Shrinks when it's colder

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Abstract:

Thermal expansion can be a leading cause of uncertainty in length metrology. A cellbased refractometer has been designed at NIST which targets 10^{-6} uncertainty in the measurement of helium refractivity; in terms of refractive index at ambient conditions, the accuracy goal is 3×10^{-11} in refractive index. To achieve this level of accuracy, the length of the 0.5 m gas cell needs to be known within 100 nm. This is achievable when cell length is measured by coordinate-measuring machine at 20 °C. However, the refractometer will operate at the thermodynamically known fixed-points of water and gallium, near 0 °C and 30 °C, respectively. The cell is made from fused quartz glass, which has a nominal thermal expansion coefficient of 0.4 (µm/m)/K. Therefore, to scale the accuracy of the dimensional metrology across 20 °C to the triple-point of water requires that the thermal expansion coefficient of fused quartz glass is known within 10 (nm/m)/K, or 2.5 %.

A method is described to measure the thermal expansion coefficient of fused quartz glass. The measurement principle is to monitor the change in resonance frequency of a Fabry–Perot cavity as its temperature changes; the Fabry–Perot cavity is made from fused quartz glass. The standard uncertainty in the measurement was less than 0.6 (nm/m)/K, or 0.15 %. The limit on performance is arguably uncertainty in the reflection phase-shift temperature dependence, because neither thermooptic nor thermal expansion coefficients of thin-film coatings are reliably known. However, several other uncertainty contributors are at the same level of magnitude, and so any improvement in performance would entail significant effort. Furthermore, measurements of three different samples revealed that material inhomogeneity leads to differences in the effective thermal expansion coefficient of fused quartz; inhomogeneity in thermal expansion among samples is 17 times larger than the measurement uncertainty in a single sample.

1 Introduction and motivation

An experimental effort is underway to measure the refractivity of helium gas at the level of $10^{-6} \cdot (n-1)$. The motivation is that a precision measurement of helium refractivity at known temperature allows a realization of the pascal, in what is sometimes called the optical pressure scale [1]. The underlying principle is the ideal gas law, which defines pressure $p = \rho RT$ in terms of density ρ and temperature T; the gas constant R is a fixed value. The interest in helium (refractivity) is because the Lorentz-Lorenz equation provides a direct link between refractivity and density $\rho = \frac{2}{3A_R}(n-1) + \cdots$ via the polarizability A_R . Polarizability is a fundamental property of a single atom, and for helium, it can be calculated [2] well-within $10^{-6} \cdot A_R$. Consequently, the realization $p = \frac{2}{3A_R}(n-1)RT$ provides a well-understood physical system, in which all input parameters are known without reference to an ancillary measurement of pressure. The ultimate accuracy of this new scale can approach the u(T)-limit; or, how well the thermodynamic temperature of the helium gas can be known.

To measure refractivity, an approach based on gas cells is being pursued. The scheme closely follows the furtive effort of Egan *et al.* [3] to determine the Boltzmann constant, with the working-principle of making window pathlength error common-mode in measurements of refractivity performed in cells of different length. Two design tweaks have improved the concept of Ref. [3]: the cells and windows have been made in fused quartz glass, and the length of the long cell is 0.5 m. These two design tweaks should achieve (uncorrected) window pathlength error at $90 \times 10^{-6} \cdot (n-1)$ for helium, and common-mode cancellation techniques are expected to reduce this by more than an order-of-magnitude. The long, medium, and short cells are photographed in Fig. 1.

At these expected levels of accuracy, uncertainty in gas temperature will become the dominant contribution to uncertainty in $p = \frac{2}{3A_R}(n-1)RT$. Therefore, the helium measurements will be performed with direct thermometry comparisons to the fixed-points of water and gallium, at 0.01 °C and 29.76 °C, respectively. However, cell length must be determined by coordinate measurement, which is performed at the 20 °C reference temperature of dimensional metrology. The objective is to scale the dimensional measurements across 20 °C and hold 100 nm, which corresponds to fractional error of $0.2 \times 10^{-6} \cdot (n-1)$ in the cell-based refractometer. Since the coefficient of thermal expansion (CTE) of fused quartz glass is approximately 0.4 (µm/m)/K, holding a 0.5 m dimension across 20 °C within 100 nm requires knowledge of CTE within 10 (nm/m)/K, or 2.5 %.

Vitreous silica was a standard reference material for thermal expansion for many decades [4]. However, the glass exhibits variations in its CTE, which have been attributed to recipe, process preparation, and thermal history [4–7]. The cell assemblies of Fig. 1



Figure 1: The three gas cells have been manufactured by potting precision bore fused quartz glass tubing into end blocks. Cell lengths are 500 mm, 258 mm, and 15 mm. Cell lengths must be dimensioned before the windows are bonded to the assembly.

are made with tubes of Type-I electric-fusion quartz glass. However, the thick wall tubes were made of a lamination of an inner and outer tube, and one therefore can not be certain of the thermal history of the material. The windows on the cell assemblies are Type-III synthetic fused silica glass (colloquially known as UV-grade fused silica). Wang, Yamada, and Okaji [8] show variations in $\int \alpha(T) dT \equiv \frac{\Delta L}{L}$ less than 1 µm/m between Type-I and Type-III glasses across 0.01 °C to 29.766 °C, but there is the possibility of a small end-effect owing to material mismatch. Another more important potential end-effect is that the tubes are bonded into end-blocks with a potting compound. The potting compound is a paste of silica powder, mixed with monoaluminum phosphate solution; the tubes were potted into the end-blocks with a slip-fit of 50 µm, and fired to 300 °C for 24 h. The potting compound has a generic mean CTE specification of 0.59 × 10⁻⁶ /K. Based on these circumstances and a potential mismatch in expansion, it was considered necessary to measure the CTE of a "mock cell" sample, upon which to base the estimate of the effective CTE of the cell assembly.

2 Measurement approach

The situation is that the lengths of the cells are measured by coordinate-measuring machine at a reference temperature $T_{\rm ref} = 293.15$ K. The actual cell length at the refractometry working temperature $T_{\rm TPW} = 273.16$ K requires that the measured length $L_{\rm ref}$ (i.e., $L_{\rm CMM}$) is corrected

$$L = L_{\rm ref} \left[1 - \int_{T_{\rm TPW}}^{T_{\rm ref}} \alpha_{\rm cell}(T) \,\mathrm{d}T \right] \tag{1}$$

for the thermal expansion coefficient $\alpha_{\text{cell}}(T)$, which is the unknown.

The mock cell samples used to infer $\alpha_{cell}(T)$ are Fabry-Perot (FP) cavities, which have spacers made from tubes of the same material batch as used in the cell assemblies; that is, Type-I fused quartz glass. The FP cavities were formed by polishing the end faces of the tubes parallel, and silicate-bonding mirrors to each end. The measurement principle [9] is to track changes in the resonance frequency of each cavity as a function of temperature, relative to the reference frequency of an iodine-stabilized laser. As such, the metrology scheme closely resembles that of Ref. [10], comprising a pair of tunable HeNe lasers, dither-locked to the resonance peaks of each cavity, and some frequency metrology. The metrology assembly is sketched in Fig. 2, and is entirely high-vacuum compatible. The two FP cavities were placed side-by-side in their own suspension frame/enclosure, and were suspended by 0.3 mm diameter cable at their Airy-points. The temperature of each aluminum suspension enclosure was measured with a thermistor embedded in a thermowell. These two thermistors had been calibrated on the international temperature scale of 1990 (ITS-90) relative to a standard platinum resistance thermometer. The apparatus of Fig. 2 was placed in a vacuum chamber and submerged in a 150 L stirred waterbath; fiber inputs and outputs were fedthrough the water and chamber. This paragraph completes the basic description of the method—the change in resonance frequency of the tube cavities was measured as a function of temperature—further details are left to Ref. [10].

The (approximate) resonance frequency of a FP cavity at vacuum $\nu \approx \frac{mc}{2L}$, depends on cavity length L and the integer mode number m; the speed of light in vacuum c is a constant. Consequently, changes in the length of the cavity can be inferred by measuring the change in resonant frequency via the difference equation $\frac{dL}{L} = -\frac{d\nu}{\nu}$, while updating $d\nu$ for the change in mode number Δm , caused by the changes in cavity temperature. (In these measurements, $\frac{d\nu}{dT} \approx 180$ MHz/K. For the 50 K change in the long cavity temperature, $\Delta m = 30$.) From the inferred specimen (cavity) length as a function of temperature L(T), one can deduce the instantaneous CTE $\alpha(T) = \frac{1}{L_{\rm ref}} \frac{dL}{dT}$. Since this experiment deals with specimens of different lengths, it is expedient to fit fractional length $\frac{L(T)}{L_{\rm ref}} = a_0 + \sum_{i=1}^3 \frac{a_i}{i} (T - T_{\rm ref})^i$ about the reference temperature $T_{\rm ref} = 293.15$ K. The resulting fit coefficients then describe instantaneous CTE as a functional approximation

$$\alpha(T) = \sum_{i=1}^{3} a_i (T - T_{\text{ref}})^{i-1}.$$
(2)

This simplified treatment has ignored the temperature-dependent effects in diffraction and mirror phase-shift on reflection—the latter has a temperature-dependence which is non-negligible.



Figure 2: Setup for the thermal expansion measurement. Two Fabry–Perot cavities were formed out of excess (cell) tubing material, and suspended side-by-side. The sketched assembly was placed inside an inner shell and vacuum chamber, which was submerged in a waterbath. Temperature of the waterbath was varied between -10 °C and 40 °C.

3 Results for two sets of CTE measurement

Two sets of CTE measurements were performed. The first set cycled the cavities $\text{FP}_{152}^{\text{Type-I}}$ and $\text{FP}_{333}^{\text{Type-I}}$ side-by-side, and the second set cycled $\text{FP}_{152}^{\text{potted}}$ and $\text{FP}_{333}^{\text{Type-I}}$ side-by-side. For the second set, the 152 mm cavity had been potted with slip-fit tubes, while the 333 mm cavity remained unchanged between the two sets of measurements. This second set of measurements is the basis for the estimate $\alpha_{\text{cell}}(T)$.

In Fig. 3(a) both sets of measurements are plotted as fractional change in cavity length as a function of temperature. For each cavity, the $\frac{L(T)}{L_{ref}}$ dataset was regressed to a cubic function, and produced coefficients specific to each cavity. The question of how much these sets of coefficients differed from one another is best answered by the diagnostic described in the next paragraph. Fig. 3(b) shows residuals from the fits to fractional length, where the ordinate corresponds to $\frac{L(T)}{L_{ref}} - [a_0 + \sum_{i=1}^3 \frac{a_i}{i}(T - T_{ref})^i]$. The plot shows residuals on the specimens $\mathrm{FP}_{152}^{\mathrm{Type-I}}$ and $\mathrm{FP}_{333}^{\mathrm{Type-I}}$ from the first measurement set, and specimen $\mathrm{FP}_{152}^{\mathrm{potted}}$ from the second measurement set. (For clarity in Fig. 3, the second measurement set for $\mathrm{FP}_{333}^{\mathrm{Type-I}}$ is not shown. Using the metric $\int \alpha(T) \, \mathrm{d}T$, the two measurements of $\mathrm{FP}_{333}^{\mathrm{Type-I}}$ reproduce within 0.7 nm/m across the 50 K range.) The root-mean-square error from the fits of the first measurement set are 82 pm/m for $\mathrm{FP}_{333}^{\mathrm{Type-I}}$, which is most likely an increase in cavity instability caused by the potting compound, because the $\mathrm{FP}_{333}^{\mathrm{Type-I}}$ residuals for the second set of measurements are within 15 % of the first set—it is no surprise that a potting compound increases instability compared to bulk material. The fit residuals for long and short cavities show obvious correlation, and the shape of the residuals persists in both sets of measurements, which is suggestive of a systematic error affecting the length of both cavities during the measurement cycle. The most likely cause for the correlated residuals is error in thermometry, on the order of $\delta T = \frac{1}{\alpha} \frac{\delta L}{L} \approx 0.3$ mK.

A key experimental diagnostic is the intercavity beat: that is, the < 1 GHz difference in frequency between the two lasers locked to a resonance frequency in each cavity. For cavities of identical material properties experiencing identical temperature changes, the intercavity beat would remain constant as a function of temperature. Stated more precisely,

$$f_{\rm f} - \frac{f_{\rm ref} + \left(\frac{\Delta m_{33} \cdot c}{2L_{33}} - \frac{\Delta m_{15} \cdot c}{2L_{15}}\right)}{1 + \int_{T_{\rm ref}}^{T_{\rm f}} \alpha(T) \,\mathrm{d}T} \stackrel{?}{=} 0.$$
(3)

Or, the change in intercavity beat measured at two different temperatures $T_{\rm ref} < T_{\rm f}$ when adjusted for changes in mode order of the respective cavities and scaled for their increasing length—should be zero, if the cavities have the same coefficient of thermal expansion, experience the same temperature change, and have no end-effect. Fig. 3(c) belies these assumptions and the potential equality of (3), demonstrating a linear trend of about 4.9 MHz/K in the case of $\rm FP_{152}^{Type-I}$ and $\rm FP_{333}^{Type-I}$ dataset. A temperature-related explanation for the trend is implausible—it would require an undetected change in gradient between the cavities of 1.25 K, which is more than three orders of magnitude larger than what was measured. Two possible end-effects are a mismatch in CTE between the tube and mirror substrates, and a temperature dependence on cavity roundtrip phase. Combined, these two end-effects are about a factor 30 too small to explain the changing intercavity beat. Therefore, the most likely explanation behind the trend of Fig. 3(c) is inhomogeneity in the CTE of fused quartz glass among different tubes, corresponding to $\Delta \alpha \approx \frac{1}{\nu} \frac{\Delta f_{\text{beat}}}{\Delta T}$, or 1.1×10^{-8} /K. The magnitude of this inhomogeneity—2.5 %—is somewhat surprising, but should be placed in context of a thorough work by Jacobs, Shough, and Connors [11], which compared 13 different samples of fused quartz glass, core-drilled from distant locations in 4 separate ingots (glass melts). These authors showed a locationdependent gradient in $\alpha(T)$ across each ingot, and they stated that the maximum variation in $\alpha(T)$ among three of the four melts was 5×10^{-9} /K. They chose to "remove" data from the fourth melt because "a different grade of crystalline quartz [was] used to yield a reduced bubble content." Nevertheless, this fourth melt would still be classified as a Type-I fused quartz glass, and if it is included in the data analysis, Jacobs, Shough, and Connors [11] showed variations in $\alpha(T)$ of up to 1.3×10^{-8} /K among the 13 samples from 4 separate melts. (See, in particular, their Fig. 13.) The CTE inhomogeneity of the present result in Fig. 3(c) is therefore not too surprising. However, the present result for absolute CTE in both specimens, discussed next, also requires mediation. [Fig. 3(c)



Figure 3: (a) Change in fractional cavity length as a function of temperature for Type-I fused quartz glass $\text{FP}_{333}^{\text{Type-I}}$ and $\text{FP}_{152}^{\text{Type-I}}$, and the potting-compound-modified $\text{FP}_{152}^{\text{potted}}$. The plots are offset by 3×10^{-6} in the ordinate for clarity. (b) Residuals from the fit: the $\frac{L(T)}{L_{\text{ref}}}$ dataset for each FP cavity has its own fit coefficients. (c) Anomalous change in the intercavity beat frequency, defined as (3). The slope of the trends is proportional to inhomogeneity $\Delta \alpha \approx \frac{1}{\nu} \frac{\Delta f_{\text{beat}}}{\Delta T}$ between the specimens being compared. (d) The deduced thermal expansion for Type-I fused quartz glass, as (2) in this work. Literature measurements also shown.

also reveals that a third FP cavity specimen $\text{FP}_{154}^{\text{Type-I}}$ of the same material demonstrated close agreement in $\alpha(T)$ to the specimen $\text{FP}_{333}^{\text{Type-I}}$.]

In Fig. 3(d) is plotted $\alpha(T)$ deduced from these measurements, expressed by (2). The present measurements are compared with the existing literature [4, 8, 11–14] in this temperature range. There are at least four notable aspects to the literature, worthy of anecdotal consideration, as follows:

- The first is evident in Fig. 3(d) as the difference between the trends "Ref. [4], SRM 739," "Ref. [13], SRM 739," and "Ref. [14], SRM 739." Okaji and coworkers have consistently reported [8, 14] a bias of 2 × 10⁻⁸ /K to 3 × 10⁻⁸ /K between their measurements of SRM 739 versus what was originally reported by Hahn and Kirby [4]. This bias is within the mutual standard uncertainties for α(T), which was 3 × 10⁻⁸ /K for Hahn and Kirby and 2 × 10⁻⁸ /K for Okaji and coworkers. Drotning [13], whose measurements of α(T) in SRM 739 are also lower than Hahn and Kirby, does not make a clear uncertainty statement, but says that the 2.7 × 10⁻⁸ /K standard deviation on their measurements was "near the estimated device uncertainty." The anecdotal conclusion of this first point is that the measurement of Hahn and Kirby [4] is an overestimate of α(T) for SRM 739.
- The second notable feature of Fig. 3(d) is the offset between data for SRM 739 compared to the shaded area plot of "Ref. [8], Type-I, II, III" which covers the range of fit data reported for all types of vitreous silica glass measured by Wang, Yamada, and Okaji [8]. This offset is notable because SRM 739 is nominally a Type-I fused quartz glass, and yet its thermal expansion coefficient (i.e., Ref. [14]) is outside the range of silica glasses investigated by Wang, Yamada, and Okaji, and measured with the same apparatus. The origin of this offset is not clear, but Wang, Yamada, and Okaji suggest that it may be related to the manufacturing process. (A subsequent article by the same Okaji group [15] extends the valid temperature range, and its findings are consistent with Refs. [8, 14].) The anecdotal conclusion of this second point is that the recipe/process/preparation of SRM 739 produces a Type-I fused quartz glass with α(T) relatively higher than other Type-I glasses.
- The third notable aspect of Fig. 3(d) is the 9 × 10⁻⁸ /K offset between Ref. [12] and Ref. [8] for Type-III fused silica glass; when these two Groups performed a bilateral comparison [16] with the same SRM 739 specimen, they had agreement within 2 × 10⁻⁸ /K, and claimed standard uncertainties on the order of 0.9 × 10⁻⁸ /K. The anecdotal conclusion of this third point is that either (i) the same process/preparation (of Type-III fused silica glass) can produce variability as large as the range of all data in Fig. 3(d), or (ii) key comparisons are most satisfying when the participants know the answer in advance.
- Finally, the work of Jacobs, Shough, and Connors [11] must be mentioned. (Ref. [11] does not list fit coefficients, but a plot digitizer [17] was used to interpolate their Fig. 1.) Jacobs and coworkers pioneered the FP cavity based approach to CTE measurement for at least three decades [9, 11, 18–20]. Arguably, Ref. [11] was the acme in their body of work; in Ref. [11] they demonstrate reproducibility at the level of 1×10⁻⁹ /K, and make a thermometry-limited uncertainty claim of 4.5×10⁻⁹ /K in the measurement of a Type-I fused quartz glass. The anecdotal conclusion of this fourth point is that the hitherto most accurate claim on α(T) for a Type-I fused quartz glass is notably lower than older data in Fig. 3(d).

• Parenthetically, also mentioned are two older sets of measurements which are not plotted in Fig. 3(d). Berthold and Jacobs [18] do not give fit coefficients, but their $\alpha(T)$ plot for a Type-III specimen appears in good agreement with the Type-III measurements of Birch [12]; however, despite claimed precision of 1×10^{-9} /K, one of Berthold and Jacobs' three specimens differed by 4×10^{-8} /K from the others. The second older measurement for a Type-III specimen was by Bennett [21], and is also in good agreement with Birch [12]; however, Bennett's result for a Type-I specimen is 4×10^{-8} /K below the shaded area Wang, Yamada, and Okaji [8]; at 293.15 K, Bennett reports a difference in $\alpha(T)$ of 15×10^{-8} /K between Type-I and Type-III silica glasses.

Taken together, the preceding comments about the literature and Fig. 3(d) convey what motivated the present CTE measurements of fused quartz glass: neither measurements on the same glass recipe, nor estimates of variability among recipes, appear reliable at the ± 10 % level in the temperature range of interest (273 < T < 303) K. Despite these staging remarks, the present measurement result for $\alpha(T)$ is no less surprising: the present work is clearly "on the low side," and furthermore, the claimed uncertainty is 5.4×10^{-10} /K, or 0.15 %. It is a struggle to coherently place the present "low" measurement in the context of historical reports. Regarding the Type-I fused quartz glass SRM 739, mutual consistency can only be claimed with the work of Drotning [13]; the present result is approximately 2.9σ lower than Okaji and coworkers [14], and 2.7σ lower than the foundational work of Hahn and Kirby [4]. [Here, σ refers to the combined standard uncertainty of two measurements, and the quantity being compared is $\int_{263}^{313} \alpha(T) \, dT$.] It is emphasized that the Type-I definition into which the tube material and SRM 739 are categorized is nominal, and it therefore may be misleading to read too much into discrepant results. Moreover, Okaji and coworkers' most recent measurements [8] cover five different recipes of fused quartz glass including Type-I, compared to which they find $\alpha(T)$ of SRM 739 to be anomalously high. Notably, the present result is mutually consistent with all five recipes reported by Okaji and coworkers [8, 15], encompassing Type-I, II, III silica glasses. Finally, it appears that the hitherto most accurate claim on a CTE measurement for a Type-I fused quartz glass was by Jacobs, Shough, and Connors [11]: the present measurements are only consistent with Ref. [11] above 295 K, if allowance is made for the 2% to 3% inhomogeneity observed in both cases.

The present CTE measurements are among the most accurate to date. For this reason, and with the stipulation that the tubes have been laminated, the glass recipe and supplier¹ are specified [22]. The present measurements may serve as reference data for a specific blend/process of Type-I electric-fusion quartz glass. Taking the average of the FP_{333}^{Type-I} and FP_{152}^{Type-I} measurements, reference data coefficients for (2)

$$a_1 = 4.01076(4) \times 10^{-7} \text{ /K} | a_2 = 3.0346(4) \times 10^{-9} \text{ /K}^2 | a_3 = -1.302(3) \times 10^{-11} \text{ /K}^3,$$

are valid in the range (263.15 < T_{90} < 313.15) K for p < 40 mPa. The stated uncertainties on the fit parameters are statistical only, and refer to the square-root of the diagonal elements in the covariance matrix. The combined standard uncertainty for the measurement of $\alpha(T)$ is 5.4 × 10⁻¹⁰ /K, and is listed in Table 1 (details elsewhere). It must be

¹Heraeus Quarzglas, Lamp Quality HLQ270, electric fusion process. Typical impurities: Li = $0.05 \ \mu\text{g/g}$, Na = $0.05 \ \mu\text{g/g}$, K = $0.1 \ \mu\text{g/g}$, Mg = $0.05 \ \mu\text{g/g}$, Ca = $0.5 \ \mu\text{g/g}$, Fe = $0.1 \ \mu\text{g/g}$, Cu < $0.05 \ \mu\text{g/g}$, Cr < $0.05 \ \mu\text{g/g}$, Mn < $0.05 \ \mu\text{g/g}$, Al = $15 \ \mu\text{g/g}$, Ti = $1.1 \ \mu\text{g/g}$.

component	$u[\alpha(T)] / (10^{-10} \text{ K}^{-1})$
$L_{\rm ref}, 12 \ \mu {\rm m}$	0.2
frequency 4 kHz	0.1
instability 15 pm/d	0.1
mirror mismatch 80 pm/K	0.8
roundtrip phase 76 kHz/K	2.4
free-spectral range 17 kHz/K	0.4
dT_{00}	0.4
calibration, 0.1 mK	0.5
stability, 0.5 mK	2.5
selfheat, 0.3 mK	1.5
nonlinearity, 0.3 mK	1.5
gradients, 0.5 mK	2.5
residual gas, 10 mPa	0.2
regression, 149 pm/m	1.5
combined $(k=1)$	5.4

Table 1: Standard uncertainty in measurement of the coefficient of thermal expansion for a nominal fused quartz glass specimen. No coverage for inhomogeneity [11] is included.

noted that inhomogeneity among specimens [Fig. 3(c)] is more than 17 times larger than the claimed measurement uncertainty.

4 Conclusion

To conclude by clarification: this work does not advocate for fused quartz glass as a standard of thermal expansion—Fig. 3(d) is a strong case against such an endeavor. The thermal expansion measurements reported here are merely an attendant outcome of the quest to know L_{cell} at 273.16 K, with the goal of establishing an optical pressure scale at the accuracy level of 1 µPa/Pa. The quest has been thoroughly justified: if cell length had been scaled for thermal expansion using the mean of Refs. [4, 13, 14], the consequence for the optical pressure scale would have been a bias error of at least 1.5 µPa/Pa, with an underestimated uncertainty for $u(L_{cell})$.

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- [22] Any mention of commercial products is for information only; it does not imply recommendation or endorsement by NIST nor does it imply that the products mentioned are necessarily the best available for the purpose.