Theoretical and Applied Fracture Mechanics 128 (2023) 104139

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

A study of ductile crack corrections for elastic-plastic fracture toughness tests

Enrico Lucon *

Applied Chemicals and Materials Division, National Institute of Standards and Technology (NIST), Boulder, CO, United States

ARTICLE INFO

Keywords:
ASTM E1820
Critical toughness
Ductile crack growth (DCG) correction
Crack resistance
Elastic Compliance tests
Incremental crack growth correction
ISO 12,135
Multi-specimen procedure
Tearing modulus

ABSTRACT

This paper critically examines existing ductile crack growth (DCG) corrections for *J*-integral values calculated with the multi-specimen and elastic compliance approaches, as well as two novel/alternative DCG corrections. The benchmark is represented by the incremental DCG correction prescribed by ASTM E1820-23a for Elastic Compliance tests. Considering calculations performed at both end-of-test and intermediate conditions, most of the existing and proposed approaches tend to under-correct *J* values, by an amount proportional to the percent of initial cracked ligament. For one of the proposed novel approaches (Pseudo-Resistance Curve Procedure), a tendency to overcorrection was observed. Nevertheless, when values of critical toughness (J_Q, J_{Ic}) and ductile crack resistance (expressed in terms of tearing modulus, T_M) are considered, differences are smaller than the typical uncertainties associated with elastic–plastic fracture toughness test results. Therefore, it does not appear necessary to modify the DCG corrections currently prescribed in the two most widely used fracture toughness test standards (ASTM E1820 and ISO 12135).

1. Introduction

The most widely used test standards for fracture toughness testing of metallic materials in the elastic–plastic (ductile) regime are ASTM E1820-23a [1] and ISO 12135:2002 [2]. Both standards also cover the determination of quasi-static fracture toughness in other fracture regimes, such as linear elastic (brittle – only ISO 12135) and ductile-to-brittle transition (mixed ductile and brittle – both standards).

In case of fully ductile behavior, the results obtained from a fracture test, in terms of *J*-integral[1], are the critical value near the onset of crack growth, and the crack resistance (*J-R*) curve (also known as *J-R* curve). Provided the critical value of *J* fulfils a number of validity requirements, it qualifies as plane-strain fracture toughness (J_{Ic} according to E1820), which is defined as the crack extension resistance under conditions of crack-tip plane strain. An non-qualified J_{Ic} is labeled J_Q. In ISO 12135, two critical values of *J*-integral are defined, namely J_i and $J_{0.2BL}$. In this paper, we will primarily refer to ASTM E1820.

Critical *J* values and crack resistance curves can be obtained via two different approaches:

- Multi-specimen approach (called *Basic Procedure* in E1820), which involves testing several nominally identical specimens without the use of crack extension measurement equipment. Each specimen is loaded to a selected displacement level, corresponding to different values of *J*–integral and amounts of stable (ductile) crack extension, Δa. Each specimen provides an individual [J,Δa] data point that will be used to obtain the *J-R* curve and the critical toughness value.
- Single-specimen approach (denominated *Resistance Curve Procedure* in E1820), whereby crack extension measurement equipment is used to obtain a full *J-R* curve (and corresponding critical toughness) from each specimen tested. Crack size can be inferred from the elastic compliance measured during unloading/reloading cycles performed at equally-spaced intervals. This is by far the most popular single-specimen technique, and is denominated Elastic (or Unloading) Compliance (EC) Technique [9]. Other single-specimen techniques covered in [1] and [2] are Electric Potential Difference Methods and the Normalization Data Reduction technique (this latter only in E1820).

Regardless of the approach used, when ductile behavior is observed,

* Corresponding author.
E-mail address: enrico.lucon@nist.gov.

[1] The *J*-integral is a line or surface integral that encloses the crack front from one surface to another, and characterizes the local stress–strain field at the crack tip [3].

https://doi.org/10.1016/j.tafmec.2023.104139
Received 31 May 2023; Received in revised form 14 August 2023; Accepted 11 October 2023
Available online 14 October 2023
0167-8442/Published by Elsevier Ltd.

J-integral values must be corrected for stable crack growth, as the total work absorbed by the specimen does not correspond to the instantaneous driving force if crack extension has occurred. Various ductile crack growth (DCG) corrections have been analyzed and compared in this study, including a couple of novel formulations that are not included in ASTM E1820 or ISO 12135.

The overall objective of this investigation was to establish whether the various available DCG corrections are consistent with each other, or whether the introduction of new corrections could improve such consistency, irrespective of test standard (ASTM or ISO) and approach used (multi-specimen or single-specimen).

2. Short historical review of J-controlled crack growth

The root problem caused by using the J-integral concept, originally proposed by Rice [3], in the presence of significant ductile crack growth was soon identified in the significant difference between the area under the force/displacement test record for a stationary crack (upon which the definition of J is based) and for a growing crack [4,5]. It was therefore recognized that a ductile crack growth correction had to be incorporated into the J evaluation procedure in order to obtain accurate J values with increasing crack growth from the measured force/displacement records.

During the 1970s, considerable effort was spent in developing experimental and analytical procedures to calculate J using the smallest possible number of specimens, as opposed to the multi-specimen approach, originally introduced by Begley and Landes [6]. Of particular importance were the analyses of Rice *et al.* [7], Merkle and Corten [8], and Clarke *et al.* [9], which allowed calculating J from a single force–displacement test record for bend-type and compact-type specimen configurations, respectively. At the same time, J had been tentatively considered the controlling parameter in the presence of crack growth, and the concept of J-R curve, or crack resistance curve, had been introduced.

In the late 1970s, the existence of a J-controlled crack growth regime for limited amounts of crack extension was demonstrated by Hutchinson and Paris [10], and the method proposed therein was later generalized by Ernst *et al.* [11]. The formulae presented were regarded as exact from an analytical point of view, but also required a considerable computational effort. As a result, Ernst *et al.* [12] were able to develop simpler formulae that could be used to establish J in the presence of stable crack growth, following the actual path of the force/displacement test record. The formula proposed is basically the same currently adopted by ASTM E1820 for the Resistance Curve Procedure. In [12], the authors also demonstrated that the crack growth-uncorrected J expression proposed in [8] tends to overestimate J, whereas the proposed corrected formula was in very good agreement with the exact procedure in [10].

Other J expressions, corrected for stable crack growth, were proposed around the same time frame (1970s and 1980s) by Garwood *et al.* [13], as well as by Andrews (quoted as “private communication” in [12]).

It’s important to note here that the various J correction schemes mentioned above hold true for crack resistance curves measured using load-line displacement (LLD) data, but become questionable if displacement is measured in terms of crack mouth opening displacement (CMOD), as customarily done for single-edge bend, SE(B), and single-edge tension, SE(T), specimens. Cravero and Ruggieri [14] developed a ductile crack growth correction for J with increased loading when using laboratory measurements of force and CMOD data, based on a constant relationship between the plastic components of LLD and CMOD. Analytical expressions for the non-dimensional plastic factors η and γ in the case of CMOD measurements and SE(B) specimens were provided by Zhu *et al.* [15]. The results of these investigations were fully adopted in the 2009 edition of the ASTM E1820 standard.

3. Existing and proposed DCG corrections

3.1. ASTM

Before the first edition of E1820 was published in 1998, other ASTM standards were available for elastic–plastic fracture toughness testing: ASTM E813 [16] and ASTM E1152 [17] were used for the establishment of J_{Ic} and the determination of J-R curves, respectively, until 1997, when they were both withdrawn. In the same year, these two standards were replaced by ASTM E1737 [18], which covered both J_{Ic} and crack resistance curves, but was in turn discontinued in 1998.

While E1152 only covered the elastic compliance technique, both E813 and E1737 included both the multi-specimen and single-specimen approaches. In E1737, the reference method was the elastic compliance technique, while multi-specimen testing was covered in Annex A4 and only restricted to the determination of the critical toughness J_{Ic}.

All three standards required the calculation of two separate J-integral components, elastic and plastic, with $J = J_{el} + J_{pl}$. The same partitioning scheme was retained by E1820.

All ASTM standards, both withdrawn and current, covered three specimen configurations: single-edge bend, SE(B), compact tension, C(T), and disk-shaped compact tension, DC(T). Only the first two specimen types, SE(B) and C(T), are considered in this study.

3.1.1. Multi-Specimen (Basic Procedure) tests

In both E813 and E1737, the elastic and plastic components of the J-integral were given respectively by:

$$J_{el} = \frac{K^2(1-\nu^2)}{E} \tag{1}$$

and

$$J_{pl} = \frac{\eta A_{pl}}{B_N b_o} \tag{2}$$

where:

- K is the stress intensity factor corresponding to the final force/displacement point of the test before the force is returned to zero;
- ν is the material’s Poisson’s ratio;
- E is the material’s Young’s modulus at the test temperature;
- η is a geometrical factor that depends on the specimen configuration and the initial crack size, a_o;
- A_{pl} is the plastic area (obtained by subtracting the elastic area[2] from the total area under the force/displacement curve);
- B_N is the net thickness of a side-grooved specimen, which corresponds to the original thickness B for a plane-sided specimen; and
- b_o is the original ligament size, given by $b_o = W - a_o$, with W = specimen width.

In E813 and E1737, Eq. (2) for the plastic J component did not include a DCG correction, which represented an inconsistency with respect to the Elastic Compliance technique, as will be seen below.

When E1820 replaced E813 and E1737 in 1998, the same eqs. (1) and (2) were retained until the 2008 edition, when a DCG correction was finally added to J_{pl}, based on work performed and published by Wallin and Laukkanen in 2004 [19]:

$$J_{pl} = \frac{J_{pl0}}{1 + \left(\frac{\alpha - m}{\alpha + m}\right)\frac{\Delta a}{b_o}} \tag{3}$$

where J_{pl0} is the uncorrected plastic J-integral given by Eq. (2), α = 1 for

[2] The elastic area is calculated using the initial elastic compliance (corresponding to a_o).

SE(B) specimens or 0.9 for C(T) and DC(T) specimens, and Δa is the ductile crack extension measured at the end of the test. The factor m was established by a three-step correction procedure provided in Annex A16, consisting of:

(a) Obtaining preliminary DCG-corrected J values from eq. (3) with $m = 0.5$.
(b) Fitting a power law of the form $J = J_{1mm}\Delta a^m$ to the corrected data for $\Delta a/b_o \geq 0.05$.
(c) Establishing the final DCG-corrected J values from eq. (3) with the value m calculated from the previous step (exponent of the power law).

In the 2018 version of E1820, the above procedure was removed, and the DCG-corrected value of J_{pl} for the Basic Procedure was simply expressed as:

$$J_{pl} = \frac{J_{pl0}}{1 + \left(\frac{\alpha - 0.5}{\alpha + 0.5}\right)\frac{\Delta a}{b_o}} \tag{4}$$

that is, assuming $m = 0.5$ in all cases. Eqs. (1) and (4) are currently used to calculate J-integral values for the Basic Procedure in E1820-23a. The same DCG correction is used irrespective of specimen type used.

3.1.2. Single-specimen (Resistance curve procedure/Elastic compliance) tests

E813 used identical formulations, eqs. (1) and (2), for both multi-specimen and single-specimen tests. In 1995, E1152 introduced a new DCG-corrected expression for the plastic component of the J–integral in case of Elastic Compliance tests:

$$J_{pl(i)} = \left[J_{pl(i-1)} + \frac{\eta_i}{b_i}\cdot\frac{A_{pl(i)} - A_{pl(i-1)}}{B_N}\right]\left[1 - \gamma_i\frac{a_i - a_{i-1}}{b_i}\right] \tag{5}$$

where the subscripts i and i-1 refer to the current and previous measurements of crack size. γ is another geometrical factor that depends, like η, on the specimen configuration. The same expression is currently used in E1820.

Eq. (5) was adapted from the original formulation of Ernst, Paris, and Landes [12], by replacing J_i and A_i with $J_{pl(i)}$ and $A_{pl(i)}$ respectively. The DCG correction is represented by the second term between square brackets in the right-hand member, and it applies to the sum of the previous plastic J plus the increment of plastic J between i-1 and i.

When E1737 replaced both E813 and E1152, eq. (5) for Elastic Compliance tests was replaced by:

$$J_{pl(i)} = \left[J_{pl(i-1)} + \frac{\eta_{i-1}}{b_{i-1}}\cdot\frac{A_{pl(i)} - A_{pl(i-1)}}{B_N}\right]\left[1 - \gamma_{i-1}\frac{a_i - a_{i-1}}{b_i}\right] \tag{6}$$

which is effectively the same as eq. (5), except that η_i, b_i, and γ_i are now replaced by η_{i-1}, b_{i-1}, and γ_{i-1}.[3] Eq. (6) was transferred to E1820 in 1999 (with the typo corrected), and has remained unchanged until today. The DCG correction in eq. (6), which is incremental by nature, is a number between 0 and 1, which implies that ductile crack growth reduces the value of J-integral with respect to a stationary crack.

3.2. ISO

The precursors to ISO 12135, first published in 2002, were the ESIS P1 and P2 Procedures ([20,21]), prepared by the European Structural Integrity Society and published in January 1992. With respect to the ESIS documents, ISO 12135 introduced the partitioning of J into elastic and plastic components. The calculation formula for the plastic J-integral in ISO 12135 is:

$$J_{pl} = \left(\frac{\eta A_{pl}}{b_o B_N}\right)\left(1 - \frac{\Delta a}{b_o}\right) \tag{7}$$

irrespective of the approach used (multi-specimen or single-specimen). In this case, the DCG correction is represented by the second factor of the right-hand member of Eq. (7). The geometrical factor η is the same as in the ASTM standards. As can be seen, according to ISO 12135 the J-integral is not calculated incrementally for an Elastic Compliance test.

3.3. Novel (Proposed) DCG corrections

3.3.1. Modified ASTM Basic Procedure Correction

In the first proposed correction, the elastic component is identical to the E1820 Basic Procedure, eq. (1), while the plastic component uses parameters calculated with respect to the end of test instead of the start of test (the final ligament size, b_i, is calculated from the final crack size a_i, and A_{pl} is evaluated used the compliance corresponding to the last unloading). This appears, at least to the author, a more logical approach than making reference to start-of-test conditions.

3.3.2. Pseudo-Resistance Curve Procedure Correction

In the second proposed correction, the end-of-test J value is corrected using an incremental approach, whereby the [i-1] point, point "A", corresponds to a generic point within the linear elastic portion of the test, while the [i] point, point "B", corresponds to the end of the test. The elastic component of the J–integral is the same as for the E1820 method, eq. (1), while the plastic component is given by:

$$J_{pl(B)} = \left[J_{pl(A)} + \frac{\eta_A}{b_A}\cdot\frac{A_{pl(B)} - A_{pl(A)}}{B_N}\right]\left[1 - \gamma_A\frac{a_B - a_A}{b_B}\right] \tag{8}$$

Since the point A is located within the linear elastic region, where plasticity is absent by definition, both $J_{pl(A)}$ and $A_{pl(A)}$ are equal to zero. On the other hand, η_A, b_A, and γ_A are calculated from a_o, and $a_A = a_o$. As for point B, $A_{pl(B)}$ uses the final compliance and a_B is the final estimated crack size. Therefore, eq. (8) transforms into:

$$J_{pl,PRCP} = \left[\frac{\eta_o}{b_o}\cdot\frac{A_{pl(B)}}{B_N}\right]\left[1 - \gamma_o\frac{\Delta a}{b_o}\right] \tag{9}$$

4. Analyses performed

Several Elastic Compliance tests, performed on different steels using SE(B) or C(T) specimens, were analyzed from a multi-specimen perspective, in order to assess the consistency of the DGC corrections that ASTM E1820 prescribes for the Basic and Resistance Curve Procedures respectively. Specifically, two rounds of analyses were performed and will be presented here:

(a) For 17 EC tests (7 on SE(B) and 10 on C(T) specimens), the J-integral values at the end of test were compared, as calculated for the ASTM Resistance Curve Procedure (J_{RCP}) with the incremental DCG correction, and the ASTM Basic Procedure (J_{BP}), using various DCG corrections (existing and novel).
(b) For 5 of the 17 EC tests above, J-integral values were calculated corresponding to selected intermediate unloadings inside the ASTM region of qualified data (delimited by J_{limit}[4] and the 0.15 mm and 1.5 mm offset exclusion lines). These J values ($J_{i,RCP}$), all incrementally corrected according to E1820, were compared with equivalent multi-specimen values ($J_{i,BP}$), calculated for the same

[3] Interestingly, an obvious typo was overlooked in E1737-96: the first factor inside the first set of square brackets was printed as $J_{pl(i)}$ instead of $J_{pl(i-1)}$.

[4] $J_{limit} = \frac{b_o \sigma_Y}{7.5}$, with σ_Y = average between yield strength and tensile strength at test temperature.

specimens using the ASTM Basic Procedure formulae and DCG-corrected by means of existing and novel formulations.

More details on the analyses performed will be provided in the following sections.

4.1. Round 1: Comparison of End-of-Test Values

The 17 single-specimen (EC) tests considered in this study are listed in Table 1, along with the J–integral values corresponding to the final unloading (J_{RCP}). As a reminder, J_{RCP} values in Table 1 are calculated in accordance with ASTM E1820-23a using eqs. (1) and (6). Table 1 also presents values of $\Delta a/b_o$ (percent of cracked ligament), a parameter that appears in every Basic Procedure DCG correction.

For each of the tests listed in Table 1, the following Basic Procedure DCG-corrected values (J_{BP}) were calculated:

- $\boldsymbol{J_{BP,ASTM}}$ – In accordance with ASTM E1820-23a, eqs. (1) and (4). Note that the elastic component is evaluated with respect to the end-of-test values of force and displacement, while the uncorrected plastic component, J_{pl0}, is calculated based on initial test conditions (b_o, η calculated from a_o, and $A_{pl} = A_{tot} - A_{el}$, where A_{tot} is the total area under the force/displacement curve and A_{el} is calculated using the final force value and the original loading slope). For the tests in Table 1, the initial crack size, a_o, corresponds to the a_{oq} value predicted by EC, while the original loading slope is the average slope of the initial unloading cycles, performed in the linear elastic region in accordance with the E1820 procedure.
- $\boldsymbol{J_{BP,ISO}}$ – In accordance with ISO 12135:2021, eqs. (1) and (7).
- $\boldsymbol{J_{BP,ASTM\_mod}}$ – Using the first novel DCG correction proposed (modified ASTM Basic Procedure correction).
- $\boldsymbol{J_{PRCP}}$ – Using the second novel DCG correction proposed (Pseudo-Resistance Curve Procedure correction).

Comparisons between end-of-test J values are summarized in Table 2 and illustrated in Fig. 1.[5] For both current Basic Procedure approaches (ASTM and ISO), values are significantly higher than for the Resistance Curve Procedure. However, the overestimation is significantly larger for ASTM (16.1 % on average) than for ISO (10.0 % on average). As for the novel approaches, the modified ASTM correction somewhat improves the agreement with respect to the original formulation (the mean difference decreases from 16.1 % to 12.7 %). The Pseudo-Resistance Curve Procedure is the only approach that tends to over-correct J values (mean difference = −17.6 %). A possible explanation is the underestimation of the actual true value of A_{pl}, ensuing from the necessarily crude partitioning of the test into just two "virtual unloadings".

Differences between J_{RCP} (used here as the benchmark) and DCG-corrected Basic Procedure values are plotted in Fig. 2 as a function of percent cracked ligament, $\Delta a/b_o$. Deviations (both positive and negative) tend to increase with $\Delta a/b_o$, although they remain within acceptable limits (±20 %) for $\Delta a/b_o < 50$ %. For the PRCP method, only data with $\Delta a/b_o < 60$ % were fitted, as the trend changes considerably above that threshold.

Table 1
Values of end-of-test J-integral calculated according to ASTM E1820-23 for the 17 EC tests considered.[1]

Material	Specimen type	T (°C)	Specimen id	J_{RCP} (kJ/m^2)	$\frac{\Delta a}{b_o}$
316L weld	PCCv	−196	W2-F8	453.63	28.9%
			W2-F9	404.98	35.8%
			W2-F10	438.54	36.3%
			W2-F11	426.00	40.7%
			W2-F12	383.92	27.8%
22NiMoCr37	1TC(T)	21	CGW_23	1435.88	13.4%
			CGW_24	1443.25	14.3%
			CGW_25	1459.45	12.1%
ASTM Set 1	1TC(T)	N/A	ASTM1	172.22	23.3%
ASTM Set 2			ASTM2	1271.97	12.1%
ASTM Set 3			ASTM3	1267.44	12.3%
ASTM Set 4	1TSE(B)		ASTM4	86.65	54.8%
ASTM Set 5	1TC(T)		ASTM5	73.01	61.2%
ASTM Set 6	0.5TC(T)		ASTM6	74.51	73.0%
ASTM Set 7	1TSE(B)		ASTM7	632.40	30.3%
ASTM Set 8	1TC(T)		ASTM8	567.82	67.4%
ASTM Set 9			ASTM9	729.00	14.4%

[1] Tests labeled "ASTM Set X", with X = 1 to 9, are the 9 standard data sets that the ASTM E08 Committee (*Fracture and Fatigue*) made available for verifying computer algorithms developed to implement J_{Ic} calculations. They correspond to a wide range of materials and toughness levels.

4.2. Round 2: Comparisons between elastic Compliance and Multi-Specimen J-R curves

The analyses conducted in Round 1 using end-of-test J values emphasized general trends when comparing different DCG corrections. However, most of those values could not be used in a multi-specimen perspective, as the corresponding crack extensions, Δa, exceeded the ASTM validity limits for establishing both the critical toughness, Δa_{limit}, and the J-R curve, Δa_{max}.

In Round 2, we chose 5 of the 17 Elastic Compliance tests listed in Table 1. For each test, we selected 5 or 6 unloading cycles inside the ASTM E1820 region of qualified data, and we calculated the corresponding J–integral values, J_i, using the following Basic Procedure DCG corrections:

(a) ASTM E1820-23a ($J_{i,BP}$): eq. (4) with $m = 0.5$.
(b) ASTM E1820 modified (J_{i,BP_mod}): eq. (2) with η, A_{pl}, and b calculated in reference to the end of test (final crack size and final compliance).
(c) Pseudo-Resistance Curve Procedure ($J_{i,PRCP}$): eq. (9).
(d) ISO 12,135 (J_{ISO}): eq. (7).

In every case, the elastic component of J was calculated according to eq. (1).

For each of the five selected tests and each of the four DCG corrections, a "virtual" multi-specimen data set was generated, consisting of five or six [Δa_i, J_i] data points, where the Δa_i values were those obtained from Elastic Compliance measurements. Each data set was then analyzed in accordance with Annex A9 of ASTM E1820-23a, in order to obtain a provisional value of critical fracture toughness, J_Q (corresponding to J_{Ic}, provided various qualification requirements are fulfilled).

Individual calculated J_i values from the 20 data sets are collected in Table 3 and illustrated in Fig. 3, where the benchmark values plotted on the X-axis are those provided by the Resistance Curve (Elastic Compliance) analyses, $J_{i,RPC}$. The vast majority of the multi-specimen values fall within ± 10 % of their single-specimen counterpart, with a tendency to underestimation for the Pseudo-Resistance Curve Procedure and a tendency to overestimation for the three remaining approaches.

To more directly illustrate the possible effects of the various DCG approaches on the J-R curve, Fig. 4 and Fig. 5 compare crack resistance curves for two very different materials and specimen types: ASTM-DS3 (1TC(T), high toughness) and ASTM-DS4 (1TSE(B), low toughness), respectively. In the former case, the EC curve is almost indistinguishable from the DCG-corrected multi-specimen curves, with the exception of PRC, which is significantly lower above $\Delta a \approx 1$ mm. In the latter case, all the DCG-corrected curves are similar and clearly higher than EC.

Relative differences between multi- and single-specimen J_i values are

[5] In Fig. 1 and in the rest of the paper, "acceptability lines" corresponding to ±10 % or ±20 % are used, based on the author's engineering judgement.

Table 2
Comparison between end-of-test J-integral values for the Resistance Curve and Basic Procedures (ASTM, ISO, and novel corrections).

Material	Specimen type	T(°C)	Specimen id	J_{RCP} (kJ/m^2)	$J_{BM,ASTM}$ (kJ/m^2)	Δ	$J_{BM,ASTM,mod}$ (kJ/m^2)	Δ	J_{PRCP} (kJ/m^2)	Δ	$J_{BP,ISO}$ (kJ/m^2)	Δ	$\frac{\Delta a}{b_o}$
316L weld	PCCv	−196	W2-F8	453.63	495.14	9.2%	459.29	1.2%	426.99	−5.9%	465.16	2.5%	28.9%
			W2-F9	404.98	462.39	14.2%	422.11	4.2%	378.10	−6.6%	425.56	5.1%	35.8%
			W2-F10	438.54	479.55	9.4%	440.40	0.4%	391.82	−10.7%	440.62	0.5%	36.3%
			W2-F11	426.00	493.03	15.7%	442.45	3.9%	387.03	−9.1%	446.57	4.8%	40.7%
			W2-F12	383.92	423.25	10.2%	395.29	3.0%	367.41	−4.3%	398.86	3.9%	27.8%
22NiMoCr37	1TC(T)	21	CGW_23	1435.88	1593.65	11.0%	1573.19	9.6%	1362.04	−5.1%	1577.37	9.9%	13.4%
			CGW_24	1443.25	1583.99	9.8%	1563.04	8.3%	1337.52	−7.3%	1566.43	8.5%	14.3%
			CGW_25	1459.45	1610.62	10.4%	1591.85	9.1%	1398.68	−4.2%	1595.85	9.3%	12.1%
ASTM Set 1	1TC(T)	N/A	ASTM1	172.22	203.94	18.4%	206.24	19.8%	163.84	−4.9%	200.74	16.6%	23.3%
ASTM Set 2			ASTM2	1271.97	1290.60	1.5%	1284.95	1.0%	1141.01	−10.3%	1282.08	0.8%	12.1%
ASTM Set 3			ASTM3	1267.44	1293.04	2.0%	1287.05	1.5%	1139.20	−10.1%	1283.80	1.3%	12.3%
ASTM Set 4	1TSE(B)		ASTM4	86.65	110.73	27.8%	105.81	22.1%	79.34	−8.4%	95.31	10.0%	54.8%
ASTM Set 5	1TC(T)		ASTM5	73.01	109.48	49.9%	115.51	58.2%	43.68	−40.2%	102.82	40.8%	61.2%
ASTM Set 6	0.5TC(T)		ASTM6	74.51	112.66	51.2%	111.32	49.4%	14.80	−80.1%	103.46	38.8%	73.0%
ASTM Set 7	1TSE(B)		ASTM7	632.40	684.76	8.3%	645.19	2.0%	584.58	−7.6%	641.19	1.4%	30.3%
ASTM Set 8	1TC(T)		ASTM8	567.82	671.79	18.3%	657.15	15.7%	135.49	−76.1%	623.86	9.9%	67.4%
ASTM Set 9			ASTM9	729.00	782.53	7.3%	777.18	6.6%	671.32	−7.9%	775.28	6.3%	14.4%

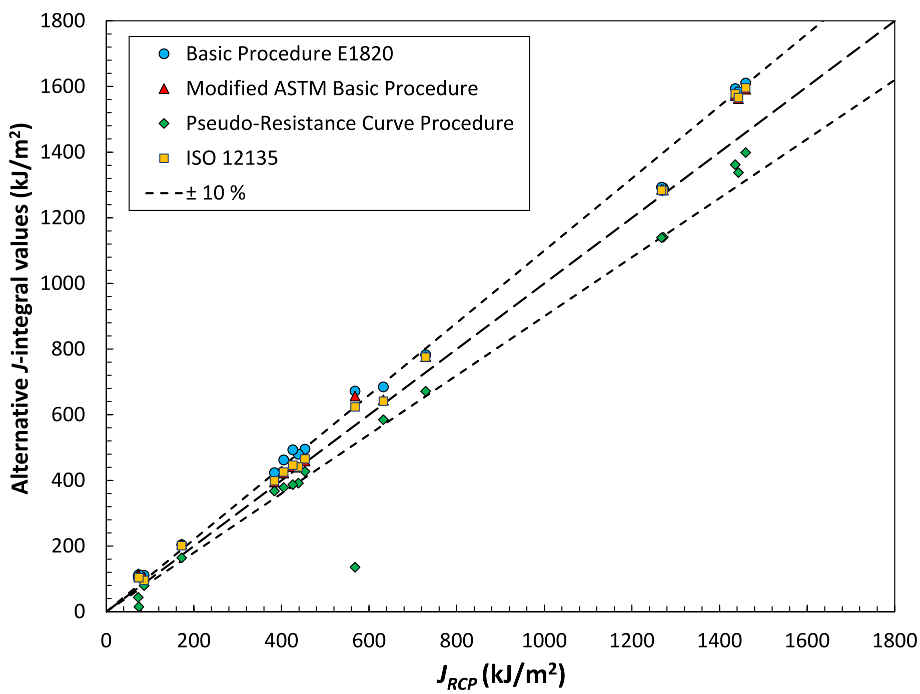

Fig. 1. Direct comparisons between end-of-test *J*-integral values calculated with the ASTM E1820 Resistance Curve Procedure and various Basic Procedure DCG-corrected values, with ±10 % reference lines.

plotted in Fig. 6 as a function of percent cracked ligament. For most DCG corrections, differences tend to increase with increasing $\Delta a/b_o$. In the case of the PRCP approach, data points are considerably scattered between positive and negative differences depending on the specific data set. Despite the different scales of the X-axes, these results substantially confirm the findings illustrated in Fig. 2. Note that the maximum crack extension capacity for a specimen, Δa_{max}, according to Annex A8 of ASTM E1820-23a, corresponds to 25 % of the initial uncracked ligament. Only two of the 29 values of $\Delta a/b_o$ reported in Table 3 are higher than 25 %, corresponding to the smallest specimen considered in Round 2 (precracked Charpy-V specimen of 316L weld material). For the four remaining specimens, the selected data points can be used both to determine J_Q and to construct the *J-R* curve.

After calculating sets of multi-specimen J_i values corresponding to 4 different DCG corrections, each set was analyzed in accordance with Annex A9 of E1820-23a. The regression function used for the data points (all located, by construction, inside the E1820 region of qualified data) is a power law of the form:

$$J = C_1 \Delta a^{C_2} \tag{10}$$

where C_1 and C_2 are coefficients that are established by least square fitting. The provisional value of plane strain fracture toughness, J_Q, and the corresponding crack extension, Δa_Q, are found by intersecting the fitting curve, eq. (10), with the 0.2 mm-offset construction line having the form:

$$J = 2\sigma_Y(\Delta a - 0.2\mathrm{mm}) \tag{11}$$

where σ_Y is the material's flow stress (average of yield and tensile stresses) at the test temperature. The slope in eq. (11), $2\sigma_Y$, is supposed to represent the material's behavior in the early stages of the test, when crack tip blunting occurs and stable crack extension hasn't initiated yet. Multi-specimen J_Q values are qualified as size-independent fracture

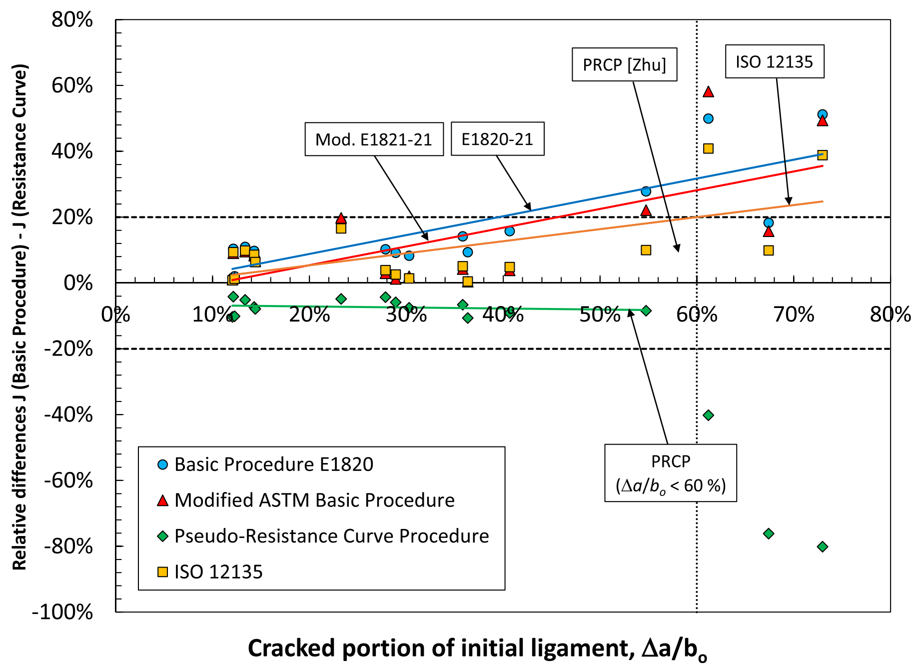

Fig. 2. Relative (percent) differences between Basic and Resistance Curve Procedure values of J-integral vs. cracked portion of initial ligament. Fitting lines refer to data symbols of the same color; dashed lines correspond to ± 20 % acceptability limits.

Table 3
Multi-specimen J_i values calculated for 5 specimens and various DCG correction approaches.

Material/Data set	Specimentype	SpecimenID	Unloading#	J_i (kJ/m^2)					$\frac{\Delta a}{b_o}$
				RPC (EC)	BP	BM_mod	PRCP	ISO	
316L weld	PCCv	W2-F11	24	261.47	275.84	271.90	268.19	272.14	8.3%
			27	295.92	309.22	300.82	293.39	302.02	13.3%
			31	336.51	358.71	344.28	331.39	346.57	18.2%
			33	352.72	379.14	361.25	344.81	363.88	20.9%
			37	378.60	419.87	392.63	366.50	396.47	27.1%
			40	385.10	437.25	402.03	366.38	406.24	32.7%
22NiMoCr37	1TC(T)	CGW_25	41	872.35	943.30	937.26	897.36	940.47	4.6%
			46	1003.48	1086.96	1079.18	1021.02	1082.79	5.7%
			50	1103.92	1202.61	1192.83	1111.55	1196.69	7.1%
			53	1173.89	1286.50	1274.92	1173.94	1278.91	8.3%
			57	1280.62	1399.90	1386.23	1261.73	1390.64	9.2%
			60	1343.45	1479.53	1463.54	1311.02	1468.02	10.6%
ASTM-DS3	1TC(T)	DS3	30	460.34	464.46	463.78	452.13	463.82	3.1%
			42	682.46	688.87	687.02	657.46	687.19	5.0%
			54	900.14	911.04	908.90	849.85	907.58	7.3%
			61	1027.77	1037.91	1032.72	954.68	1033.26	8.3%
			68	1143.59	1161.30	1154.19	1046.91	1154.76	10.1%
			74	1236.10	1263.06	1254.01	1116.01	1254.47	11.8%
ASTM-DS4	1TSE(B)	DS4	8	41.52	43.36	43.53	43.41	43.29	1.9%
			9	45.62	47.94	48.18	47.97	47.82	2.9%
			10	48.96	52.02	52.36	52.00	51.80	4.3%
			11	52.19	55.93	56.34	55.79	55.59	5.6%
			12	54.55	59.24	59.77	58.95	58.73	7.4%
ASTM-DS8	1TC(T)	DS8	6	96.95	98.82	100.34	99.91	98.80	1.3%
			7	128.16	130.94	132.98	131.73	130.87	2.2%
			8	156.61	159.40	161.61	159.54	159.30	2.6%
			9	181.28	187.25	190.17	185.74	187.01	4.2%
			10	203.41	210.61	213.76	207.22	210.26	5.1%
			11	231.62	245.82	249.95	236.82	245.08	7.9%

toughness values, J_{Ic}, if the following requirements are met:

(1) Power law coefficient, $C_2 < 1.0$.
(2) Specimen thickness, $> 10\frac{J_Q}{\sigma_Y}$.
(3) Initial ligament size, $b_o > 10\frac{J_Q}{\sigma_Y}$.

An additional parameter that was calculated is the tearing modulus T_M, which is defined as [12,22]:

$$T_M = \frac{E}{\sigma_Y}\cdot\left(\frac{dJ}{da}\right)_Q \tag{12}$$

where $\left(\frac{dJ}{da}\right)_Q$ is the slope of the power law fit, eq. (10), in the point with coordinates $[\Delta a_Q, J_Q]$. This parameter is not included in E1820, but can provide an indication of the material's resistance to crack propagation at

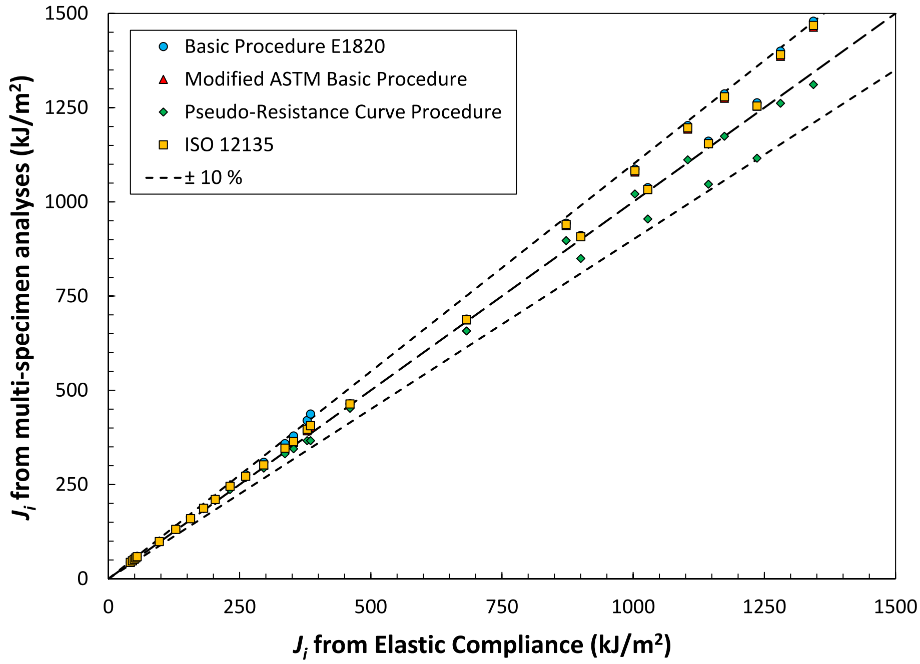

Fig. 3. Comparison between DCG-corrected single-specimen and multi-specimen J_i values.

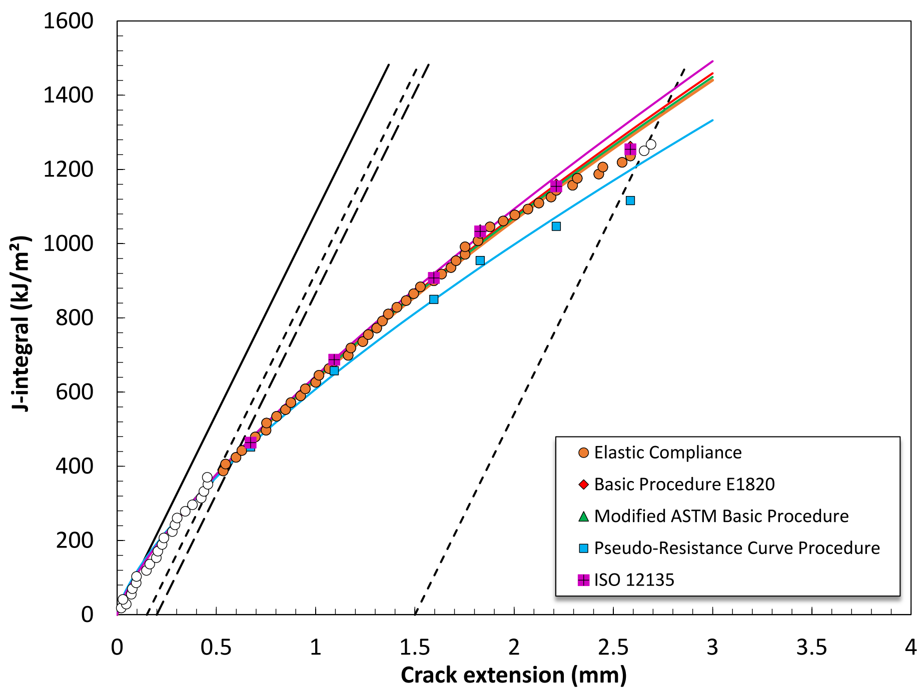

Fig. 4. ASTM-DS3 (1TC(T), high toughness): crack resistance (*J-R*) curves for elastic compliance and DCG-corrected approaches. Empty symbols represent Elastic Compliance data points outside the ASTM E1820 region of qualified data.

the onset of stable crack propagation (the higher T_M, the higher the resistance to crack propagation).

Values of critical fracture toughness, J_Q or J_{Ic}, and tearing modulus, T_M, are listed in Table 4 and compared in Fig. 7 (critical toughness) and Fig. 8 (tearing modulus).

Irrespective of DCG correction approach, excellent agreement between single-specimen (Elastic Compliance) and multi-specimen results was observed. All differences were within ± 10 % for J_Q/J_{Ic} (Fig. 7) and within ± 20 % for T_M (Fig. 8). The smallest average differences were calculated for the Pseudo-Resistance Curve for both J_Q/J_{Ic} and T_M, while the largest were observed for the modified Basic Procedure (J_Q/J_{Ic}) and for the E1820-21 Basic Procedure (T_M).

The mean differences observed in terms of critical toughness were all larger than zero (between 2.7 % and 4.7 %), indicating that DCG corrections for the Basic Procedure tend to slightly overestimate the values yielded by Elastic Compliance. However, the PRCP approach exhibited significant scatter, corresponding to overestimation or underestimation depending on the specific data set analyzed. In the case of the tearing modulus, mean differences are all larger than zero (between 5.8 % and 9.4 %), except for the Pseudo-Resistance Curve Procedure (–3.5 %).

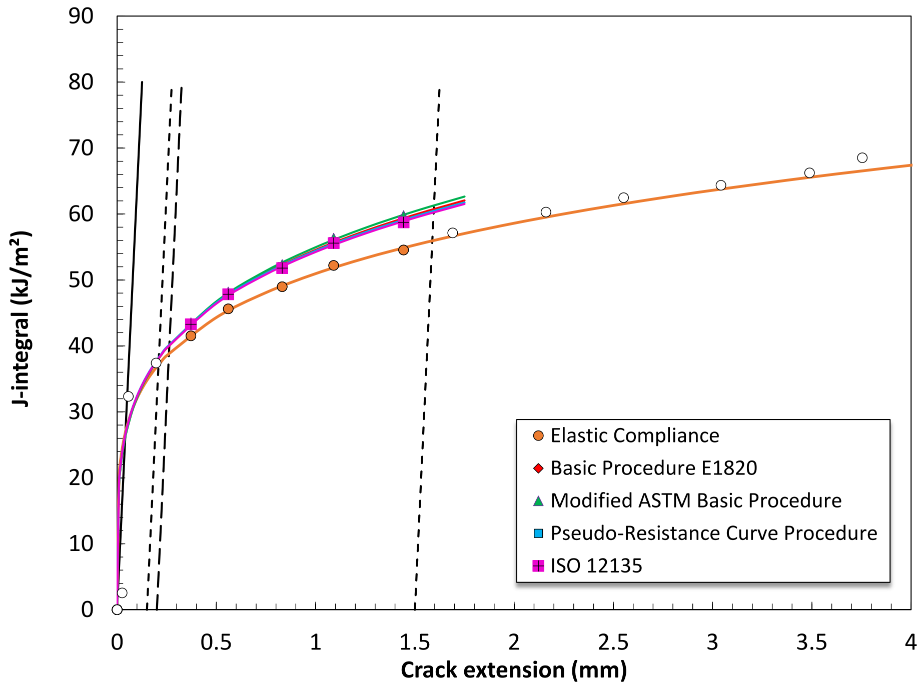

Fig. 5. ASTM-DS4 (1SE(B), low toughness): crack resistance (*J-R*) curves for elastic compliance and DCG-corrected approaches. Empty symbols represent Elastic Compliance data points outside the ASTM E1820 region of qualified data.

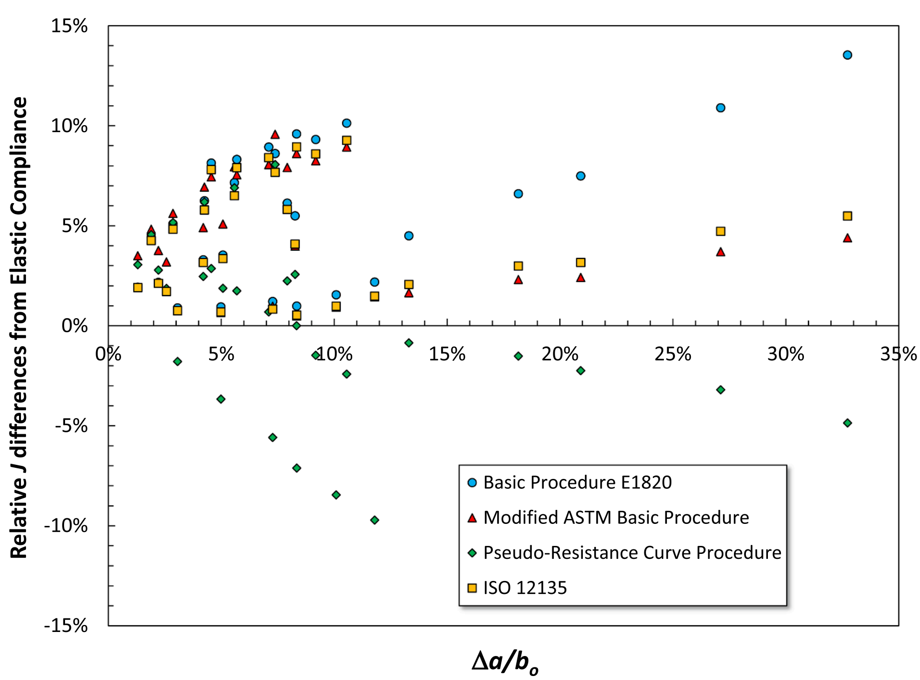

Fig. 6. Relative (percent) differences between Basic Procedure and Elastic Compliance values of *J*-integral vs. cracked portion of initial ligament.

Furthermore, it's interesting to note that, while all 20 multi-specimen critical toughness values could be qualified as size-independent J_{Ic}, this was the case only for 2 or the 5 single-specimen (Elastic Compliance) results – see Table 3. In all cases, the causes of invalidity were related to the determination of the predicted initial crack size, a_{oq}, from data points preceding maximum force, which obviously does not apply to a multi-specimen scenario.

From a Fitness-for-Service (FFS) assessment point of view, it might also be interesting to evaluate the influence of the various DCG corrections on multi-specimen *J*-integral values corresponding to specific amounts of stable tearing, such as 1 mm and 2 mm (Table 5 and Fig. 9).

In all cases, the influence of the different DCG corrections for the multi-specimen values is within a reasonable accuracy interval of ± 10 %, irrespective of DCG approach or toughness level.

5. Conclusions

This investigation examined two current and two novel ductile crack growth (DCG) corrections for multi-specimen *J*-integral values, comparing them to the well-established incremental correction

Table 4
Values of critical toughness and tearing modulus calculated from the multi-specimen analyses. NOTE: critical toughness values qualified as J_{Ic} are indicated in **bold font**; non-qualified values are indicated in *italic font*.

Specimen ID	J_Q or J_{Ic} (kJ/m^2)					T_M (MPa)				
	RPC (EC)	BP	BM_mod	PRCP	ISO	RPC (EC)	BP	BM_mod	PRCP	ISO
W2-F11	**238.23**	**251.18**	**252.01**	**254.27**	**251.80**	49.2	55.8	48.1	39.5	49.2
CGW_25	*824.38*	**903.16**	**896.13**	**852.85**	**899.74**	318.8	351.5	347.1	292.9	347.5
DS3	**427.71**	**434.12**	**434.86**	**413.25**	**433.72**	380.8	384.3	382.0	357.7	382.2
DS4	*38.85*	**40.11**	**40.22**	**40.27**	**40.15**	21.8	25.5	25.9	25.1	24.9
DS8	*104.21*	**105.66**	**107.49**	**107.58**	**105.67**	80.7	85.1	86.3	81.8	84.8

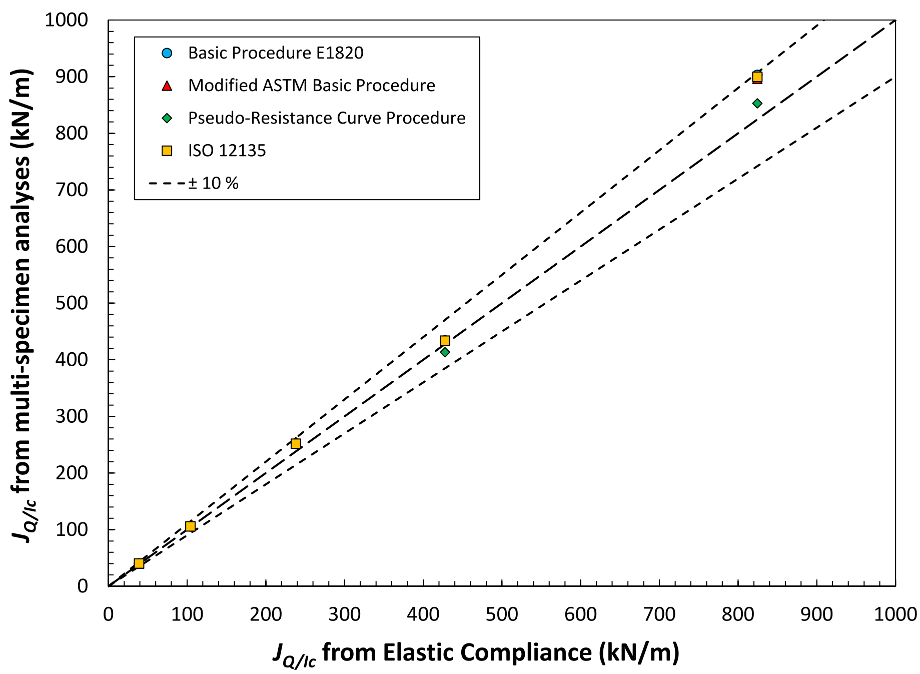

Fig. 7. Comparison between single- and multi-specimen critical toughness values.

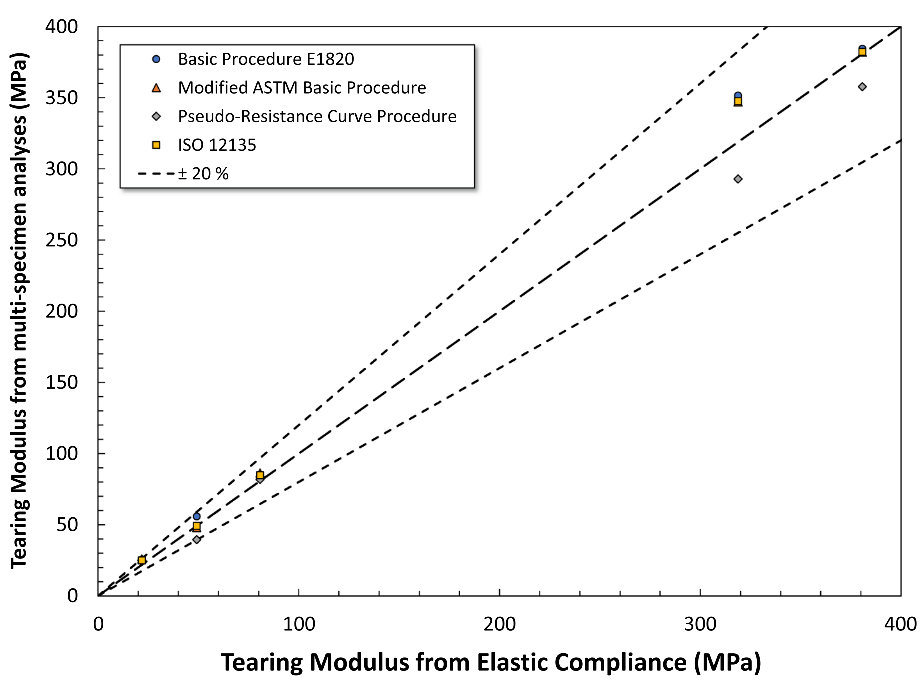

Fig. 8. Comparison between single- and multi-specimen tearing modulus values.

Table 5
Values of *J*-integral corresponding to 1 mm (J_{1mm}) and 2 mm (J_{2mm}) ductile crack growth for single-specimen (EC) and DCG-corrected multi-specimen analyses.

	J_{1mm} (kN/m)					J_{2mm} (kN/m)				
Data set	**EC**	**BP**	**BM_mod**	**PRCP**	**ISO**	**EC**	**BP**	**BM_mod**	**PRCP**	**ISO**
DS3	631.58	637.38	635.84	607.91	635.81	1062.63	1074.83	1069.17	997.43	1069.28
DS4	50.95	54.56	54.95	54.43	54.23	58.59	63.99	64.62	63.65	63.39
DS8	194.24	202.07	205.23	198.50	201.70	272.00	287.21	291.84	277.09	286.40
CGW_25	819.06	862.39	857.94	836.59	860.94	1149.71	1245.35	1234.45	1140.53	1238.41
W2-F11	339.52	368.41	350.16	332.52	352.63	424.29	470.97	432.49	395.25	437.71

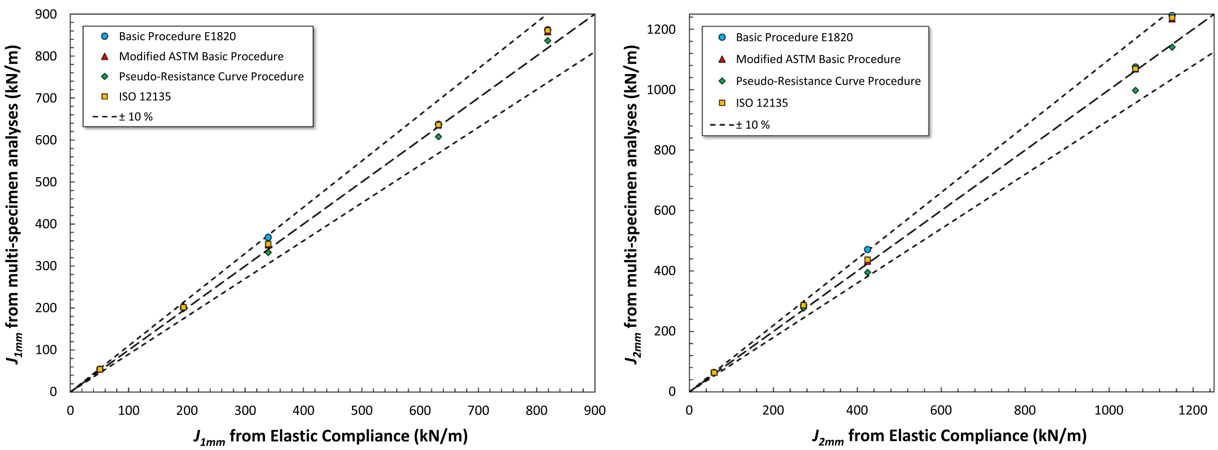

Fig. 9. Values of *J*-integral corresponding to 1 mm (left) and 2 mm (right) of stable crack growth.

prescribed by ASTM E1820 for single-specimen Elastic Compliance tests. Using this latter as benchmark, differences between individual *J* values, J_i, and critical toughness values, J_Q or J_{Ic}, were calculated.

Calculations and comparisons were performed on 17 Elastic Compliance tests from various materials and specimen configurations, in reference to end-of-test conditions (final force, displacement, crack size, and elastic compliance). Three of the four corrections were found to overestimate most J_i values, the exception being the novel Pseudo-Resistance Curve Procedure, which in most cases produces lower J_i values than Elastic Compliance. In all cases, deviations tend to increase with the percentage of cracked ligament, becoming larger than ± 20 % when ductile crack extension is more than 50 % of the initial uncracked ligament.

For all the specimens examined, however, end-of-test values of crack extension largely exceeded the limits set by ASTM or ISO for the determination of critical toughness (Δa_{limit}) or the establishment of the crack resistance curve (Δa_{max}). Hence, we selected 5 of the 17 Elastic Compliance tests, and calculated DCG-corrected J_i values for 5 or 6 elastic unloadings per test, all located inside the ASTM E1820 region of qualified data. From the multi-specimen data sets thus obtained, critical toughness and tearing modulus values were calculated and compared with the corresponding single-specimen (Elastic Compliance) J_Q/J_{Ic} and T_M values. Across the board, differences were found to be small compared to the typical uncertainty of J_{Ic} test results: less than ± 10 % for critical toughness and less than ± 20 % for tearing modulus.

All the comparative analyses presented in this paper clearly demonstrate that ductile crack growth corrections are only impactful when the crack has extended significantly above 2 mm. By and large, whether a more or less accurate DCG correction should be selected, or whether a correction should be used at all, ultimately depends on the expected amount of ductile tearing. Indeed, some fracture assessment codes, such as DNV-ST-F101 [23] and DNV-RP-F108 [24], state that a ductile crack growth correction is not necessary, since the maximum amount of ductile tearing allowed in these codes is in the order of 1 mm – 1.5 mm.

In the specific case of critical toughness values (J_Q or J_{Ic}), where the associated ductile crack growth is typically in the range 0.2 mm – 0.5 mm, the differences observed between the different approaches most likely arise from small discrepancies in the power law regressions, coupled with some statistical variability.

All things considered, urgent changes in the current elastic–plastic fracture mechanics test standards (ASTM E1820 or ISO 12135) appear unwarranted.

CRediT authorship contribution statement

Enrico Lucon: Investigation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request by contacting the author.

References

[1] ASTM E1820-23a, Standard Test Method for Measurement of Fracture Toughness, ASTM International, West Conshohocken, PA.
[2] ISO 12135:2021, Metallic Materials - Unified Method of Test for the Determination of Quasistatic Fracture Toughness, International Standards Organization, Geneva, Switzerland.
[3] J.R. Rice, A path independent integral and the approximate analysis of strain concentration by notches and cracks, J. Appl. Mech. 35 (1968) 379–386.
[4] M.F. Kanninen, C.H. Popelar, Advanced Fracture Mechanics, Oxford University Press, New York, 1985.
[5] T.L. Anderson, Fracture mechanics: fundaments and applications, third Edition, 2005, CRC Press, Boca Raton, FL.
[6] J.A. Begley, J.D. Landes in Fracture Toughness, Proceedings of the 1971 National Symposium on Fracture Mechanics, Part II, ASTM STP 514, H. T. Corten and J. P. Gallagher, eds., ASTM, Philadelphia, PA, 1972, pp. 1-23 and 24-39.
[7] J.R. Rice, P.C. Paris, J.G. Merkle, Some Further Results of J-Integral Analysis and Estimates, in: J. Kaufman, J. Swedlow, H. Corten, J. Srawley, R. Heyer, E. Wessel,

G. Irwin (Eds.), Progress in Flaw Growth and Fracture Toughness Testing, ASTM STP 536, Philadelphia, PA, ASTM, 1973, pp. 231–245.
[8] J.G. Merkle, H.T. Corten, A *J* integral analysis for the compact specimen, considering axial force as well as bending effects, J. Pressure Vessel Technol. 96 (4) (1974) 286–292.
[9] G.A. Clarke, W.A. Andrews, P.C. Paris, D.W. Schmidt, Single Specimen Tests for J_{Ic} Determination, in: J.R. Rice, P.C. Paris (Eds.), Mechanics of Crack Growth, ASTM STP 590, Philadelphia, PA, ASTM, 1976, pp. 27–42.
[10] J.W. Hutchinson, P.C. Paris, Stability Analysis of *J*-Controlled Crack Growth, in: J. D. Landes, J.A. Begley, G.A. Clarke (Eds.), Elastic-Plastic Fracture, ASTM STP 668, Philadelphia, PA, ASTM, 1979, pp. 37–64.
[11] H. Ernst, P. C. Paris, M. Rossow, J. W. Hutchinson, Analysis of Load-Displacement Relationship to Determine J-R Curve and Tearing Instability Material Properties, in: Fracture Mechanics: Proceedings of the Eleventh National Symposium on Fracture Mechanics: Part I, ASTM STP 677, C. W. Smith, ed., ASTM, Philadelphia, PA, 1979, pp. 581-599.
[12] H.A. Ernst, P.C. Paris, J.D. Landes, Estimations on J-Integral and Tearing Modulus *T* from a Single Specimen Test Record, in: R. Roberts (Ed.), Fracture Mechanics: Thirteenth Conference, ASTM STP 743, American Society for Testing and Materials, Philadelphia, PA, 1981, pp. 476–502.
[13] S.J. Garwood, J.N. Robinson, C.E. Turner, The measurement of crack growth resistance curves (R-curves) using the J integral, Int. J. Fract. 11(3), pp. 528-530.
[14] S. Cravero, C. Ruggieri, Further developments in *J* evaluation procedure for growing cracks based on LLD and CMOD data, Int. J. Fract. 148 (2007) 378–400.
[15] X.-K. Zhu, B. N. Leis, J.A. Joyce, Experimental Estimation of J-R Curves from Load-CMOD Record for SE(B) Specimens, J. ASTM Int. 5(5), 2008, Paper ID JAI101532.
[16] ASTM E813-89$^{\varepsilon 1}$, "Standard Test Method for J_{IC}, A Measure of Fracture Toughness," ASTM, Philadelphia, PA (discontinued 1997).
[17] ASTM 1152-95, "Standard Test Method for Determining *J-R* Curves," ASTM, Philadelphia, PA (discontinued 1997).
[18] ASTM E1737-96, "Standard Test Method for J-integral Characterization of Fracture Toughness," ASTM, Philadelphia, PA (discontinued 1997).
[19] K. Wallin, A. Laukkanen, Improved crack growth corrections for *J-R*-curve testing, Eng. Fract. Mech. 71 (2004) 1601–1614.
[20] ESIS P1-92, "ESIS Recommendations for Determining the Fracture Resistance of Ductile Materials," European Structural Integrity Society, TC1 Subcommittee on Fracture Mechanics Testing Standards, ESIS Publishing House, 1992.
[21] ESIS P2-92, "ESIS Procedure for Determining the Fracture Behavior of Materials," European Structural Integrity Society, TC1 Subcommittee on Fracture Mechanics Testing Standards, January 1992.
[22] P.C. Paris, H. Tada, A. Zahoor, H. Ernst, A Treatment of the Subject of Tearing Instability, U.S. Nuclear Regulatory Commission, Washington, DC, Report No. NUREG-0311, 1977.
[23] Det Norske Veritas, Submarine pipeline systems, DNV-ST-F101, Edition 2021-08 – Amended 2021-12.
[24] Det Norske Veritas, Assessment of flaws in pipeline and riser girth welds, DNV-RP-F108, Edition 2019-09 – Amended 2021-09.