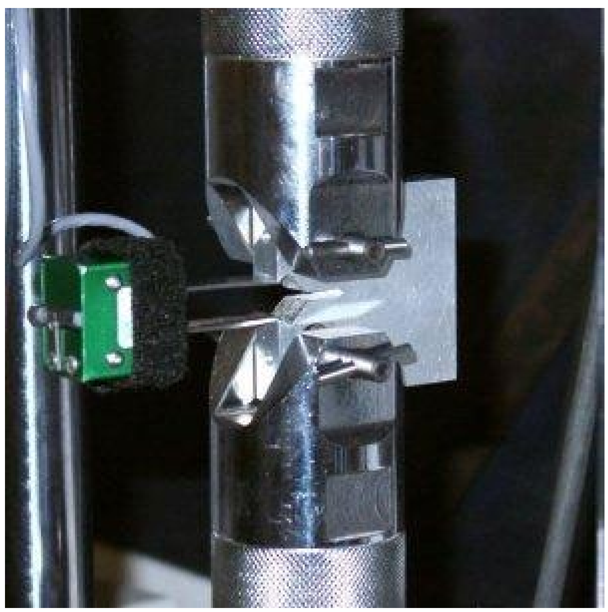
NISTIR 8421

Spreadsheet-Based Software for the Analysis of Unloading Compliance Fracture Toughness Tests in Accordance with ASTM E1820



Enrico Lucon

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Material Measurement Laboratory

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Abstract

NIST has developed a user-friendly spreadsheet-based software for the analysis of elastic-plastic fracture toughness tests conducted according to ASTM E1820-20b on either Compact Tension or Single-Edge Bend specimens using the Unloading/Elastic Compliance (UC/EC) single-specimen technique. The software consists of multiple spreadsheets, which feature several macros that automate most calculations. Complete user's instructions are provided in this report.

The software has been successfully validated using nine sample data sets that are available through ASTM, covering both specimen configurations and various fracture toughness levels.

As in the case of previous software packages developed by the Fatigue and Fracture Group of NIST Boulder, the spreadsheet will be made freely available to the public by contacting the author of this report.

Key words

ASTM E1820-20b; elastic compliance technique; elastic-plastic fracture toughness test; spreadsheet-based software; unloading compliance technique.

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1. Introduction

ASTM E1820, "Standard Test Method for Measurement of Fracture Toughness" (current version: 2020b) [1] covers procedures and guidelines for the determination of fracture toughness of metallic materials, using different fracture parameters (stress intensity factor, K, J-integral, and crack-tip opening displacement, δ). Although the standard addresses material behavior in different fracture regimes (brittle, ductile-to-brittle transitional, ductile), this Test Method is primarily used for conducting and analyzing elastic-plastic fracture toughness tests for characterizing a material's resistance to ductile crack propagation, where the critical parameter used is J-integral at (or close to) the onset of stable crack extension.

There are two approaches that can be followed to characterize the elastic-plastic (ductile) fracture toughness of metallic materials in E1820-20b:

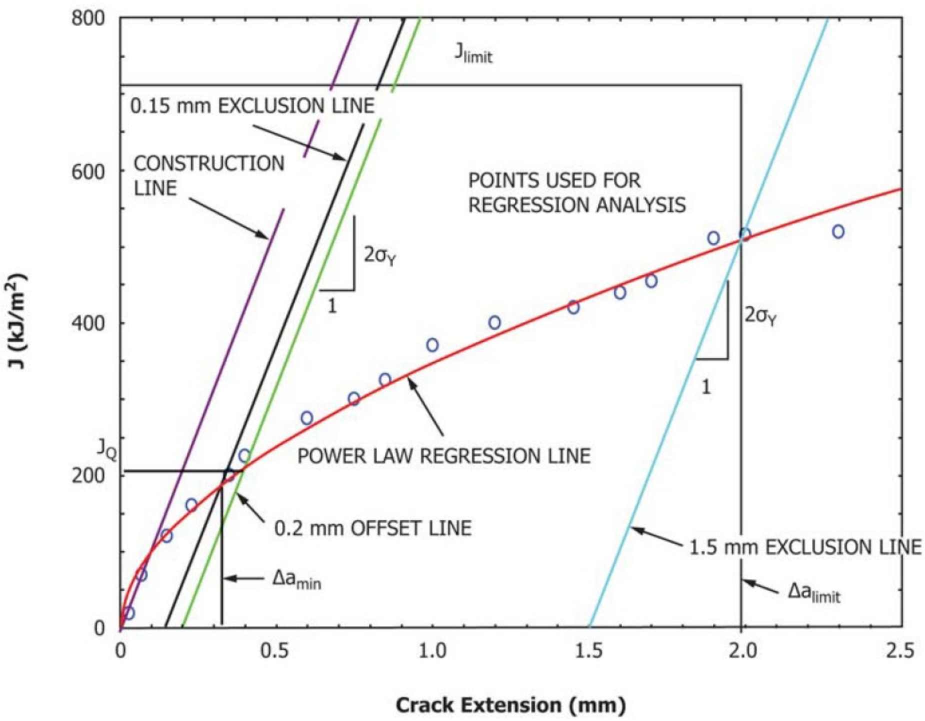
- Multiple-specimen technique (referred to as *basic procedure* in E1820): several, nominally identical, specimens are tested to different amounts of ductile crack extension, in order to obtain a crack resistance curve (J-integral as a function of crack extension – J-R curve) where each data point corresponds to an individual test specimen.
- Single-specimen technique (denominated *resistance curve procedure* in E1820): a complete crack resistance (J-R) curve is obtained from one individual tested specimen, where the amount of crack extension is inferred by monitoring a specific test parameter.

By far, the most commonly used single-specimen technique is the **Unloading** or **Elastic Compliance** (UC or EC) technique [2], whereby crack propagation is inferred by measuring the specimen compliance during small unloadings (less than 15 % of the maximum force) conducted at regular intervals throughout the test. The slope of these unloadings (expressed as displacement/force ratio) can be analytically correlated to the crack size for standard specimen geometries.

Other single-specimen techniques used are the Electric Potential Difference Method [3-5] (crack size is inferred from the voltage difference measured across the crack plane while electric current flows through the specimen) and the Normalization Data Reduction Technique [6,7], whereby the J-R curve is obtained analytically from the force/displacement record and the initial and final crack size measurements taken from the fracture surface.

Once the crack resistance curve has been established via a multiple- or single-specimen technique, the critical toughness (J_Q or J_{Ic})[1] is obtained from the intersection between a power law curve that fits qualified $J/\Delta a$ data points and a construction line with an offset of 0.2 mm (Figure 1).

[1] The plane-strain size-independent critical fracture toughness is denominated J_{Ic}, while J_Q is a size-dependent value that cannot be validated as J_{Ic} according to the E1820 requirements.



This report covers the use of spreadsheet-based software, developed at NIST, for the analysis of elastic-plastic fracture toughness tests conducted with the UC/EC technique in accordance with ASTM E1820-20b.

It consists of four separate macro-enabled MS Excel[2] spreadsheets (two for C(T) specimens and two for SE(B) specimens), which can be used to accomplish the following tasks:

(a) Spreadsheet *Elastic unloading analysis – XXX[3] specimen*: determines the slope of an elastic unloading, and calculates the corresponding crack size, as well as ancillary parameters (plastic load-line displacement, standard error of the compliance) that are used in subsequent analyses.

(b) Spreadsheet *Unloading compliance E1820 analyses – XXX[3] specimen*: performs a complete test analysis in accordance with E1820-20b, including the establishment of the *J-R* curve and the determination of the critical fracture toughness (J_Q or J_{Ic}).

This software is the latest in a series of programs for the obtainment of various mechanical test results that has been developed at NIST [8-10] and can be requested free of charge by contacting the author of this report (enrico.lucon@nist.gov).

[2] Trade names and manufacturers are mentioned in this report only to accurately describe NIST activities. Such inclusion neither constitutes not implies endorsement by NIST or by the U.S. government.

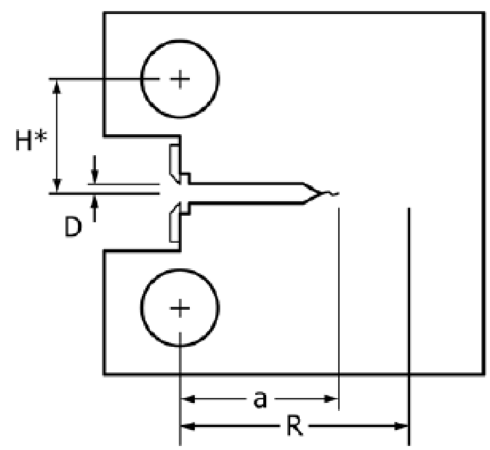
[3] XXX represents the test specimen geometry. Individual spreadsheets are available for the two most commonly used fracture toughness specimen configuration: Compact Tension, C(T), and Single-Edge Bend, SE(B).

2. Calculation of Elastic Compliances and Crack Sizes: Spreadsheet *Elastic unloading analysis – XXX specimen.xlsm*

2.1. Sheet *Unloading analysis*

The first step is for the user to input basic information about specimen dimensions and material's properties in the block of cells {O1:T5} of sheet *Unloading analysis*. **NOTE:** here and in the remaining spreadsheets, cells that require direct input from user (rather than display calculated results) are highlighted in yellow. The information required here is the following:

- Specimen thickness (B), width (W), and net thickness (B_N)[4], in mm.
- Original/initial (fatigue) crack size (a_0) measured on the fracture surface, in mm.
- Young's elastic modulus at test temperature (E), in GPa.
- Only for C(T) specimens: half-distance between the displacement measurement points (D) and half-distance between the centers of the pin holes (H^*), in mm (Figure 2).
- Only for SE(B) specimens: support span, or distance between the loading points on the lower side of the specimen (S), in mm.



Next, the user must input raw displacement[5] (v) and force (F) data from the elastic unloading that is being evaluated in the columns starting at {A9,B9}. The spreadsheet allows entering a maximum of 1000 raw data points.

In accordance with Appendix X3, X3.2, of ASTM E1820-20b: "*The start of an unload is defined as the point at which the crack opening displacement first decreases. (…)*

[4] For side-grooved specimens. For plane-sided specimens, $B_N = B$.

[5] In the case of a C(T) specimen, displacement v corresponds to load-line displacement (in mm). For a SE(B) specimen, displacement v corresponds to crack mouth opening displacement, CMOD (in mm).

The end of an unload is defined as the point at which crack opening displacement first increases again to start reloading."

The selection of the *v-F* data points that are used for calculating elastic compliance (in mm/kN) occurs in columns {C,D}. Section X3.2 recommends removing the first and last 5 % of the unloading, to avoid nonlinearities in the test record, caused, for example, by stress relaxation and ductile tearing. Data selection is based on the force at the start of the unloading (P_1) and the force at the end of the unloading (P_2). Should the user wish to change the percentage of discarded data points, for example to 10 %, "0.05" must be changed in the formula of cell {N12}, for example to "0.1".

Calculation results are displayed starting with cell {F9}, namely:

- C(T) specimen: according to E1820, compliances measured need to be corrected for rotation, to account for crack opening displacement. The parameters D and H^*, along with the uncorrected crack size a_i, are needed for this geometrical correction.
  - Uncorrected compliance, C_i, the corresponding intermediate function u_i, and the uncorrected crack size, $a_{i,uncorr}$, are shown in cells {F9-H9}.
  - The radius of rotation of the crack centerline, R_i (Figure 2), and the angle of rotation about the broken midsection line, ϑ_i, are presented in cells {I9,J9}.
  - Rotation corrected compliance, $C_{corr,i}$, u function, $u_{i,corr}$, and crack size, $a_{i,corr}$, are displayed in cells {K9-M9}.
  - Finally, cells {N9-Q9} display additional parameters related to the estimation of the uncertainty in J_Q/J_{Ic}: intercept of the linear fit (β_0), number of data points used in the regression (n), average of selected force values ($\bar{x}$), and standard error of the compliance ($S_{\beta 1}$).
- SE(B) specimen: for this specimen configuration, compliance does not need to be corrected. Therefore, the results displayed in cells {F9-L9} are: C_i, u_i, a_i, β_0, n, $\bar{x}$, and $S_{\beta 1}$.

Other parameters associated to the current unloading and needed for subsequent test analyses[6] are: force F_i (cell {G12}), displacement v_i (cell {G13}), and plastic displacement $v_{i,pl}$ (cell {G14}).

Once calculations for the current unloading are completed, the main results (unloading number, displacement, force, plastic displacement, compliance, crack size, number of data points in the unloading, and standard error of the compliance) are written in the sheet *Output data* by clicking the button **WRITE**.

The number of the unloading associated with the results is in cell {I3}, and coincides with the contents of cell {J3} (automatic option: the unloading number of the last row in *Output data* + 1) or cell {J4} (manual option: number input by the user). If cell {J4} is left empty, then the unloading number is that provided by the automatic option in cell {J3}.

Clicking the button **CLEAR** erases all force/displacement data points in the *Unloading analysis* sheet.

Figure 3 shows a screenshot of the *Unloading analysis* sheet for a C(T) specimen.

[6] The force/displacement data point associated to the unloading, according to ASTM E1820-20b, is the one immediately preceding the start of the unloading. **Therefore, the user should paste in the *Unloading analysis* sheet force/displacement data points starting with the data point immediately before the force P_1 corresponding to the start of the unloading, up to the data point corresponding to P_2.**

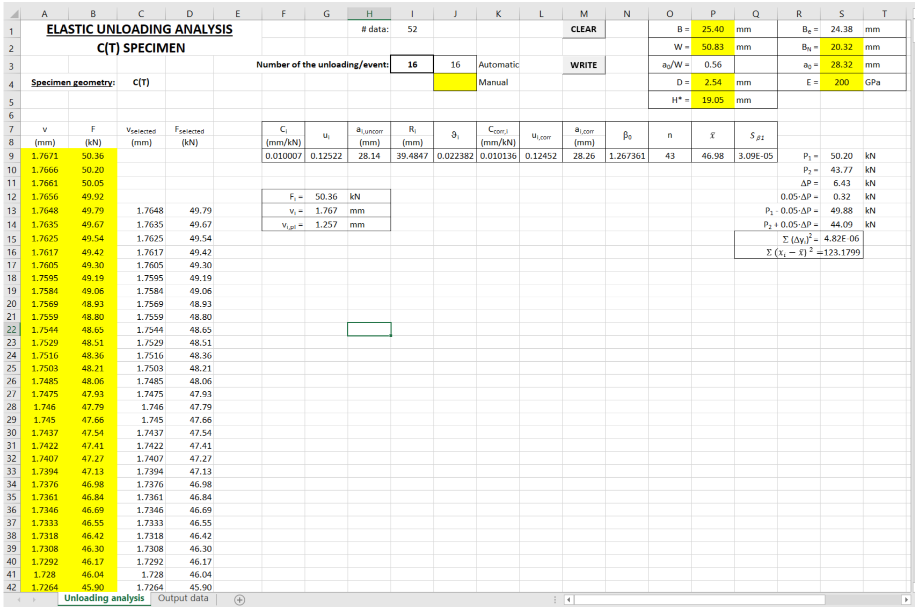


Figure 3 - Sheet *Unloading analysis* for a C(T) specimen (unloading #16).

2.2. Sheet *Output data*

Besides calculation results in columns {A-H}, the *Output data* sheet also displays:

- The average number of data points used in compliance calculations, cell {M3}.
- A plot of force and crack size values as a function of displacement.

By clicking the button **CLEAR**, the user can erase all calculation results from the sheet.

A screenshot of the *Output data* sheet is shown in Figure 4.

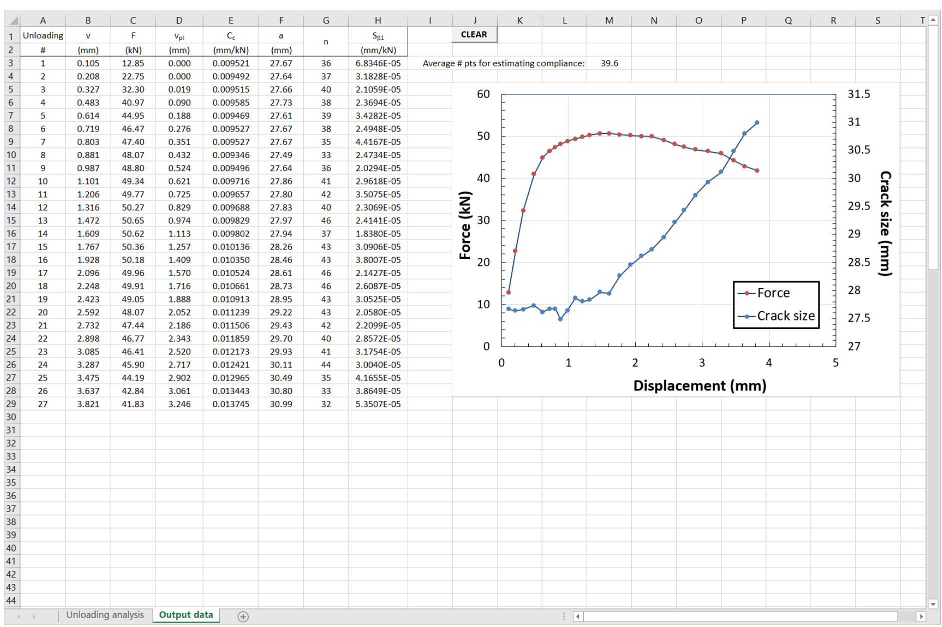


Figure 4 - Sheet *Output data*.

3. Analysis of Elastic-Plastic Fracture Toughness Test: Spreadsheet *Unloading compliance E1820 analyses – XXX specimen.xlsm*

3.1. Sheet *Input data*

Specimen dimensions, measured crack sizes, and tensile properties are entered in this sheet (Figure 5). For a C(T) specimen, these include H^* and D; for a SE(B) specimen, S.

The following parameters are automatically calculated: effective thickness, B_e, and measured crack extension, $\Delta a_{p,meas}$.

Clicking on the button **CLEAR** erases all input data.

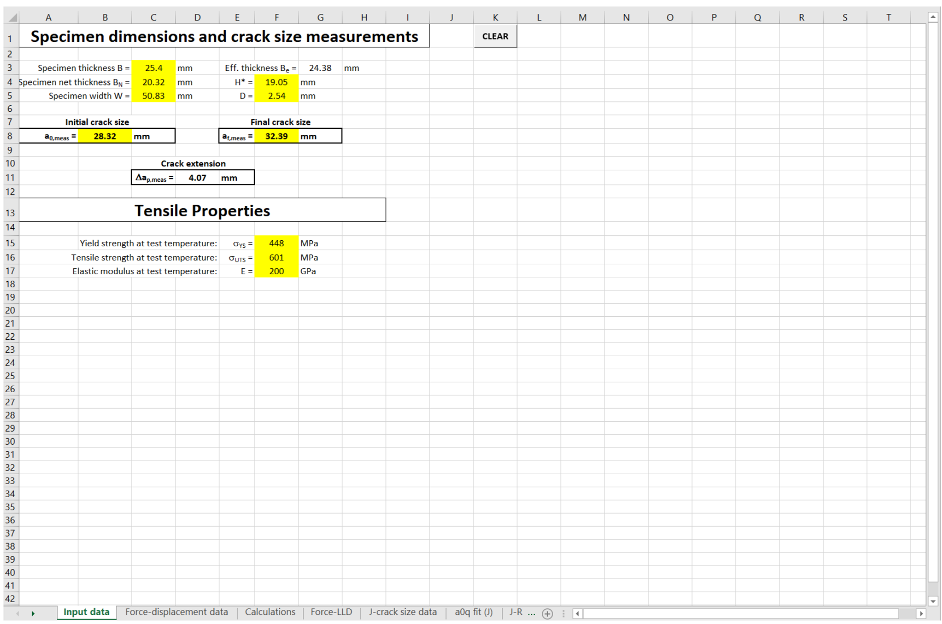


Figure 5 - Sheet *Input data* for a C(T) specimen.

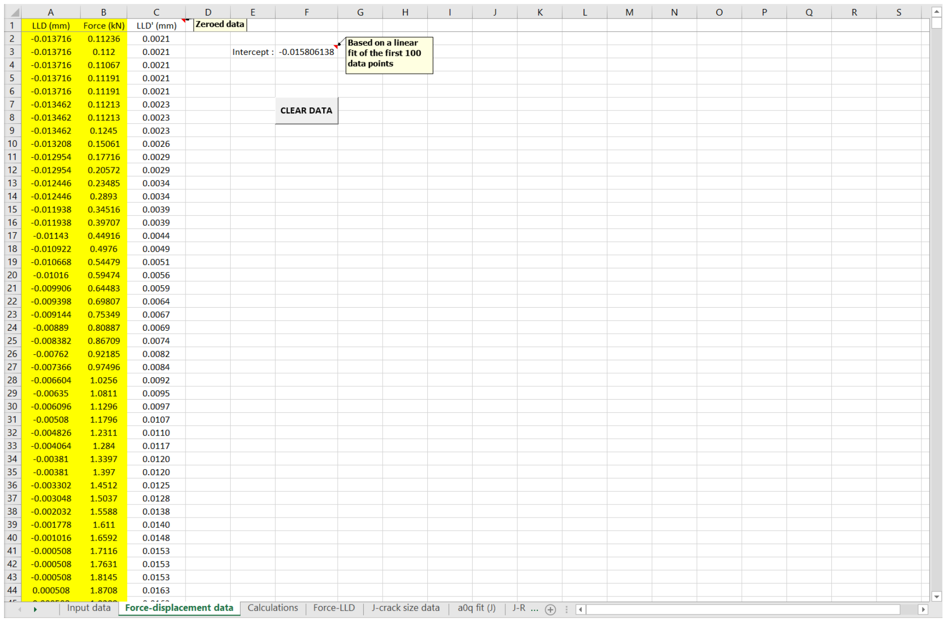
3.2. Sheet *Force-displacement data*

Here, the user inputs raw force/displacement[7] values in columns {A,B}. By default, the first 100 data points are linearly fitted in order to set to zero the plotted test record. The calculated intercept (data shift) is shown in cell {F3}, while column {C} displays zeroed displacement data (LLD' or CMOD').

The current spreadsheet accommodates a maximum of 50000 data points, but could be easily modified to allow larger data sets. A screenshot is provided in Figure 6.

Clicking on the button **CLEAR DATA** erases all force/displacement values.

[7] Load-line displacement, LLD, for a C(T) specimen and crack mouth opening displacement, CMOD, for a SE(B) specimen.



	LLD (mm)	Force (kN)	LLD' (mm)
2	-0.013716	0.11236	0.0021
3	-0.013716	0.112	0.0021
4	-0.013716	0.11067	0.0021
5	-0.013716	0.11191	0.0021
6	-0.013716	0.11191	0.0021
7	-0.013462	0.11213	0.0023
8	-0.013462	0.11213	0.0023
9	-0.013462	0.1245	0.0023
10	-0.013208	0.15061	0.0026
11	-0.012954	0.17716	0.0029
12	-0.012954	0.20572	0.0029
13	-0.012446	0.23485	0.0034
14	-0.012446	0.2893	0.0034
15	-0.011938	0.34516	0.0039
16	-0.011938	0.39707	0.0039
17	-0.01143	0.44916	0.0044
18	-0.010922	0.4976	0.0049
19	-0.010668	0.54479	0.0051
20	-0.01016	0.59474	0.0056
21	-0.009906	0.64483	0.0059
22	-0.009398	0.69807	0.0064
23	-0.009144	0.75349	0.0067
24	-0.00889	0.80887	0.0069
25	-0.008382	0.86709	0.0074
26	-0.00762	0.92185	0.0082
27	-0.007366	0.97496	0.0084
28	-0.006604	1.0256	0.0092
29	-0.00635	1.0811	0.0095
30	-0.006096	1.1296	0.0097
31	-0.00508	1.1796	0.0107
32	-0.004826	1.2311	0.0110
33	-0.004064	1.284	0.0117
34	-0.00381	1.3397	0.0120
35	-0.00381	1.397	0.0120
36	-0.003302	1.4512	0.0125
37	-0.003048	1.5037	0.0128
38	-0.002032	1.5588	0.0138
39	-0.001778	1.611	0.0140
40	-0.001016	1.6592	0.0148
41	-0.000508	1.7116	0.0153
42	-0.000508	1.7631	0.0153
43	-0.000508	1.8145	0.0153
44	0.000508	1.8708	0.0163

Figure 6 - Sheet *Force-displacement data* for a C(T) specimen.

3.3. Sheet *Calculations*

Data calculated for each unloading by means of the *Elastic unloading analysis – XXX specimen* spreadsheet must be input here, using the columns highlighted in yellow. Namely:

- Column A: displacement values (the corresponding zeroed values are automatically displayed in column B).
- Column C: force values.
- Column D: elastic compliance values (rotation corrected for a C(T) specimen).
- Column F: calculated crack sizes.
- Column M: plastic displacements.
- Column R: standard errors of the compliance.

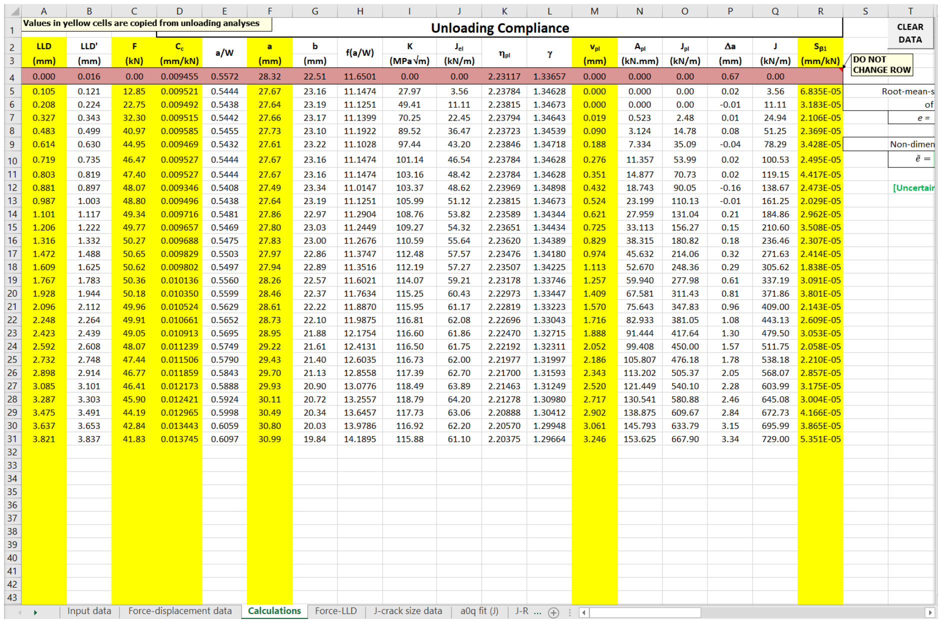
The remaining columns automatically display calculation results of fracture toughness parameters according to E1820-20b, Annex A1 for SE(B) specimens or Annex A2 for C(T) specimens. Note that the right side of the sheet (columns {AA-AI}) details calculation steps for the incremental calculation of the *J*-integral, equations (A1.9) and (A2.9) for SE(B) and C(T) specimens, respectively.

NOTE – Cells {A4-R4}, highlighted in red, represent "point zero" for the test (*i.e*, zero fracture toughness), and contain predetermined values that should not be changed.

The non-dimensional rms standard error, $\tilde{e}$, is provided in cell {U10}, based on the root-mean-square of the standard error of the compliance, e (cell {U7}). This, in turn, is obtained from the standard errors of the data points selected in the power law regression that establishes J_Q, column {Y}. In accordance with E1820-20b section X3.5.3, if $\tilde{e} < 400$, the uncertainty in J_{Ic} due to noise in the unload/reload data is less than 4 % {11}. In that case, cell {U10} turns green; otherwise, the cell turns red.

The current spreadsheet accommodates a maximum of 100 data points (unloadings) but could be easily modified to allow larger data sets. A screenshot of the left side of this sheet is provided in Figure 7.

Clicking on the button **CLEAR DATA** erases all values in the yellow columns.



Row	LLD (mm)	LLD' (mm)	F (kN)	C_c (mm/kN)	a/W	a (mm)	b (mm)	f(a/W)	K (MPa√m)	J_{el} (kN/m)	η_{pl}	γ	v_{pl} (mm)	A_{pl} (kN.mm)	J_{pl} (kN/m)	Δa (mm)	J (kN/m)	$S_{\beta 1}$ (mm/kN)
4	0.000	0.016	0.00	0.009455	0.5572	28.32	22.51	11.6501	0.00	0.00	2.23117	1.33657	0.000	0.000	0.00	0.67	0.00	
5	0.105	0.121	12.85	0.009521	0.5444	27.67	23.16	11.1474	27.97	3.56	2.23784	1.34628	0.000	0.000	0.00	0.02	3.56	6.835E-05
6	0.208	0.224	22.75	0.009492	0.5438	27.64	23.19	11.1251	49.41	11.11	2.23815	1.34673	0.000	0.000	0.00	-0.01	11.11	3.183E-05
7	0.327	0.343	32.30	0.009515	0.5442	27.66	23.17	11.1399	70.25	22.45	2.23794	1.34643	0.019	0.523	2.48	0.01	24.94	2.106E-05
8	0.483	0.499	40.97	0.009585	0.5455	27.73	23.10	11.1922	89.52	36.47	2.23723	1.34539	0.090	3.124	14.78	0.08	51.25	2.369E-05
9	0.614	0.630	44.95	0.009469	0.5432	27.61	23.22	11.1028	97.44	43.20	2.23846	1.34718	0.188	7.334	35.09	-0.04	78.29	3.428E-05
10	0.719	0.735	46.47	0.009527	0.5444	27.67	23.16	11.1474	101.14	46.54	2.23784	1.34628	0.276	11.357	53.99	0.02	100.53	2.495E-05
11	0.803	0.819	47.40	0.009527	0.5444	27.67	23.16	11.1474	103.16	48.42	2.23784	1.34628	0.351	14.877	70.73	0.02	119.15	4.417E-05
12	0.881	0.897	48.07	0.009346	0.5408	27.49	23.34	11.0147	103.37	48.62	2.23969	1.34898	0.432	18.743	90.05	-0.16	138.67	2.473E-05
13	0.987	1.003	48.80	0.009496	0.5438	27.64	23.19	11.1251	105.99	51.12	2.23815	1.34673	0.524	23.199	110.13	-0.01	161.25	2.029E-05
14	1.101	1.117	49.34	0.009716	0.5481	27.86	22.97	11.2904	108.76	53.82	2.23589	1.34344	0.621	27.959	131.04	0.21	184.86	2.962E-05
15	1.206	1.222	49.77	0.009657	0.5469	27.80	23.03	11.2449	109.27	54.32	2.23651	1.34434	0.725	33.113	156.27	0.15	210.60	3.508E-05
16	1.316	1.332	50.27	0.009688	0.5475	27.83	23.00	11.2676	110.59	55.64	2.23620	1.34389	0.829	38.315	180.82	0.18	236.46	2.307E-05
17	1.472	1.488	50.65	0.009829	0.5503	27.97	22.86	11.3747	112.48	57.57	2.23476	1.34180	0.974	45.632	214.06	0.32	271.63	2.414E-05
18	1.609	1.625	50.62	0.009802	0.5497	27.94	22.89	11.3516	112.19	57.27	2.23507	1.34225	1.113	52.670	248.36	0.29	305.62	1.838E-05
19	1.767	1.783	50.36	0.010136	0.5560	28.26	22.57	11.6021	114.07	59.21	2.23178	1.33746	1.257	59.940	277.98	0.61	337.19	3.091E-05
20	1.928	1.944	50.18	0.010350	0.5599	28.46	22.37	11.7634	115.25	60.43	2.22973	1.33447	1.409	67.581	311.43	0.81	371.86	3.801E-05
21	2.096	2.112	49.96	0.010524	0.5629	28.61	22.22	11.8870	115.95	61.17	2.22819	1.33223	1.570	75.643	347.83	0.96	409.00	2.143E-05
22	2.248	2.264	49.91	0.010661	0.5652	28.73	22.10	11.9875	116.81	62.08	2.22696	1.33043	1.716	82.933	381.05	1.08	443.13	2.609E-05
23	2.423	2.439	49.05	0.010913	0.5695	28.95	21.88	12.1754	116.60	61.86	2.22470	1.32715	1.888	91.444	417.64	1.30	479.50	3.053E-05
24	2.592	2.608	48.07	0.011239	0.5749	29.22	21.61	12.4131	116.50	61.75	2.22192	1.32311	2.052	99.408	450.00	1.57	511.75	2.058E-05
25	2.732	2.748	47.44	0.011506	0.5790	29.43	21.40	12.6035	116.73	62.00	2.21977	1.31997	2.186	105.807	476.18	1.78	538.18	2.210E-05
26	2.898	2.914	46.77	0.011859	0.5843	29.70	21.13	12.8558	117.39	62.70	2.21700	1.31593	2.343	113.202	505.37	2.05	568.07	2.857E-05
27	3.085	3.101	46.41	0.012173	0.5888	29.93	20.90	13.0776	118.49	63.89	2.21463	1.31249	2.520	121.449	540.10	2.28	603.99	3.175E-05
28	3.287	3.303	45.90	0.012421	0.5924	30.11	20.72	13.2557	118.79	64.20	2.21278	1.30980	2.717	130.541	580.88	2.46	645.08	3.004E-05
29	3.475	3.491	44.19	0.012965	0.5998	30.49	20.34	13.6457	117.73	63.06	2.20888	1.30412	2.902	138.875	609.67	2.84	672.73	4.166E-05
30	3.637	3.653	42.84	0.013443	0.6059	30.80	20.03	13.9786	116.92	62.20	2.20570	1.29948	3.061	145.793	633.79	3.15	695.99	3.865E-05
31	3.821	3.837	41.83	0.013745	0.6097	30.99	19.84	14.1895	115.88	61.10	2.20375	1.29664	3.246	153.625	667.90	3.34	729.00	5.351E-05

Figure 7 - Sheet *Calculations* (left side).

3.4. Chart *Force-LLD (C(T) specimen) or Force-CMOD (SE(B) specimen)*

This chart (Figure 8) uses data from both sheets *Force-displacement data* and *Calculations*.

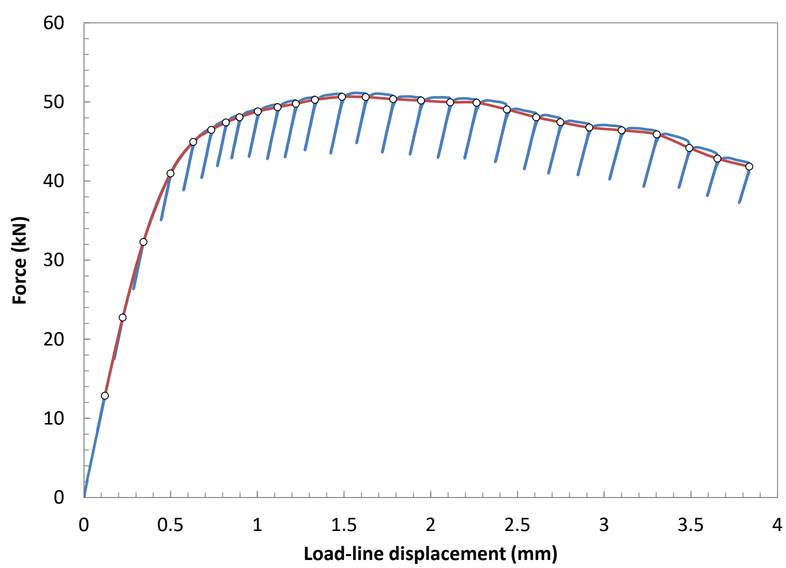


Figure 8 - Force/load-line displacement chart for a C(T) specimen. The round symbols correspond to the first points of each elastic unloading.

3.5. Sheet *J-crack size data*

This sheet contains data that are used for the calculation of the original crack size estimated from compliance, a_{oq}, namely force, crack size, and J-integral, all copied from the *Calculations* sheet. In addition, the maximum value of force, F_{max}, and the corresponding data point number, n_{Fmax}, are reported in cells {E2,F2} (only data points before F_{max} are used to calculated a_{oq}), while the minimum crack size, a_{min}, and its corresponding data point number, n_{amin}, are given in cells {E6,F6}. F_{max} and a_{min} are also highlighted in columns {B,C} in red and green respectively. Cell {E9} shows the J-integral value of the data point corresponding to the minimum crack size, J_{amin}, while cell {E8} contains the corresponding crack extension due to crack tip blunting[8], given by $\Delta a_{bl,amin} = J_{amin}/2\sigma_Y$, with σ_Y = flow stress (average of yield and ultimate tensile stresses at test temperature).

On the right side of the sheet, a chart is displayed showing J vs. crack size data points. In the lower part of the sheet, the calculated value of a_{oq} is shown in cell {H29}, as well as the remaining two fitting coefficients (B, C) in the following fitting curve (equation A9.1 in E1820):

$$a = a_{oq} + \frac{J}{2\sigma_Y} + BJ^2 + CJ^3 \quad . \tag{1}$$

Tensile properties (yield, tensile, and flow stresses) are shown in cells {H33-H35}, the correlation coefficient value in cell {H37}, and the number of points used in the regression in cell {H38}. Cell {H40} displays the measured value of original crack size, $a_{o(meas)}$.

It is common for the early stages of an unloading compliance test to display the so-called "apparent negative crack growth" [12-14], whereby some early data points exhibit a non-physical trend of decreasing compliance (*i.e.*, decreasing crack size) with increasing J. Various explanations have been offered for this behavior, including friction, specimen rotation, misalignments, but also compressive stresses and strain hardening caused by the development of a plastic zone ahead of the blunting crack tip.

ASTM E1820 does not provide guidance on how to treat such occurrences, but many researchers tend to ignore all data points before the one corresponding to the shortest crack size and only fit the remaining data points preceding F_{max} by means of eq. (1). Other authors [13,14] have suggested shifting all data points so that a_{min} is associated to a crack extension corresponding to the blunting of the crack tip ($\Delta a_{bl,amin}$ defined above).

This sheet provides the option to select the preferred method via a drop-down menu on the right side of the J-crack size plot, columns {S-U}; the three available choices are:

(a) *Rigorous E1820 (all data before Fmax)* (all data points before F_{max} are used for the fit).
(b) *Starting from minimum crack size* (only data points between a_{min} and a_{Fmax} are used for the fit, and the data point corresponding to a_{min} is given $\Delta a = 0$).
(c) *Blunting-corrected minimum crack size* (only data points between a_{min} and a_{Fmax} are used for the fit, and their crack sizes are incremented by $\Delta a_{bl,amin}$).

Depending on the selected option, the calculated value of a_{oq} in cell {G29} will change slightly.

[8] Crack tip blunting is a phenomenon by which the tip of the crack slightly extends due to plastic deformation caused by applied force, before stable/ductile crack growth actually occurs. According to ASTM E1820, the relationship between J and Δa during blunting is given by $J = 2\sigma_Y\Delta a$.

Section A9.5 of the E1820-20b standard requires the final *J*-integral values to be recalculated using the adjusted a_{oq} value obtained from eq. (1). This is accomplished by clicking the button **UPDATE J CALCULATIONS USING aoq**, located below the *J*-crack size plot.

A screenshot of this sheet, showing an example of apparent negative crack growth, is given in Figure 9.

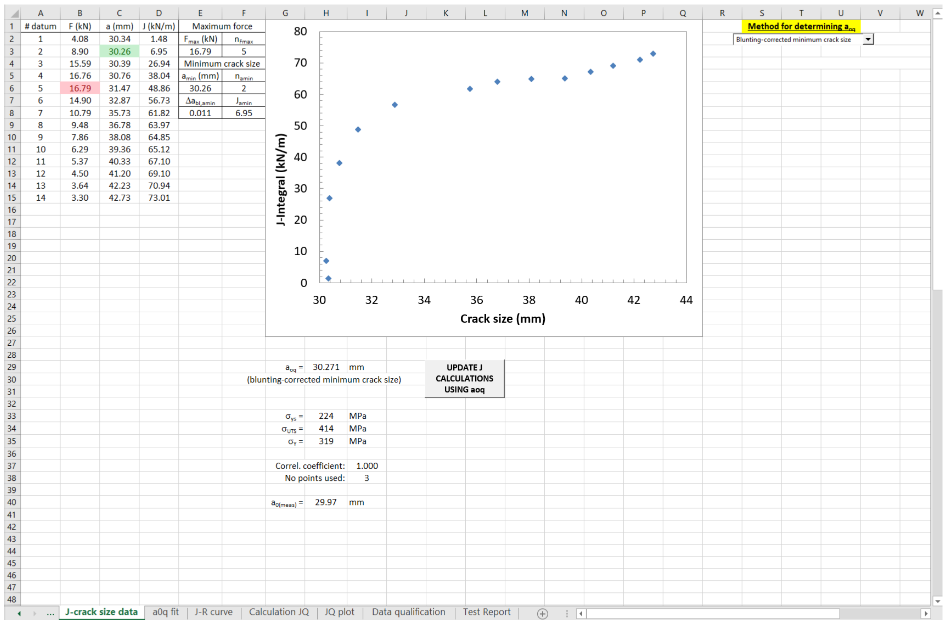


Figure 9 - Sheet *J-crack size data*, showing evidence of apparent negative crack growth and with the third calculation option (*Blunting-corrected minimum crack size*) selected.

3.6. Sheet *a0q fit*

This sheet is where a_{oq} is actually calculated, following the recommendations provided in Appendix X1 of ASTM E1820-20b. The equation (X1) that needs to be solved using the method of least squares is:

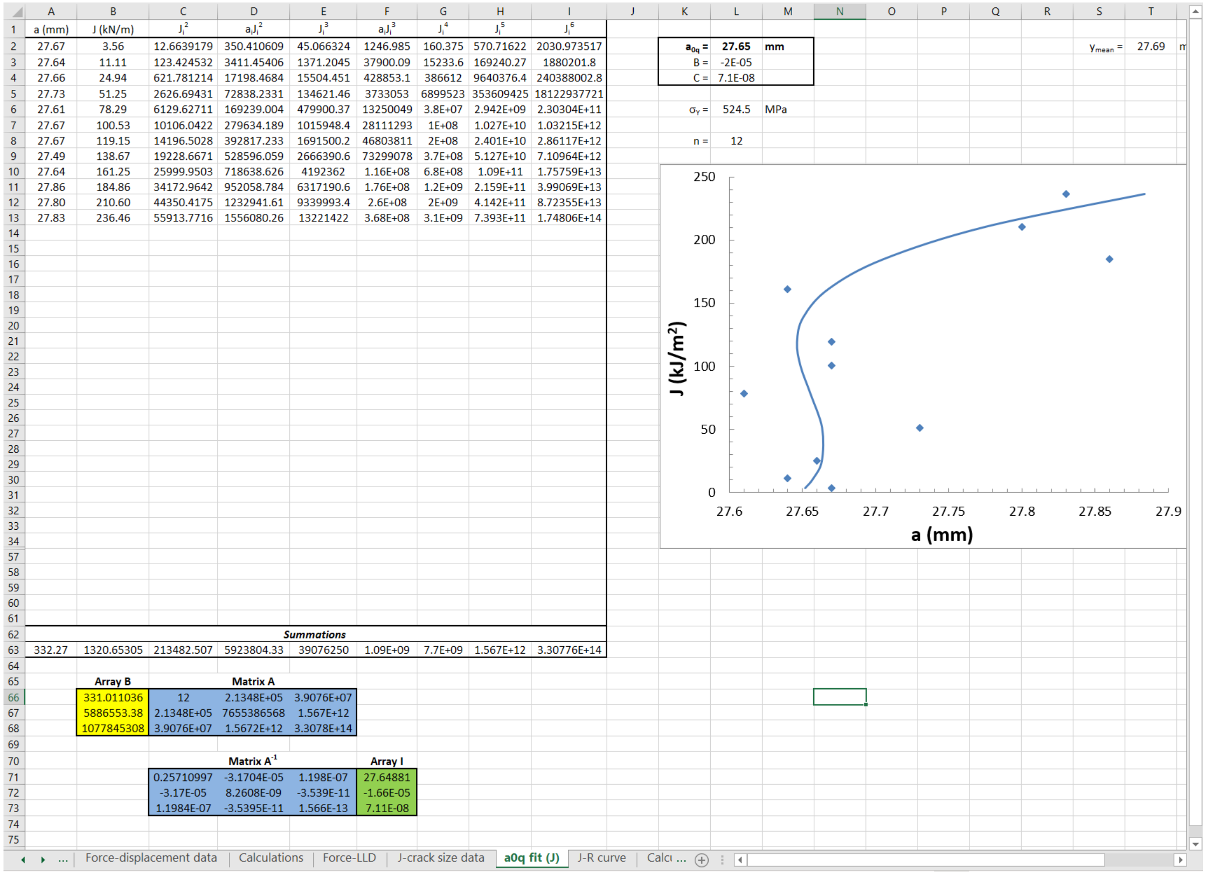
$$\begin{Bmatrix} \sum a_i - \dfrac{\sum J_i}{2\sigma_Y} \\ \sum a_i J_i^2 - \dfrac{\sum J_i^3}{2\sigma_Y} \\ \sum a_i J_i^3 - \dfrac{\sum J_i^4}{2\sigma_Y} \end{Bmatrix} = \begin{bmatrix} n & \sum J_i^2 & \sum J_i^3 \\ \sum J_i^2 & \sum J_i^4 & \sum J_i^5 \\ \sum J_i^3 & \sum J_i^5 & \sum J_i^6 \end{bmatrix} \begin{Bmatrix} a_{oq} \\ B \\ C \end{Bmatrix} \tag{2}$$

The parameters used in the analysis are shown in columns {A-I} for up to 60 data points, while the arrays and the matrices used in the calculations are shown in rows {66-73} below.

On the right side of the sheet, the values of the fitting coefficients (a_{oq}, B, and C) are listed, while a chart showing the fitted data points[9] and the obtained regression curve, equation (1), is shown below.

[9] The specific data points displayed depend on the option selected by the user in the previous sheet (all data points before F_{max} or only data starting from a_{min}).

A screenshot of this sheet is given in Figure 10.



	a (mm)	J (kN/m)	J_i^2	$a_iJ_i^2$	J_i^3	$a_iJ_i^3$	J_i^4	J_i^5	J_i^6
2	27.67	3.56	12.6639179	350.410609	45.066324	1246.985	160.375	570.71622	2030.973517
3	27.64	11.11	123.424532	3411.45406	1371.2045	37900.09	15233.6	169240.27	1880201.8
4	27.66	24.94	621.781214	17198.4684	15504.451	428853.1	386612	9640376.4	240388002.8
5	27.73	51.25	2626.69431	72838.2331	134621.46	3733053	6899523	353609425	18122937721
6	27.61	78.29	6129.62711	169239.004	479900.37	13250049	3.8E+07	2.942E+09	2.30304E+11
7	27.67	100.53	10106.0422	279634.189	1015948.4	28111293	1E+08	1.027E+10	1.03215E+12
8	27.67	119.15	14196.5028	392817.233	1691500.2	46803811	2E+08	2.401E+10	2.86117E+12
9	27.49	138.67	19228.6671	528596.059	2666390.6	73299078	3.7E+08	5.127E+10	7.10964E+12
10	27.64	161.25	25999.9503	718638.626	4192362	1.16E+08	6.8E+08	1.09E+11	1.75759E+13
11	27.86	184.86	34172.9642	952058.784	6317190.6	1.76E+08	1.2E+09	2.159E+11	3.99069E+13
12	27.80	210.60	44350.4175	1232941.61	9339993.4	2.6E+08	2E+09	4.142E+11	8.72355E+13
13	27.83	236.46	55913.7716	1556080.26	13221422	3.68E+08	3.1E+09	7.393E+11	1.74806E+14
62	Summations								
63	332.27	1320.65305	213482.507	5923804.33	39076250	1.09E+09	7.7E+09	1.567E+12	3.30776E+14

Array B	Matrix A		
331.011036	12	2.1348E+05	3.9076E+07
5886553.38	2.1348E+05	7655386568	1.567E+12
1077845308	3.9076E+07	1.5672E+12	3.3078E+14

Matrix A^{-1}			Array I
0.25710997	-3.1704E-05	1.198E-07	27.64881
-3.17E-05	8.2608E-09	-3.539E-11	-1.66E-05
1.1984E-07	-3.5395E-11	1.566E-13	7.11E-08

Figure 10 - Sheet *a0q fit*. NOTE: several rows have been hidden so that all relevant content could be displayed.

3.7. Chart *J-R curve*

This chart (Figure 11) plots the experimental *J*-Δ*a* data points after the calculation of the original crack size based on compliance, a_{oq}. Crack extension values are given by $\Delta a_i = a_i - a_{oq}$, where a_i is the crack size calculated from the compliance of the i^{th} unloading.

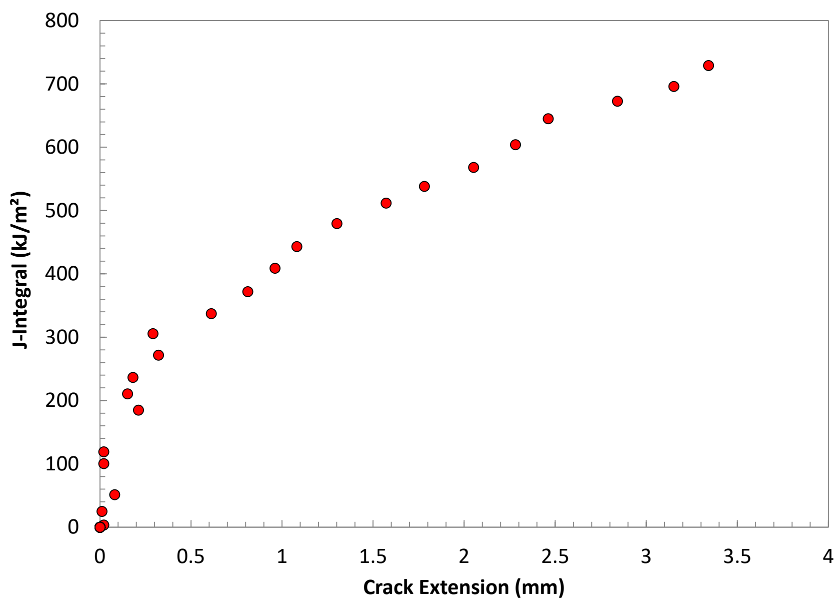


Figure 11 - Chart *J-R curve*.

3.8. Sheet *Calculation JQ*

The size-independent plane-strain fracture toughness, J_{Ic} (or its size-dependent counterpart, J_Q) is calculated in this sheet in accordance with Annex A9 of E1820-20b. Calculations are performed by clicking the button **CALC** in cell {U1}, by identifying the coordinates of the intersection point between the power law fitting curve $J = C_1 \Delta a^{C_2}$ and the 0.2 mm-offset construction line $J = M\sigma_Y(\Delta a - 0.2\ mm)$, where:

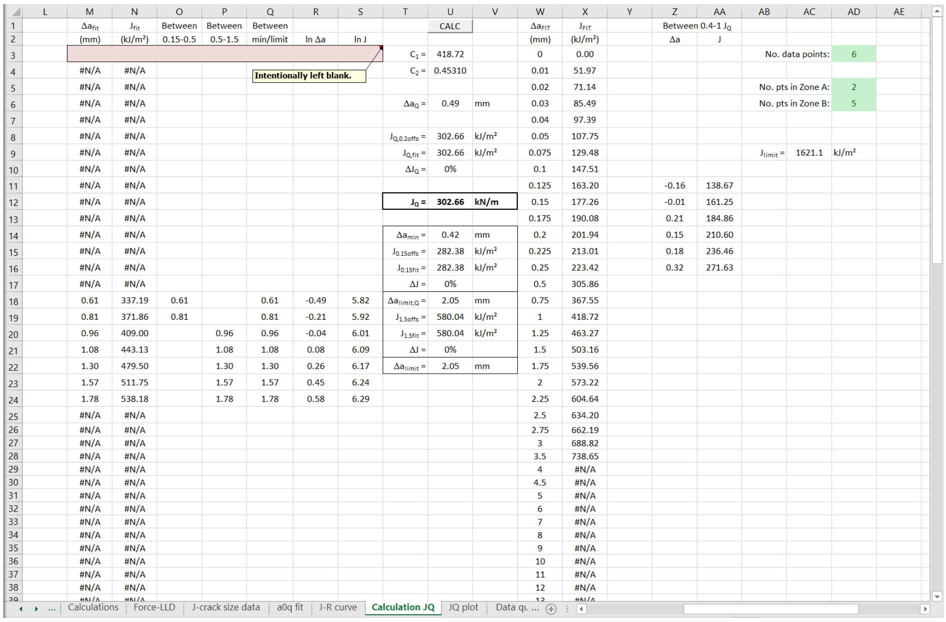
- C_1 and C_2, coefficients of the regression curve, are found in cells {U3,U4};
- M, the slope of the construction line, is by default equal to 2 (eq.(A9.4) in E1820-20b), but can be modified by the user by changing the value in cell {B1}.

The calculated value of J_Q is displayed in cell {U12}. The corresponding crack extension, Δa_Q, appears in cell {U6}. The calculated intersections between the power law fitting curve and the 0.15 mm and 1.5 mm-offset exclusion lines (Figure 1) are shown below, in cells {U14-U21}. Finally, the values of Δa_{limit} and J_{limit}, as defined in A9.6.6.5 of E1820-20b, are provided in cells {U22} and {AC9}, respectively.

The upper limit of the construction and offset lines, for plotting purposes, is automatically calculated as 1.1 times the J value of the intersection between the fitting curve and the 1.5 mm-exclusion line in cell {E2}, but can also be freely modified by the user if needed.

The sheet also checks several validity requirements mentioned in Annex A9, such as the data points distribution (cell {E17}), the number of qualified data points (cell {E18}), the number of data points between 0.4 J_Q and J_Q (cell {AD3}), and the number of data points in zones A and B (cells {AD5,AD6}).

A screenshot of the right side of this sheet, which includes the results of the critical fracture toughness calculations, is given in Figure 12.



Row	L	M	N	O	P	Q	R	S	T	U	V
1		Δa_{fit}	J_{fit}	Between	Between	Between				CALC	
2		(mm)	(kJ/m²)	0.15-0.5	0.5-1.5	min/limit	ln Δa	ln J			
3									C_1 =	418.72	
4		#N/A	#N/A						C_2 =	0.45310	
5		#N/A	#N/A								
6		#N/A	#N/A						Δa_Q =	0.49	mm
7		#N/A	#N/A								
8		#N/A	#N/A						$J_{Q,0.2offs}$ =	302.66	kJ/m²
9		#N/A	#N/A						$J_{Q,fit}$ =	302.66	kJ/m²
10		#N/A	#N/A						ΔJ_Q =	0%	
11		#N/A	#N/A								
12		#N/A	#N/A						**J_Q =**	**302.66**	**kN/m**
13		#N/A	#N/A								
14		#N/A	#N/A						Δa_{min} =	0.42	mm
15		#N/A	#N/A						$J_{0.15offs}$ =	282.38	kJ/m²
16		#N/A	#N/A						$J_{0.15fit}$ =	282.38	kJ/m²
17		#N/A	#N/A						ΔJ =	0%	
18		0.61	337.19	0.61		0.61	-0.49	5.82	$\Delta a_{limit,Q}$ =	2.05	mm
19		0.81	371.86	0.81		0.81	-0.21	5.92	$J_{1.5offs}$ =	580.04	kJ/m²
20		0.96	409.00		0.96	0.96	-0.04	6.01	$J_{1.5fit}$ =	580.04	kJ/m²
21		1.08	443.13		1.08	1.08	0.08	6.09	ΔJ =	0%	
22		1.30	479.50		1.30	1.30	0.26	6.17	Δa_{limit} =	2.05	mm
23		1.57	511.75		1.57	1.57	0.45	6.24			
24		1.78	538.18		1.78	1.78	0.58	6.29			
25		#N/A	#N/A								
26		#N/A	#N/A								
27		#N/A	#N/A								
28		#N/A	#N/A								
29		#N/A	#N/A								
30		#N/A	#N/A								
31		#N/A	#N/A								
32		#N/A	#N/A								
33		#N/A	#N/A								
34		#N/A	#N/A								
35		#N/A	#N/A								
36		#N/A	#N/A								
37		#N/A	#N/A								
38		#N/A	#N/A								

Intentionally left blank.

Row	W	X	Y	Z	AA	AB	AC	AD	AE
1	Δa_{FIT}	J_{FIT}		Between 0.4-1 J_Q					
2	(mm)	(kJ/m²)		Δa	J				
3	0	0.00					No. data points:	6	
4	0.01	51.97							
5	0.02	71.14					No. pts in Zone A:	2	
6	0.03	85.49					No. pts in Zone B:	5	
7	0.04	97.39							
8	0.05	107.75							
9	0.075	129.48				J_{limit} =	1621.1	kJ/m²	
10	0.1	147.51							
11	0.125	163.20		-0.16	138.67				
12	0.15	177.26		-0.01	161.25				
13	0.175	190.08		0.21	184.86				
14	0.2	201.94		0.15	210.60				
15	0.225	213.01		0.18	236.46				
16	0.25	223.42		0.32	271.63				
17	0.5	305.86							
18	0.75	367.55							
19	1	418.72							
20	1.25	463.27							
21	1.5	503.16							
22	1.75	539.56							
23	2	573.22							
24	2.25	604.64							
25	2.5	634.20							
26	2.75	662.19							
27	3	688.82							
28	3.5	738.65							
29	4	#N/A							
30	4.5	#N/A							
31	5	#N/A							
32	6	#N/A							
33	7	#N/A							
34	8	#N/A							
35	9	#N/A							
36	10	#N/A							
37	11	#N/A							
38	12	#N/A							

Figure 12 - Right side of sheet *Calculation JQ*.

3.9. Chart *JQ plot*

This chart (Figure 13) illustrates the analyses for the determination of J_Q. “Qualified” data points (within the 0.15 mm and 1.5 mm-exclusion lines and below J_{limit}), which are fitted by a power law function, are displayed as green round symbols.

The user must manually position the text boxes corresponding to J_Q, Δa_{limit}, and J_{limit} (NOTE: in most cases, J_{limit} lies beyond the maximum value of the ordinate axis and is therefore not visualized).

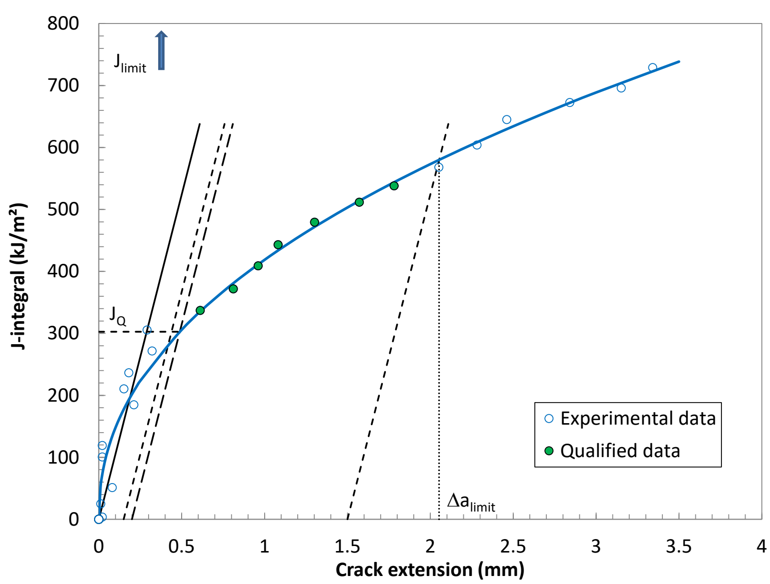


Figure 13 - Chart *JQ plot*.

3.10. Sheet *Data qualification*

This sheet summarizes several validity requirements that are scattered throughout the ASTM E1820-20b standard, including:

- Differences between individual values and the average value of original crack size, as predicted from at least three elastic unloadings, performed at the beginning of the test.[10]
- Comparison between measured and predicted final crack extension.
- Requirements for the qualification of data (section A9.9), some of which are specific to the elastic compliance procedure.
- Requirements for the qualification of J_Q as J_{Ic} (section A9.10).

Fulfilled requirements are highlighted in green, while those not fulfilled are highlighted in red.

A screenshot of this sheet is provided in Figure 14.

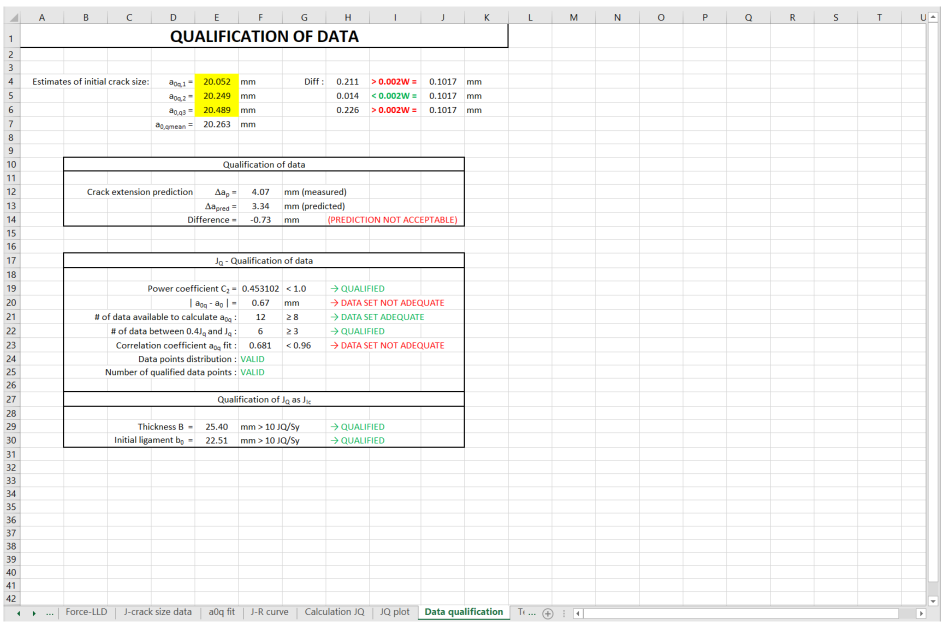


Figure 14 - Sheet *Data qualification*.

3.11. Sheet *Test Report*

This last sheet summarizes all analysis main results, as well as specimen information and dimensions, and tensile properties.

The calculated value of critical fracture toughness is shown in cell {E16}. If all the requirements in the previous sheet *Data qualification* are fulfilled, cell {D6} reports "J_Ic ="; otherwise, the cell reports "J_Q =".

[10] This requirement does not contribute to the qualification of J_Q as J_{Ic}.

By clicking the button **PRINT TEST RESULTS**, the user can print the following sheets and charts on the default system printer[11]:

- Sheet *Test Report*.
- Sheet *Data qualification*.
- Chart *Force-LLD* (C(T) specimen) or *Force-CMOD* (SE(B) specimen).
- Chart *JQ plot*.

A screenshot is provided in Figure 15 for a C(T) specimen.

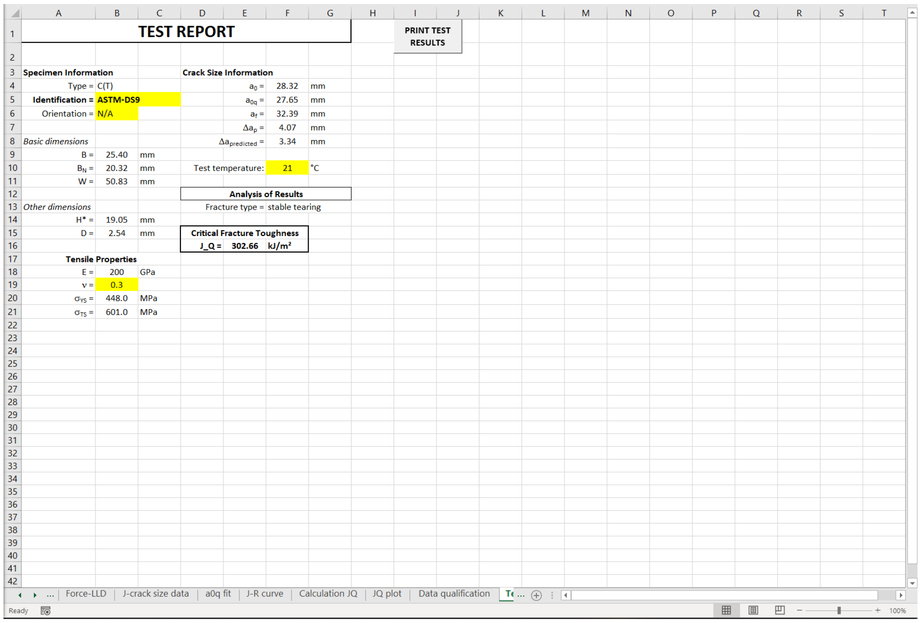


Figure 15 - Sheet *Test Report* for a C(T) specimen.

4. Software Validation: E1820-20b Standard Data Sets

The current version of the ASTM E1820 standard mentions the availability of a collection of nine standard data sets, which can be used for verifying computer algorithms developed to implement the calculations to evaluate J_{Ic}. These datasets are available for download from ASTM at https://www.astm.org/COMMIT/E1820 Data Sets DS1-DS9.7z (nine ASCII text files in a compressed archive). Seven of the data sets are for C(T) specimens, the remaining two are for SE(B) specimens.

These data sets were used in an analytical round-robin that involved four participating labs (including NIST, under the supervision of the author). The results of this round-robin, which showed good agreement among the laboratories in terms of individual *J*-integral calculations and crack size estimates, were published in [15].

[11] If the user wants to print using a different printer, the printer selection must be made before clicking the button **PRINT TEST RESULTS**.

In order to validate the spreadsheet-based software described in this report, we analyzed all nine data sets and compared the results with the "Expected Results" and corresponding standard deviations (σ_{ASTM}) provided in the data files, namely:

- J_Q;
- Validity of J_{Ic} (TRUE/FALSE);
- Predicted final crack extension, Δa_{pred};
- Number of data points in zone A;
- Number of data points in zone B;
- Number of qualified data points in the power law fit;
- Fitting coefficients C_1 and C_2.
- Predicted original crack size, a_{oq};
- Absolute difference between measured and predicted original crack size.
- Number of data points used to establish a_{oq};
- Number of data points between $0.4J_Q$ and J_Q;
- Correlation coefficient for a_{oq} fit;
- $B_{qual}, b_{qual} = 10\frac{J_Q}{\sigma_Y}$ (used in the qualification of J_Q as J_{Ic});
- Non-dimensionalized rms standard error of the compliances, $\tilde{e}$;
- Average number of points used for calculating compliances.

The results of the comparisons are detailed in Table 1 (seven C(T) specimens) and Table 2 (two SE(B) specimens). If the NIST result is within the ASTM values ± one standard deviation (σ_{ASTM}), the cell is highlighted in green; if not, the cell is highlighted in red.

Table 1 - Comparison between "Expected Results" (reference values) for the ASTM sample data sets and NIST software outcome for seven C(T) specimens.[12]

Data set	J_Q or J_{Ic} (kJ/m^2)			Validity of J_{Ic}		Δa_{pred} (mm)			Data points in A			Data points in B			Points in power law fit		
id	ASTM	σ_{ASTM}	NIST	ASTM	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST
DS1	85.8	0.8	85.3	TRUE	TRUE	5.118	N/A	5.117	5	N/A	5	8	N/A	8	13	N/A	13
DS2	442.7	3.0	442.1	FALSE	FALSE	2.55	0.01	2.5	22	1	22	33	+0/-1	33	54	+1/-0	54
DS3	428.6	5.8	427.7	TRUE	TRUE	2.70	0.01	2.69	17	+1/-0	17	33	+0/-1	32	49	1	49
DS5	33.2	1.1	32.4	FALSE	FALSE	12.42	0.02	12.42	1	0	1	1	0	1	2	0	2
DS6	33.9	0.8	33.3	FALSE	FALSE	7.47	0.01	7.47	4	+0/-1	4	5	0	5	9	+0/-1	9
DS8	103.8	1.7	104.2	FALSE	FALSE	13.72	0.01	13.71	3	0	3	3	0	3	6	0	6
DS9	301.9	4.6	302.7	FALSE	FALSE	3.37	0.01	3.34	2	0	2	5	0	5	7	0	7

Data set	Coefficient C_1 (kJ/m^2)			Coefficient C_2 (kJ/m^2)			a_{0q} (mm)			$\|a_0 - a_{0q}\|$ (mm)			Data points for a_{0q} fit		
id	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST
DS1	123.6	N/A	123.6	0.2961	N/A	0.2960	28.00	0.01	28.00	0.004	N/A	0.004	25	N/A	23
DS2	644.7	5.1	645.7	0.762	0.003	0.762	29.62	0.01	29.62	0.63	0.01	0.63	45	1	45
DS3	630.9	4.6	631.6	0.749	0.004	0.751	28.70	0.01	28.69	0.42	0.01	0.42	47	+0/-1	46
DS5	47.5	0.8	47.0	0.261	0.009	0.268	30.31	0.01	30.31	0.34	0.01	0.34	4	+1/-0	4
DS6	49.7	0.6	49.1	0.278	0.011	0.282	15.08	0.01	15.08	0.58	0.01	0.58	7	+0/-1	6
DS8	193.6	1.4	194.2	0.486	0.007	0.486	30.40	0.01	30.40	0.32	0.01	0.33	6	+1/-0	6
DS9	417.8	4.1	418.7	0.454	0.004	0.453	27.64	0.01	27.65	0.68	0.01	0.67	13	+0/-1	12

Data set	Data points between 0.4 and $1.0J_Q$			Correl. coeff. for a_{0q} fit			B_{qual}, b_{qual} (mm)			rms standard error			Average points unloadings		
id	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST
DS1	9	N/A	10	0.9999	N/A	0.9999	1.722	N/A	1.705						
DS2	16	0	16	0.999	0.001	0.999	8.18	0.08	8.16	238	2	241	94.9	0.2	94
DS3	15	+0/-1	15	0.999	0.001	0.999	7.91	0.08	7.90	153	2	178	123.0	0.8	121.9
DS5	1	0	1	0.987	0.013	0.991	1.04	0.03	1.02	129	5	132	78.4	3.6	79
DS6	3	1	3	0.825	0.060	0.808	1.07	0.03	1.04	167	10	169	28.2	1.0	28.1
DS8	2	0	2	0.994	0.001	0.994	1.54	0.02	1.55	253	6	263	41.9	0.8	41.3
DS9	6	1	6	0.695	0.081	0.681	5.73	0.04	5.77	261	1	271	40.4	0.4	39.6

[12] Some results (rms standard error, average number of points for compliance calculation) are missing in data set DS1. In the same data set, most standard deviations (σ_{ASTM}) are also missing.

Table 2 - Comparison between "Expected Results" (reference values) for the ASTM sample data sets and NIST software outcome for two SE(B) specimens.

Data set	J_Q or J_{Ic} (kJ/m^2)			Validity of J_{Ic}		Δa_{pred} (mm)			Data points in A			Data points in B			Points in power law fit		
id	ASTM	σ_{ASTM}	NIST	ASTM	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST
DS4	39.7	1.2	38.9	FALSE	FALSE	10.71	0.01	10.70	2	0	2	3	+1/-0	3	5	+1/-0	5
DS7	204.7	3.1	206.0	TRUE	TRUE	7.06	0.01	7.05	4	+0/-1	4	5	+1/-0	5	9	+0/-1	9

Data set	Coefficient C_1 (kJ/m^2)			Coefficient C_2 (kJ/m^2)			a_{0q} (mm)			$\|a_0 - a_{0q}\|$ (mm)			Data points for a_{0q} fit		
id	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST
DS4	52.3	2.4	50.9	0.207	0.015	0.202	31.25	0.01	31.26	2.20	0.01	2.19	7	+0/-1	6
DS7	27.56	0.01	27.6	0.370	0.004	0.367	27.56	0.01	27.56	0.35	0.01	0.35	11	+0/-1	10

Data set	Data points between 0.4 and 1.0J_Q			Correl. coeff. for a_{0q} fit			B_{qual}, b_{qual} (mm)			rms standard error			Average points unloadings		
id	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST	ASTM	σ_{ASTM}	NIST
DS4	4	+0/-1	4	0.982	0.011	0.983	1.24	0.04	1.22	238	2	235	23.4	0.4	23.1
DS7	5	0	5	0.982	0.005	0.982	3.04	0.05	3.06	185	1	187	36.7	0.4	36.0

Examination of Table 1 and Table 2 shows that:

(a) For both specimen configurations, all critical toughness values (J_Q or J_{Ic}) calculated by the NIST software were found to be in agreement with the ASTM reference values (within $\pm 1\sigma_{ASTM}$). All critical values were also correctly identified as valid or invalid. This represents substantial validation of the NIST software.

(b) Most disagreements were observed for the rms standard error (5 data sets out of 8) and the average number of points used to calculate compliance (4 out of 8). All the calculations for these two parameters were carefully reviewed, and no errors were found. The source of the disagreements is therefore unknown, although in the case of $\tilde{e}$, it is suspected that minor differences between NIST and ASTM in the individual compliances can quickly add up and eventually cause a larger difference in the overall rms standard error. It is hypothesized that differences in the average number of data points used for compliance calculations could be due to discrepancies in the selection of the start/end points of the unloading cycles.[13]

(c) The two remaining disagreements are both for Δa_{pred}, and in both cases differences between the NIST value and the ASTM lower limit ($\Delta a_{pred,ASTM} - \sigma_{ASTM}$) is very small: less than 0.01 mm for DS7 and 0.02 mm for DS9. Once again, these calculations were reviewed, and no errors were found.

Altogether, the validation of the NIST software can be considered successful. For the complete analysis of a typical unloading compliance test with a total number of unloading cycles between 40 and 50 using the two developed spreadsheets, less than 30 minutes are required.

5. Conclusions

NIST has developed a macro-enabled, spreadsheet-based software for the analysis of elastic-plastic fracture toughness tests conducted on C(T) or SE(B) specimens with the elastic/unloading compliance single-specimen procedure, in accordance with the current version of the relevant ASTM standard (E1820-20b). Detailed instructions for the use of this software were provided in this report.

[13] The ASTM sample data files do not detail which data points were fitted for determining compliances, so it was not possible to identify specific discrepancies in the data point selection process (specifically, the exclusion of first and last 5 % for each unloading).

The software was successfully validated by comparison with nine sample data sets that ASTM recently made available for this specific purpose. Only a few discrepancies were observed, and all for secondary parameters. All critical toughness values were found in agreement between ASTM and NIST.

References

[1] ASTM E1820-20b, *Standard Test Method for Measurement of Fracture Toughness*, ASTM International, West Conshohocken, PA, 2020.

[2] Willoughby, A. A., and Garwood, S. J., "On the Loading Compliance Method of Deriving Single Specimen *R*-Curves in Three Point Bending," *Elastic-Plastic Fracture: Second Symposium, Volume II—Fracture Resistance Curves and Engineering Applications*, ASTM STP 803, C. F. Shih and J. P. Gudas, eds., ASTM, 1983, pp. II-372–II-397. https://doi.org/10.1520/STP803V2-EB

[3] Dover, W. D., et al., "A.C. Field Measurement—Theory and Practice," from *The Measurement of Crack Length and Shape During Fracture and Fatigu*e, Beevers, C. J., Ed., EMAS, Cradley Heath, UK, 1980, pp. 222-260.

[4] Catlin, W. R., Lord, D. C., Prater, T. A., and Coffin, L. F., "The Reversing D-C Electrical Potential Method," *Automated Test Methods for Fracture and Fatigue Crack Growth*, ASTM STP 877, Cullen, W. H., Landgraf, R. W., Kaisand, L. R., and Underwood, J. H., Eds., ASTM, 1985, pp. 67–85. https://doi.org/10.1520/STP877-EB

[5] Hartman, G. A., and Johnson, D. A., "D-C Electric Potential Method Applied to Thermal/Mechanical Fatigue Crack Growth," *Experimental Mechanics*, March 1987, pp. 106–112. http://dx.doi.org/10.1007/BF02318872

[6] Herrera, R., and Landes, J. D., "Direct J-R Curve Analysis: A Guide to the Methodology," *Fracture Mechanics: Twenty-First Symposium*, ASTM STP 1074, J.P. Gudas, J.A. Joyce, and E.M. Hackett, Eds., ASTM, Conshohocken, PA, 1990, pp 24–43. https://doi.org/10.1520/STP1074-EB

[7] Landes, J. D., Zhou, Z., Lee, K., and Herrera, R., "Normalization Method for Developing J-R Curves with the LMN Function," *Journal of Testing and Evaluation*, JTEVA, Vol 19, No. 4, July 1991, pp. 305–311. https://doi.org/10.1520/JTE12574J

[8] E. Lucon, "Use and Validation of the Slope Determination by the Analysis of Residuals (SDAR) Algorithm," NIST Technical Note 2050, June 2019. https://doi.org/10.6028/NIST.TN.2050

[9] Lucon, E., Benzing, J., and Hrabe, N., "Development and Validation of Small Punch Testing at NIST," NIST Internal Report 8303, April 2020. https://doi.org/10.6028/NIST.IR.8303

[10] Lucon, E., Splett, J., Koepke, A., and Newton, D., "NIST Software Package for Obtaining Charpy Transition Curves," NIST Technical Note 2158, May 2021. https://doi.org/10.6028/NIST.TN.2158

[11] Graham, S. M., "Uncertainty in Ductile Fracture Initiation Toughness (J_{Ic}) Resulting From Compliance Measurement," Application of Automation Technology in Fatigue and Fracture Testing and Analysis, STP 1571, P. C. McKeighan and A. A. Braun, Eds., pp. 134 – 152, ASTM International, West Conshohocken, PA, 2014. https://doi.org/10.1520/STP1571-EB

[12] Voss, B. and Mayville, R. A., “The Use of the Partial Unloading Compliance Method for the Determination of J_I-R Curves and J_{Ic},” *Elastic-Plastic Fracture Test Methods: The User’s Experience*, ASTM STP 856, E. T. Wessel and F. J. Loss, Eds., ASTM, Philadelphia, 1985, pp. 177-130. https://dx.doi.org/10.1520/STP856-EB

[13] J. H. Underwood, E. J. Troiano, and R. T. Abbott, “Simpler J_{Ic} Test and Data Analysis Procedures for High-Strength Steels,” *Fracture Mechanics: Twenty-Fourth Volume*, ASTM STP 1207, J. D. Landes, D. E. McCabe, and J. A. M. Boulet, Eds., ASTM, Philadelphia, 1994, pp. 410-421. https://dx.doi.org/10.1520/STP1207-EB

[14] C. -S. Seok, “Correction methods of an apparent negative crack growth phenomenon,” *International Journal of Fracture* **102**, 2000, pp. 259-269. https://doi.org/10.1023/A:1007680608587

[15] Link, R. E., “Round-Robin Analysis of Standard Data Sets for Standard for Fracture Toughness Evaluation in ASTM E1820,” *Journal of Testing and Evaluation*, JTEVA, Vol 43, No. 1, January 2015, pp. 159-170. https://dx.doi.org/10.1520/JTE20130143