bar{\text{T}}_i^{\text{DELAY}}$	The Minimum delay in until the repair of damaged buildings can start in neighborhood $i \in V^{\text{N}}$
$\bar{N}_{ibs}^0$	The Initial number of buildings of archetype $b \in B$ in neighborhood $i \in V^{\text{N}}$ with retrofit strategy $s \in S$

$\overline{AH}_i^0$	The average number of households per building in neighborhood $i \in V^{\mathrm{N}}$
$\bar{R}_{ibs}^{\mathrm{BUILD}}$	The hazard resistance of buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ with strategy $s \in S$
$\bar{W}_{ibsfe}^{\mathrm{DS}}$	Binary parameter: =1, if buildings of archetype $b \in B$ with strategy $s \in S$ in neighborhood $i \in V^{\mathrm{N}}$ fall into damage state $f \in F$ under event $e \in E$ =0, otherwise
$\bar{W}_{ibf}^{\mathrm{DMDL}}$	Binary parameter: =1, if buildings of archetype $b \in B$ and damage state $f \in F$ in neighborhood $i \in V^{\mathrm{N}}$ reach dislocation threshold due to building damage =0, otherwise
$\bar{W}_{ibf}^{\mathrm{DMRO}}$	Binary parameter: =1, if damage state $f \in F$ is eligible for reoccupation for buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ =0, otherwise
$\bar{\mathrm{T}}_{bff'}^{\mathrm{REPAIR}}$	The time requirement for repairing a building of archetype $b \in B$ from damage state $f \in F$ to $f' \in F$
$C_i^{\mathrm{RESISTANCE}}$	The variable cost of adding unit resistance to node $i \in V$
C_i^{INSTALL}	The fixed monetary installation cost for protective node $i \in V^{\mathrm{PR+}}$
C_i^{RECOVERY}	The fixed cost of recovering failed node $i \in V$

C_i^{STARTUP}	The fixed monetary startup cost for node $i \in V$
C_{ik}^{STORAGE}	The variable cost of adding unit backup storage capacity of product $k \in K$ to node $(i, k) \in V_k^{\text{UT}}$
$C_{bss'}^{\text{RFIT}}$	The cost of upgrading retrofitting of a building of archetype $b \in B$ form mitigation strategy $s \in S$ to strategy $s' \in S$
$C_{bff'}^{\text{REPAIR}}$	The cost of repairing to improve functionality level of a building of archetype $b \in B$ form damage state $f \in F$ to $f' \in F$
C_{ik}^{NF}	The cost of losing the functionality of node $i \in V_k^{\text{UT}}, \forall k \in K$
C_i^{DELAY}	The cost of unit time delay in the service restoration in neighborhood $i \in V^{\text{N}}$
C_i^{PDL}	The cost of permanent dislocation of each household from neighborhood $i \in V^{\text{N}}$
C_i^{TDL}	The cost of temporary dislocation of each household from neighborhood $i \in V^{\text{N}}$
C_i^{OTDL}	The cost of dislocation of each household due to service outage from neighborhood $i \in V^{\text{N}}$
$\overline{\mathcal{B}}$	Total available budget
ρ	Discount rate
α	Confidence level
γ	Risk weight parameter

$\bar{M}$	A large positive number

Decision variables

Decision levers, i.e., recommended actions for the community entities to achieve resilience, are introduced as model variables. The variables are defined over the elements of one or several sets. The variables, their definitions, corresponding sets, and index notations are given here. Two broad categories of variables are present in the model: event-independent mitigation variables and recovery variables that are defined, among others, over the set of hazard events.

Mitigation decision variables

These decisions are event-independent and are made without knowing the hazard loading.

$z_i^{\mathrm{N}} \in \{0,1\}$	$= 1$, if protective node $i \in V^{\mathrm{PR+}}$ is to be installed $= 0$, otherwise
$z_i^{\mathrm{SP}} \in \{0,1\}$	$= 1$, if node $i \in V$ is self-protected $= 0$, if parent-protected
$r_i \geq 0$	The resistance added to node $i \in V$
$r_i^{\mathrm{PM}} \geq 0$	Post-mitigation resistance of node $i \in V$
$r_i^{\mathrm{EFF}} \geq 0$	The effective resistance of node $i \in V$
$s_{ik} \geq 0$	Increase in backup storage of product $k \in K$ at utility network node $i \in V_k^{\mathrm{UT}}$
$s_{ik}^{\mathrm{PM}} \geq 0$	Post-mitigation backup storage of product $k \in K$ at utility network node $i \in V_k^{\mathrm{UT}}$
$n_{ibss'}^{\mathrm{RFIT}} \geq 0$	Number of buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ with retrofit strategy $s \in S$ to be upgraded to $s' \in S$

$n_{ibs}^{\mathrm{PM}} \geq 0$	Post-mitigation number of buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ with retrofit strategy $s \in S$ after

Recovery decision variables

Event-dependent recovery variables are as follows:

$y_{ie}^{\mathrm{S}} \in \{0,1\}$	= 1, if node $i \in V$ survives under event $e \in E$ = 0, otherwise
$y_{ie}^{\mathrm{R}} \in \{0,1\}$	= 1, if failed node $i \in V$ is recovered under event $e \in E$ = 0, otherwise
$y_{ie}^{\mathrm{A}} \in \{0,1\}$	= 1, if dormant node $i \in V$ is activated under event $e \in E$ = 0, otherwise
$y_{ike}^{\mathrm{F}} \in \{0,1\}$	= 1, if node $i \in V_k^{\mathrm{UT}}$ remains functional throughout under event $e \in E$ = 0, otherwise
$y_{ijke}^{\mathrm{FLOW}} \in \{0,1\}$	= 1, if arc $(i,j) \in A_k^{\mathrm{UT}}, \forall k \in K$ is used under event $e \in E$ = 0, otherwise
$x_{ie}^{\mathrm{OT}} \in \{0,1\}$	= 1, if neighborhood $i \in V^{\mathrm{N}}$ experience service outage under event $e \in E$ = 0, otherwise
$x_{ibff'e}^{\mathrm{REPAIR}} \in \{0,1\}$	= 1, if buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ with damage state $f \in F$ are repaired up to damage state $f' \in F$ under event $e \in E$

	= 0, otherwise
$q_{ijke}^{\mathrm{FLOW}} \geq 0$	Flow amount of product $k \in K$ sent along network arc $(i,j) \in A_k^{\mathrm{UT}}$ under event $e \in E$
$q_{ike}^{\mathrm{SUPPLY}} \geq 0$	Generated supply of product $k \in K$ at node $i \in V_k^{\mathrm{UT}}$ under event $e \in E$
$q_{ike}^{\mathrm{DELIVER}} \geq 0$	Amount of product $k \in K$ delivered at node $i \in V_k^{\mathrm{UT}}$ under event $e \in E$
$n_{ibfe}^{\mathrm{DS}} \geq 0$	Number of buildings of archetype $b \in B$ with damage state $f \in F$ in neighborhood $i \in V^{\mathrm{N}}$ under event $e \in E$ before repair
$n_{ibfe}^{\mathrm{PR}} \geq 0$	Number of buildings of archetype $b \in B$ with damage state $f \in F$ in neighborhood $i \in V^{\mathrm{N}}$ under event $e \in E$ after repair
$n_{ibe}^{\mathrm{DMDL}} \geq 0$	Number of buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ experiencing dislocation under event $e \in E$ due to building damage
$n_{ie}^{\mathrm{OTDL}} \geq 0$	Number of buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ experiencing dislocation due to service outage under event $e \in E$
$n_{ibff'e}^{\mathrm{REPAIR}} \geq 0$	Number of buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ with damage state $f \in F$ that are repaired to damage state $f' \in F$ under event $e \in E$
$n_{ibfe}^{\mathrm{DMRO}} \geq 0$	Number of buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ with post-repair damage state $f \in F$ that are reoccupied under event $e \in E$

$t_{ie}^{\mathrm{RECOVER}} \geq 0$	The time to recover the node $i \in V$ under event $e \in E$
$t_{ike}^{\mathrm{RESTORE}} \geq 0$	The time to restore the availability of product $k \in K$ at the node $i \in V_k^{\mathrm{UT}}$ under event $e \in E$
$t_{ike}^{\mathrm{U}} \geq 0$	The latest time of service restoration in neighborhood $i \in V^{\mathrm{N}}$ under event $e \in E$
$t_{ibfe}^{\mathrm{REPAIR}} \geq 0$	End time of repair of buildings of archetype $b \in B$ in neighborhood $i \in V^{\mathrm{N}}$ with damage state $f \in F$ under event $e \in E$
η	α-quantile of the random variable for expected total cost distribution (value-at-risk)
$v_e \geq 0$	Expected total cost value exceeding value-at-risk under event $e \in E$

With the sets, parameters, and variables defined above, Eq. (A1)- (A48) present the mathematical programming model.

$$Minimize\ \left[(1+\gamma)Mitigation\_Cost + \left(\frac{1}{\rho}\right)\sum_{e \in E} \lambda_e (Recourse\_Cost)_{\boldsymbol{e}} + \left(\frac{\gamma}{\rho}\right)\left(\eta + \frac{1}{1-\alpha}\sum_{e \in E} \lambda_e v_e\right)\right] \tag{A1}$$

Subject to:

$$\begin{aligned} Mitigation\_Cost &= \sum_{i \in V} \bar{C}_i^{\mathrm{RESISTANCE}}\, r_i + \sum_{k \in K} \sum_{i \in V_k^{\mathrm{UT}}} \bar{C}_i^{\mathrm{STORAGE}}\, s_{ik} \\ &+ \sum_{i \in V^{\mathrm{PR+}}} \bar{C}_i^{\mathrm{INSTALL}}\, z_i^{\mathrm{NEW}} + \sum_{i \in V^{\mathrm{N}}} \sum_{b \in B} \sum_{s,s' \in S} \bar{C}_{bss'}^{\mathrm{RFIT}} n_{ibss'}^{\mathrm{RFIT}} \end{aligned} \tag{A2}$$

$$
\begin{aligned}
(Restoration\_Cost)_e &= \sum_{i \in V} \left[C_i^{\mathrm{RECOVERY}} y_{ie}^{\mathrm{R}} + C_i^{\mathrm{STARTUP}}\, y_{ie}^{\mathrm{A}} \right] \\
&+ \sum_{i \in V^{\mathrm{N}}} \sum_{b \in B} \sum_{f, f' \in F} n_{ibff'e}^{\mathrm{REPAIR}}\, C_{ibff'e}^{\mathrm{REPAIR}}, \qquad \forall e \in E \qquad \text{(A3)}
\end{aligned}
$$

$$
\begin{aligned}
(Recourse\_Cost)_e &= (Restoration\_Cost)_e \\
&+ \sum_{i \in V^{\mathrm{N}}} \Bigg\{ C_i^{\mathrm{DELAY}}\, t_{ie}^{\mathrm{U}}\, \bar{N}_{ibs}^{0} + C_i^{\mathrm{OTDL}} n_{ie}^{\mathrm{OTDL}} \\
&+ \sum_{b \in B} \left(C_{ib}^{\mathrm{PDL}} \left[n_{ibe}^{\mathrm{DMDL}} - n_{ibe}^{\mathrm{DMRO}} \right] + C_{ib}^{\mathrm{TDL}} n_{ibe}^{\mathrm{DMRO}} \right) \Bigg\} \overline{AH}_i^{0} \\
&+ \sum_{k \in K} \sum_{i \in V_k^{\mathrm{UT}}} C_{ik}^{\mathrm{NF}} \left(\bar{Y}_i^{0} - y_{ike}^{\mathrm{F}} \right), \qquad \forall e \in E \qquad \text{(A4)}
\end{aligned}
$$

$$v_e \geq (Recourse\_Cost)_e - \eta, \qquad \forall e \in E \qquad \text{(A5)}$$

$$Mitigation\_Cost + (Restoration\_Cost)_e \leq \bar{\mathcal{B}}, \qquad \forall e \in E \qquad \text{(A6)}$$

$$z_i^{\mathrm{N}} \leq 1 - \bar{Z}_i^{0}, \qquad \forall i \in V^{\mathrm{PR+}} \qquad \text{(A7)}$$

$$r_i \leq \bar{R}_i^{\mathrm{U}} \left(z_i^{\mathrm{N}} + \bar{Z}_i^{0} \right), \qquad \forall i \in V^{\mathrm{PR+}} \qquad \text{(A8)}$$

$$r_i \leq \bar{R}_i^{\mathrm{U}} \qquad \forall i \in V \backslash V^{\mathrm{PR+}} \qquad \text{(A9)}$$

$$r_i^{\mathrm{EFF}} \leq \bar{R}_i^{0} \left(z_i^{\mathrm{N}} + \bar{Z}_i^{0} \right) + r_i, \qquad \forall i \in V^{\mathrm{PR+}} \qquad \text{(A10)}$$

$$r_i^{\mathrm{EFF}} \leq \bar{R}_i^0 + r_i + \bar{M}\left(1 - z_i^{\mathrm{SP}}\right), \qquad \forall i \in V^{\mathrm{PR}-} \tag{A11}$$

$$r_j^{\mathrm{EFF}} \leq r_i^{\mathrm{EFF}} + \bar{M} z_j^{\mathrm{SP}}, \qquad \forall (i,j) \in A^{\mathrm{PR}} \tag{A12}$$

$$r_i^{\mathrm{EFF}} = \bar{R}_i^0 + r_i, \qquad \forall i \in V \backslash \left(V^{\mathrm{PR}+} \cup V^{\mathrm{PR}-}\right) \tag{A13}$$

$$s_{ik} \leq \bar{S}_{ik}^{\mathrm{U}}, \qquad \forall\, k \in K, i \in V_k^{\mathrm{UT}} \tag{A14}$$

$$s_{ik}^{\mathrm{PM}} = \bar{S}_{ik}^{0} + s_{ik} \qquad \forall\, k \in K, i \in V_k^{\mathrm{UT}} \tag{A15}$$

$$n_{ibs}^{\mathrm{PM}} = \bar{N}_{ibs}^{0} + \sum_{s' \in S} n_{ibs's}^{\mathrm{RFIT}} - \sum_{s' \in S} n_{ibss'}^{\mathrm{RFIT}}, \qquad \forall i \in V^{\mathrm{N}}, b \in B, s \in S \tag{A16}$$

$$\sum_{s \in S} \bar{N}_{ibs}^{0} = \sum_{s \in S} n_{ibs}^{\mathrm{PM}}, \qquad \forall i \in V^{\mathrm{N}}, b \in B \tag{A17}$$

$$r_i^{\mathrm{EFF}} - L_{ie} \geq \bar{M}\left(y_{ie}^{\mathrm{S}} - 1\right), \qquad \forall i \in V, e \in E \tag{A18}$$

$$y_{ie}^{\mathrm{R}} \leq 1 - y_{ie}^{\mathrm{S}}, \qquad \forall i \in V, e \in E \tag{A19}$$

$$y_{ie}^{\mathrm{A}} \leq y_{ie}^{\mathrm{S}}(1 - \bar{Y}_i^0), \qquad \forall i \in V, e \in E \tag{A20}$$

$$y_{ike}^{\mathrm{F}} \leq y_{ie}^{\mathrm{S}} \bar{Y}_i^0, \qquad \forall k \in K, i \in V_k^{\mathrm{UT}}, e \in E \tag{A21}$$

$$y_{ijke}^{\mathrm{FLOW}} \leq y_{ie}^{\mathrm{S}} \bar{Y}_i^0 + y_{ie}^{\mathrm{R}} + y_{ie}^{\mathrm{A}}, \qquad \forall k \in K, (i,j) \in A_k^{\mathrm{UT}}, e \in E \tag{A22}$$

$$y_{ijke}^{\mathrm{FLOW}} \leq y_{ie}^{\mathrm{S}} \bar{Y}_i^0 + y_{je}^{\mathrm{R}} + y_{je}^{\mathrm{A}}, \qquad \forall k \in K, (i,j) \in A_k^{\mathrm{UT}}, e \in E \tag{A23}$$

$$q_{ijke}^{\mathrm{FLOW}} \leq \bar{Q}_{ijk}^{\mathrm{U}} y_{ijke}^{\mathrm{FLOW}}, \qquad \forall\, k \in K, (i,j) \in A_k^{\mathrm{UT}}, e \in E \tag{A24}$$

$$q_{ike}^{\mathrm{SUPPLY}} \leq \bar{Q}_{ijk}^{\mathrm{S}}, \qquad \forall k \in K, i \in V_k^{\mathrm{UT}+}, e \in E \tag{A25}$$

$$\sum_{(i,j)\in A_k^{\mathrm{UT}}} q_{ijke}^{\mathrm{FLOW}} \le q_{ike}^{\mathrm{SUPPLY}} + \sum_{(j,i)\in A_k^{\mathrm{UT}}} q_{jike}^{\mathrm{FLOW}},$$

$$\forall\ k\in K,(i,j)\in A_k^{\mathrm{UT}},e\in E \qquad \text{(A26)}$$

$$\sum_{(j,i)\in A_k^{\mathrm{UT}}} q_{jike}^{\mathrm{FLOW}} - \sum_{(i,j)\in A_k^{\mathrm{UT}}} q_{ijke}^{\mathrm{FLOW}} \ge q_{ike}^{\mathrm{DELIVER}},$$

$$\forall k\in K,i\in V_k^{\mathrm{UT}-},e\in E \qquad \text{(A27)}$$

$$q_{ike}^{\mathrm{DELIVER}} \ge \bar{Q}_{ik}^{\mathrm{D}}, \qquad \forall k\in K,i\in V_k^{\mathrm{UT}-},e\in E \qquad \text{(A28)}$$

$$q_{ile}^{\mathrm{SUPPLY}} + \sum_{(j,i)\in A_l^{\mathrm{UT}}} q_{jile}^{\mathrm{FLOW}} \ge \overline{\mathrm{H}}_{ikl}\left(q_{ike}^{\mathrm{DELIVER}} + \sum_{(i,j)\in A_k^{\mathrm{UT}}} q_{ijke}^{\mathrm{FLOW}}\right)$$

$$\forall i\in V_k^{\mathrm{UT}+},l\in K,e\in E>0 \qquad \text{(A29)}$$

$$t_{ike}^{\mathrm{RESTORE}} \ge \bar{D}_i^{\mathrm{R}} y_{ie}^{\mathrm{R}} + \bar{D}_i^{\mathrm{S}} y_{ie}^{\mathrm{A}}, \qquad \forall i\in V,e\in E \qquad \text{(A30)}$$

$$t_{ike}^{\mathrm{RESTORE}} \ge t_{je}^{\mathrm{RECOVERY}} - \bar{M}\left(1 - y_{jike}^{\mathrm{FLOW}}\right)$$

$$\forall k\in K,i\in V_k^{\mathrm{UT}},e\in E \qquad \text{(A31)}$$

$$t_{je}^{\mathrm{RECOVERY}} \ge t_{ike}^{\mathrm{RESTORE}} - \bar{M}\left(y_{ike}^{\mathrm{F}} - 1\right),$$

$$\forall k\in K,(j,i)\in A_k^{\mathrm{UT}},e\in E \qquad \text{(A32)}$$

$$s_{ik}^{\mathrm{PM}} - t_{ike}^{\mathrm{RESTORE}} \ge \bar{M}\left(y_{ike}^{\mathrm{F}} - 1\right), \qquad \forall k\in K,i\in A_k^{\mathrm{UT}},e\in E \qquad \text{(A33)}$$

$$n_{ibfe}^{\mathrm{DS}} = \sum_{s\in S} n_{ibs}^{\mathrm{PM}}\,\bar{W}_{ibsfe}^{\mathrm{DS}}, \qquad \forall i\in V^{\mathrm{N}},b\in B,f\in F,e\in E \qquad \text{(A34)}$$

$$n_{ibe}^{\mathrm{DMDL}} \ge \sum_{f\in F} n_{ibfe}^{\mathrm{DS}}\bar{W}_{ibf}^{\mathrm{DMDL}} - \bar{M}y_{ie}^{\mathrm{S}}, \qquad \forall i\in V^{\mathrm{N}},b\in B,e\in E \qquad \text{(A35)}$$

$$\sum_{f\in F} n^{\text{PR}}_{ibfe} = \sum_{f\in F} n^{\text{DS}}_{ibfe}, \qquad \forall i \in V^{\text{N}}, b \in B, e \in E \tag{A36}$$

$$n^{\text{PR}}_{ibfe} = n^{\text{DS}}_{ibfe} + \sum_{f'\in F: f' > f} n^{\text{REPAIR}}_{ibf'fe} - \sum_{f'\in F: f' < f} n^{\text{REPAIR}}_{ibff'e},$$

$$\forall i \in V^{\text{N}}, b \in B, f \in F, e \in E \tag{A37}$$

$$x^{\text{REPAIR}}_{ibff'e} \le \overline{W}^{\text{DMDL}}_{ibf} \overline{W}^{\text{DMRO}}_{ibf'}, \qquad \forall i \in V^{\text{N}}, b \in B, f, f' \in F, e \in E: f' < f \tag{A38}$$

$$n^{\text{REPAIR}}_{ibff'e} \le \left(\sum_{s\in S} \overline{N}^{0}_{ibs}\right) x^{\text{REPAIR}}_{ibff'e}, \qquad \forall i \in V^{\text{N}}, b \in B, f, f' \in F, e \in E: f' < f \tag{A39}$$

$$n^{\text{DMRO}}_{ibe} \le \sum_{f\in F} \sum_{f'\in F: f' > f} n^{\text{REPAIR}}_{ibf'fe} \overline{W}^{\text{DMRO}}_{ibf},$$

$$\forall i \in V^{\text{N}}, b \in B, , e \in E \tag{A40}$$

$$t^{\text{U}}_{ie} - t^{\text{RESTORE}}_{jke} \ge 0, \qquad \forall k \in K, j \in V^{\text{UT}-}_{k}, i \in V^{\text{N}}, e \in E: \bar{I}^{\text{M}}_{ij} = 1 \tag{A41}$$

$$t^{\text{U}}_{ie} - \overline{\mathrm{T}}^{\text{TOL}}_{i} \ge \bar{M}\left(x^{\text{OT}}_{ie} - 1\right), \qquad \forall i \in V^{\text{N}}, e \in E \tag{A42}$$

$$t^{\text{U}}_{ie} - \overline{\mathrm{T}}^{\text{TOL}}_{ik} \le \bar{M} x^{\text{OT}}_{ie} \qquad \forall i \in V^{\text{N}}, e \in E \tag{A43}$$

$$n^{\text{OTDL}}_{ie} \ge \sum_{b\in B} \left(\sum_{s\in S\cap S'} n^{\text{PM}}_{ibs} - \sum_{f\in F} n^{\text{DMDL}}_{ibfe} \right) - \bar{M}\left(1 - x^{\text{OT}}_{ie}\right),$$

$$\forall i \in V^{\text{N}}, b \in B, f \in F, e \in E \tag{A44}$$

$$t^{\text{REPAIR}}_{ibfe} \ge \left(\overline{\mathrm{T}}^{\text{DELAY}}_{i} + \overline{\mathrm{T}}^{\text{REPAIR}}_{bff'}\right) x^{\text{REPAIR}}_{ibff'e},$$

$$\forall i \in V^{\text{N}}, b \in B, f \in F, f' \in F, e \in E: f' < f \tag{A45}$$

$$t_{ie}^{\mathrm{RECOVERY}} \geq t_{ibfe}^{\mathrm{REPAIR}}, \qquad \forall i \in V^{\mathrm{N}}, b \in B, f \in F, e \in E \tag{A46}$$

$$t_{ie}^{\mathrm{RECOVERY}} \geq t_{ie}^{\mathrm{U}}, \qquad \forall i \in V^{\mathrm{N}}, e \in E \tag{A47}$$

$$\eta \in \mathbb{R}, v_e \geq 0, \qquad \forall e \in E$$

$$z_i^{\mathrm{N}}, z_i^{\mathrm{SP}} \in \{0,1\}, \qquad \forall i \in V, k \in K$$

$$r_i, r_i^{\mathrm{PM}}, r_i^{\mathrm{EFF}}, s_{ik}^{\mathrm{PM}} \geq 0, \qquad \forall i \in V, k \in K$$

$$y_{ie}^{\mathrm{S}}, y_{ie}^{\mathrm{R}}, y_{ie}^{\mathrm{A}}, y_{ike}^{\mathrm{F}}, y_{ijke}^{\mathrm{FLOW}} \in \{0,1\}, \qquad \forall i \in V, k \in K, e \in E \tag{A48}$$

$$q_{ike}^{\mathrm{SUPPLY}}, q_{ike}^{\mathrm{DELIVER}}, q_{ijke}^{\mathrm{FLOW}} \geq 0, \qquad \forall k \in K, (i,j) \in A_k^{\mathrm{UT}}, e \in E$$

$$t_{ike}^{\mathrm{RESTORE}}, t_{ie}^{\mathrm{RECOVER}}, t_{ie}^{\mathrm{U}} \geq 0, \qquad \forall k \in K, i \in V, e \in E$$

$$x_{ie}^{\mathrm{OT}}, x_{ibff'e}^{\mathrm{REPAIR}} \in \{0,1\}, \qquad \forall i \in V^{\mathrm{N}}, b \in B, f \in F, f' \in F, e \in E$$

$$n_{ibs}^{\mathrm{PM}}, n_{ibfe}^{\mathrm{DS}}, n_{ibfe}^{\mathrm{PR}}, n_{ibe}^{\mathrm{DMDL}}, n_{ie}^{\mathrm{OTDL}}, n_{ibe}^{\mathrm{DMRO}}, n_{ibff'e}^{\mathrm{REPAIR}}, t_{ibfe}^{\mathrm{REPAIR}} \geq 0,$$

$$\forall i \in V^{\mathrm{N}}, b \in B, s \in S, f \in F, f' \in F, e \in E$$

Appendix B. Present value of scenario costs

Let the occurrence of the hazard corresponding to scenario $\omega \in \Omega$ follows the Poisson distribution with an annual rate λ_e. The present value, PV of the costs (C_e) of the events occurring over an infinite horizon and discount rate ρ is given by the following:

$$PV = \int_{t=0}^{t=\infty} C_\omega e^{-\rho t} \lambda_\omega dt$$

$$= \frac{C_\omega \lambda_\omega}{-\rho} \int_{t=0}^{t=\infty} e^{-\rho t}(-\rho) dt$$

$$= \frac{C_\omega \lambda_\omega}{-\rho} [e^{-\rho t}]_{t=0}^{t=\infty}$$

$$= \frac{C_\omega \lambda_\omega}{-\rho}(0-1)$$

$$= \frac{C_\omega \lambda_\omega}{\rho}$$

So, if the recourse cost of scenario ω is $\bar{\mathbf{q}}_e^T \boldsymbol{y}_\omega$, then the present value of the cost occurred following the Poisson distribution over an infinite planning horizon is ${\lambda_\omega \bar{\mathbf{q}}_e^T \boldsymbol{y}_\omega}/{\rho}$.

For the $CVaR_\alpha$ of the recourse cost, we can find the present value as :

$$\left(\frac{\gamma}{\rho}\right)\left(v + \frac{1}{(1-\alpha)}\sum_{\omega\in\Omega} \lambda_\omega u_\omega\right)$$

So the mean-risk objective function becomes the following:

$$Minimize \left\{(1+\gamma)\bar{\mathbf{c}}^{\mathrm{T}}\boldsymbol{x} + \sum_{\omega\in\Omega} \frac{\lambda_\omega \bar{\mathbf{q}}_e^T \boldsymbol{y}_\omega}{\rho} + \left(\frac{\gamma}{\rho}\right)\left(v + \frac{1}{1-\alpha}\sum_{\omega\in\Omega} \lambda_\omega u_\omega\right)\right\}$$

Appendix C. Running NIST ARC

NIST ARC consists of the Jupyter notebook (*.ipynb* extension) and accompanying Python (*.py*) files that are imported into the notebook. A zip file is available at https://www.nist.gov/services-resources/software/nist-arc-nist-alternatives-resilient-communities-tool.

Required hardware/software:

- Windows, MacOS, or Linux operating system
- Python (https://www.python.org/)
- JupyterLab (https://jupyterlab.readthedocs.io/en/stable/)
- AMPL (https://ampl.com/)
- amplpy python package (https://ampl.com/api/nightly/python/getting-started.html#installation)
- At least one linear programming solver that interfaces with AMPL (e.g., FICO XPRESS)
- Python packages are listed in the requirements.txt

C.1. Computer Hardware Requirements

Hardware requirements for running NIST ARC depend primarily on the community size for which the model will be solved. The machine should generally have a processor as fast as 1

GHz, with at least 512 MB RAM. Model run times will be greatly sped up with more memory and faster processors. Local storage of at least 2 GB is needed to store the output of calculations.

C.2. Computer Operating System (OS) Requirements

The goal of making NIST ARC publicly available is to enable decision-makers to evaluate the implications of mitigation and recovery decisions and compare different strategies at a reasonable computational cost. Thus, NIST ARC has been designed for computers running Microsoft Windows, Mac OS X, and various implementations of Unix/Linux. NIST ARC is available for Windows, Unix, Linux, and Mac OS operating systems. The current version made available is a beta version and as such, familiarity with the Python environment is beneficial for resolving any small issues, e.g., python library incompatibilities. The model's late 2022/early 2023 version will incorporate changes tested in the research version, greatly simplifying the installation process.

C.3. Installation

Suggested installation steps:

- Download the “NIST_ARC” zip file and extract to a known directory.
- If not already installed, download and run the installer for Anaconda (https://www.anaconda.com/products/individual)
- If not already installed, install AMPL software (https://ampl.com/).
- Record for later use the path to the AMPL executable (“path_to_ampl”).
- Install the AMPL Python API. Instructions at: https://ampl.com/api/nightly/python/getting-started.html#installation
- Follow the instructions for “conda”, i.e., by Anaconda, installation of JupyterLab via link: https://jupyterlab.readthedocs.io/en/stable/getting_started/installation.html#conda
- Following Anaconda instructions, install the python packages in the file requirements.txt.
- Edit myGlobals.py file to point to default locations for AMPL and an available solver:
    - In JupyterLab, to edit the myGlobals.py file, right-click and select ‘Open with’ and then ‘Editor’
    - Assign path_to_ampl to the path in which AMPL was installed.
    - Assign solver to the default solver to be used (e.g., ‘xpress’, ‘cplex’, ‘gurobi’). (If the solver was not as bundled with AMPL, place instead path to the solver for-AMPL executable as per AMPL instructions)
- If not familiar with the Juptyer notebook environment, follow: https://jupyterlab.readthedocs.io/en/stable/getting_started/starting.html
- Test the installation

- From JupyterLab, open main notebook ‘NIST ARC 0.9 (Beta).ipynb’ from the directory in which the zip file was extracted
- From Run menu, click ‘Run all cells’ (Alternatively, walk through by hitting Shift-Enter in each “cell”). This will set up the graphical user interface (GUI) elements, or “widgets”.
- Follow the instructions as laid out in the workbook itself.