PRECISION ENGINEERING FOR GRAVITATIONAL EXPERIMENTS

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INTRODUCTION

Four fundamental forces, or technically more correct interactions, are known in physics. The gravitational force is one of them and is a mysterious one. Gravity has an infinite range, just like the electromagnetic interaction. However, in contrast to electromagnetic forces, gravitational forces are always attractive and can't be shielded. In addition, the gravitational force is approximately 40 orders of magnitude weaker than the electrostatic one. These properties make it challenging to measure gravitational forces in the laboratory precisely.

So, why then study this finicky force? Most of the big questions that we have about the universe revolve around gravity. That is not surprising, given that most celestial bodies are electrically neutral, and the only long-range interaction between them is gravitational.

What are these big questions? First, we do not completely understand the rotational dynamics of galaxies, including ours, the Milky Way. The measured angular velocity of the stars around the center of the galaxy as a function of radial distance does not match the calculated value based on the density of baryonic (made from protons and neutrons) matter. One solution that has been proposed is the existence of other matter, called dark matter. We have not detected dark matter, despite decades of effort.

Things get even weirder if we zoom out and look at the dynamics of the universe as a whole. Does its expansion accelerate, or will it stop and eventually collapse? The true answer is in the middle of the two, and can not be explained by ordinary matter and dark matter. So, a new construct had to be invented, dark energy. We don't know what dark energy is either.

One can study gravity by observing the universe. But then, one is at the mercy of whatever experiment the universe presents, and often the experiment can't be repeated. One can also study gravity in the laboratory. In the labortory, it is easy to repeat an experiment.

What questions can be studied in the laboratory? Examples might be: How strong is gravity (Measurement of G [1])? Does gravity between two bodies fall off as the inverse of the separation squared (Test of the inverse square law)? Do two bodies of two different compositions fall with the same rate (Test of the equivalence principle)? The phrases in parentheses are in the jargon of the field. The presenter has worked on all three of these experiments. This talk will focus on the interaction between precision engineering and the gravitational constant measurement as an example.

THE MEASUREMENT OF G

In the laboratory, the first measurement of the gravitational constant, G, was carried out by Henry Cavendish from 1793 to 1798. He used a torsion balance that was invented by Charles A. de Coloumb about fifteen years earlier. Even today, about a quarter millennium later, the torsion balance is still the most precise instrument to measure G. That is not because there is a lack of competition. People have used an atom interferometer to measure G.

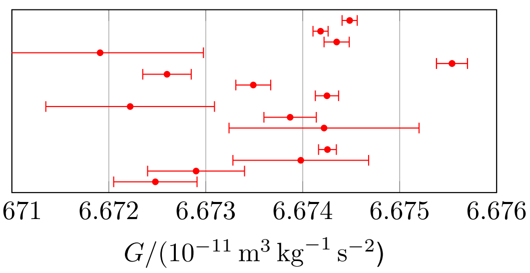

FIGURE 1 . Results of more than a dozen high precision measurements of the gravitational constant carried out over the past 40 years. The most recent ones are on the top.

In Cavendish's setup, a dumbbell (inertia) is suspended from a fine wire (spring). The combination of inertia and spring forms a torsional oscillator. The brilliant concept behind the torsion pendulum is that it hangs like a plumb bob in the Earth's gravitational field. Hence, the torsional oscillator is only sensitive to torques about the wire, which coincides with the local vertical per the definition of a plumb line.

Why are we still talking 230 years after Cavendish's experiment about the measurement of G? The reason can be seen in Fig. 1. It shows the results of G experiments carried out in the last forty years. The picture is not pretty. While some experiments report relative uncertainties in the lower 10^{-5} range, the relative scatter between the results is a few times 10^{-4}. So, the measurement of G is still an active research area, and good ideas are needed.

PRECISION ENGINEERING

Cavendish used a dumbbell (see Fig. 2), two balls at either end of a stick, as the torsion bob of his pendulum. Many of his successors stayed with this choice, mainly because the gravitational force on a sphere is easy to calculate. But then, a paradigm shift occurred at the end of the 20th century, and two interesting ideas emerged.

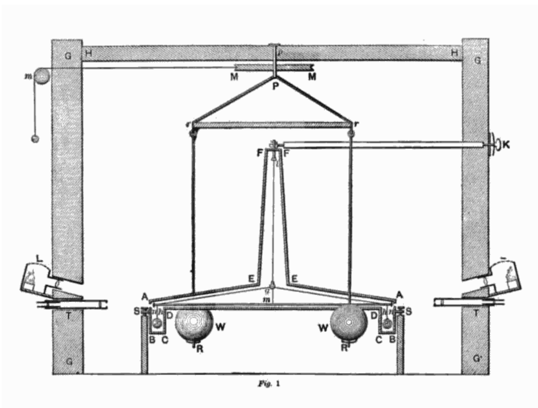

FIGURE 2. A schematic of the torsion balance that was originally used by Henry Cavendish to measure the strength of the gravitational interaction in the laboratory. Source: [2]

The first idea is using a parallel plate [3] and, second, employing torsion bobs with higher symmetry [5]. Figure 3 shows the first experiment implementing the parallel plate geometry. In the pictured experiment, the angular acceleration is compensated using a servo loop that applies an equal angular acceleration to the inner turntable. The latter is the measure of the experiment. The governing equation for the angular acceleration is

$$\ddot{\phi} = -\frac{4\pi G}{I}\sum_{l=2}^{\infty}\frac{1}{2l+1}\sum_{m=-l}^{l} m q_{lm} Q_{lm} e^{im\phi} \quad (1)$$

where q_{lm} and Q_{lm} denote the inner and outer multipole moments of the pendulum and the external field rotated by ϕ against each other. The moment of inertia of the pendulum is given by I. The researchers presenting [3] had the insight that for a thin rectangular plate of thickness t and width w, the ratio of the dominant term of the multipole expansion and the moment of inertia converges according to

$$\lim_{t\to 0}\frac{q_{22}}{I} = \lim_{t\to 0}\sqrt{\frac{15}{32\pi}\frac{w^2-t^2}{w^2+t^2}} = \sqrt{\frac{15}{32\pi}} \quad (2)$$

Hence, if $w \gg t$, the produced gravitational acceleration becomes independent of the exact geometry and mass of the plate. This cancellation is a rotational equivalent of a point mass dropping in the local gravitational field. Such a point mass will accelerate with g.

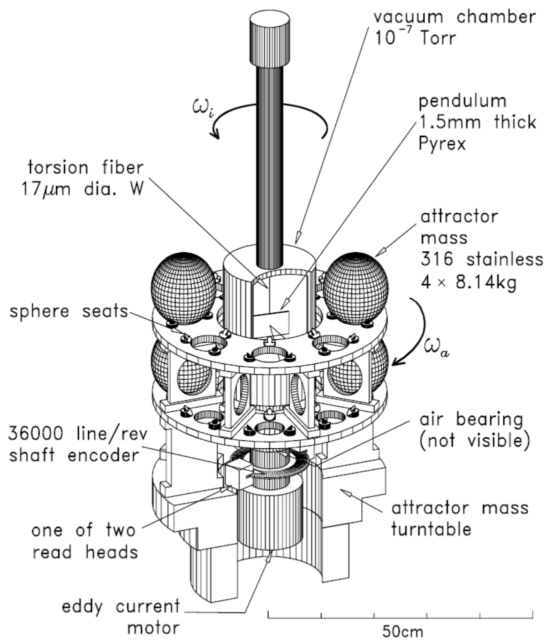

FIGURE 3. The first experiment using a parallel plate for the small mass [3].

Because of this trick, the demands on dimensional metrology of the smaller mass in the G experiment have been significantly reduced. Consequently, the relative contribution of the small mass metrology to the result is only 5×10^{-6} (less than half the total reported uncertainty), while in a previous experiment [4] it was 42×10^{-6} (almost two-thirds of the total reported uncertainty).

The second idea seems to go in the opposite direction. As we shall see, it requires much more careful metrology of the small and large masses, but it has the advantage that the gravitational coupling of the environment to the pendulum is greatly reduced

The experiment described above relies on the quadrupole coupling ($l = 2$) to produce the gravitational torque. The multipole field of a mass distribution with a typical size R (radius of a pitch circle) falls according to

$$Q_{lm} \propto \frac{1}{R^{l+1}} \quad (3)$$

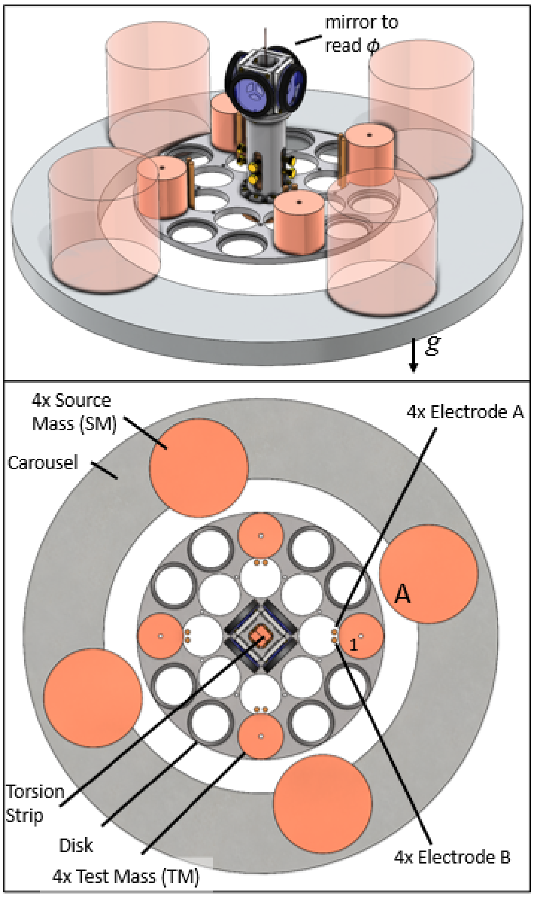

FIGURE 4. The higher order symmetry of the BIPM experiment results in reduced gravitational coupling between the pendulum and background fields. [5].

Hence, the quadrupole coupling diminishes proportionally to R^{-3}. Researchers at the BIPM (Bureau International des Poids et Mesures) opted to reduce the coupling further and decided on a four-fold symmetry, see Fig. 4. In such a hexadecapole coupling ($l = 4$) the maximal torque on the small mass set is given by

$$N = 35GMm\frac{r^4}{R^5}, \tag{4}$$

where r, R, m, and M denote the pitch circle radii and masses of the small (lower case) and large cylinders. While this geometry reduces the parasitic coupling of environmental gravity gradients, the demands on dimensional metrology are pretty stringent, as illustrated in the following example. The large pitch circle radius is $R \approx 214$ mm. Its uncertainty contribution will be below 10^{-5} if the radius can be measured with an uncertainty of 0.4 µm. This uncertainty can only be achieved by putting the experiment on a coordinate measurement machine. More details on the large pitch circle radius measurements can be found in [6].

Next to the inertial (and gravitational) mass, the torsion bob, the spring, is an essential ingredient of a torsion balance. What is the effect of the torsion spring on the measurement? Cavendish already reported anelastic observations. But, it was not until the mid-90s that people started seriously thinking about the effect of elastic loss on a G measurement. It does introduce a bias in specific measurements. The bias can be simply illustrated in the Cavendish method.

The gravitational torque acting on the pendulum (eq. (4) if inferred from the angular excursion from the equilibrium position,

$$N = \kappa(0)\,(\phi - \phi_o)\,, \tag{5}$$

where $\kappa(0)$ denotes the torsion constant at low (zero) frequency. The torsion constant is usually obtained by measuring the free angular frequency of the oscillation (ω_o) together with

$$\kappa(\omega_o) = \omega_o^2 I, \tag{6}$$

where I is the moment of inertia. Equation (6) calculates κ at $\omega = \omega_o$, but equation (5) needs κ at $\omega = 0$. The equivalence of $\kappa(0)$ and $\kappa(\omega_o)$ depends on the dissipative fraction of the elastic constant of the torsion spring. A small ratio of the imaginary to the real part of κ, leads to better agreement between $\kappa(0)$ and $\kappa(\omega_o)$.

Interesting strategies to reduce $\mathrm{Im}(\kappa)/\mathrm{Re}(\kappa)$ have been used in the last decades — experiments with quartz fibers, metal fiber at cryogenic temperatures, and strips. The latter has been employed by the BIPM experiment.

The pendulum bob, in this case, the torsion disk with four test masses, is supported by a heat-treated CuBe strip with a length $L = 120$ mm, a width of $b = 2.5$ mm, and a thickness of $t = 30$ µm.

The torsional stiffness is given by

$$\kappa = \frac{F\,bt^3}{3L} + \frac{M_p\,g\,b^2}{12L} \tag{7}$$

where F is the shear modulus of the material and $M_p\,g$ the weight of the pendulum bob. Two terms contribute to the restoring torque provided by the strip - elastic and gravitational. The first fraction in the equation above gives the elastic part of κ. The gravitational part is due to the fact that the torsion bob rises in the gravitational field as the strip is twisted, and it is given by the second fraction. The second term is about 24 times larger than the first.

The advantage of using a strip instead of a round fiber becomes apparent when contemplating the dissipative properties of the two contributions to the torsion constant. The elastic part has a loss, while the gravitational part is completely lossless. At room temperature, metallic torsion fibers with round cross sections show quality factors up to 3 000. Here, since the lossy part only contributes about 4 % of the total stiffness, a theoretical Q of 75 000 seems possible. In the literature, this effect is described as dissipation dilution.

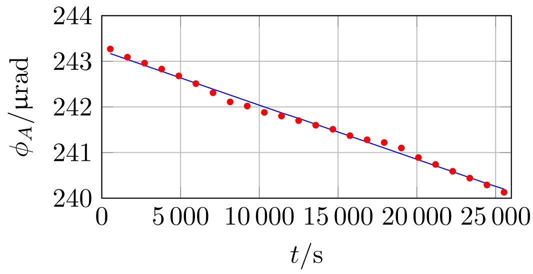

FIGURE 5. Measured free decay of the pendulum amplitude ϕ_{A} as a function of time. The period of the pendulum is 120.7 s. The decay time estimated from the blue fit line is about 24 days, corresponding to a $Q \approx 54\,000$. There is evidence that the system is limited by residual gas damping in the vacuum chamber.

Fig. 5 shows a measured decay of the free torsional amplitude of the BIPM torsion pendulum. The calculated Q is 54 000, slightly smaller than the theoretical Q mentioned above. In this case, the damping of the pendulum is caused by residual gas in the Vacuum chamber. For the measurement shown above the vacuum pressure was 1.1 mPa.

Cavendish's original measurement of the strength of gravity measured the deflection of the torsional balance under a given gravitational torque. As is discussed above, the deflection can be converted into torque using the spring constant obtained from the frequency measurement. In addition to the frequency measurement, a calibrated angle measurement had to be performed. In the mid-19th century, a new approach was invented. It relied solely on the measurement of frequencies, which are easy to perform in a traceable way. Shortly after the halfway point in the 20th century, Dicke was the first to operate a torsion balance with an electrostatic servo. The servo became a third successful method to measure G. About ten years later, Rose invented a fourth method to measure G. He mounted a torsion balance on a rotating turntable and used inertial (angular) acceleration to counteract the gravitational acceleration.

The BIPM torsion balance is the only experiment capable of measuring G with two methods: Cavendish and electrostatic servo. Having two methods offers two substantial advantages. First, if both measurements produce consistent results, the trust in the measurement is higher. Many biases would influence the two methods differently and would come apparent. However, consistent data does not guarantee that the combined result is correct because some systematics affect both results the same way, most notably the mass integration. Second, combining the two results will lead to a much smaller uncertainty due to some negative correlation. For example, the measurement of the angle, usually performed with an autocollimator, appears in the numerator in the Cavendish method and in the denominator in the Servo method.

Precision engineering and the measurement of G are fascinating windows into human ingenuity. In this talk, I will tell the story of the laboratory measurements of G from Cavendish to today. I will highlight the moments when new ideas, technology, and insights enhanced our understanding of the measurement problem. Besides these moments of leaps in progress, there is steady incremental improvement through elbow grease, sweat, and tears. Still, 250 years of incremental progress and half a dozen leaps have led to an intriguing story in precision measurements. Maybe you can learn something from that story?

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