Semiclassical Theory of Photon Echoes with Application to Pr:YSO

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Abstract: Coherent states are used to prepare a crystal using the Atomic Frequency Comb protocol for quantum memory. Here, semiclassical theory is developed and compared to experimental photon echoes of a coherent pulse.

The atomic frequency comb (AFC) protocol is a method of preparing quantum memories [1]. A large bandwidth is a desired feature of any information transmission system. In the case of the AFC, an optical signal consisting of many discrete frequencies (referred to as "teeth") spaced over the spectral band is used. The spacing of the teeth in frequency is the repetition rate of the signal.

In the particular case of Pr:YSO, one of the first-studied candidate for quantum memory, increasing the bandwidth into the regime in which the AFC tooth spacing is comparable to the spacing of the hyperfine-split ground and excited states leads to a complicated control problem. As a key tool relevant to a quantum memory, we calculate photon echoes and compare them to experiment.

The experiment and the theory used to find populations based on the actual signals used to create the AFCs has been described in a preprint [2]. Here, we confine ourselves to the part of the theory used to predict the photon echoes given the rearranged populations, the ground and excited state energy levels, and the oscillator strengths of the 9 relevant transitions. Our approach is limited to the use of coherent states both to create the AFC and for the pulse used to create the photon echo.

The atomic polarizabilities are given by

$$\alpha_i(\Delta,\Delta_0) = f_0 \sum_j \frac{f_{ij}}{\Delta_0 + \Delta_{ij}^{(0)} - \Delta - i\gamma} \tag{1}$$

where Δ_0 describes the position of a given Pr ion in the inhomogeneous band relative to the band center at $\Delta_0 = 0$, i is an index of the three ground states in the absence of a magnetic field, j is an index of the three excited states, f_{ij} is the oscillator strength normalized to sum to 1, f_0 is a constant dependent on the dipole moment of the transition, Δ is the detuning of the optical frequency relative to the band center, and γ is the homogeneous linewidth. Because the ions are well separated from each other in the sense of band theory, we may use the Clausius-Mossotti relation to find the frequency-dependent dielectric function $\varepsilon(\Delta)$ of the solid via

$$3\frac{\varepsilon(\Delta)-1}{\varepsilon(\Delta)+2} = \sum_i \frac{1}{\varepsilon_0} \int d\Delta_0\, \rho_i(\Delta_0)\, \alpha_i(\Delta,\Delta_0) \tag{2}$$

where $\rho_i(\Delta_0)$ is the density of ions for which a reference transition has detuning Δ_0.

We do not know the density of ions in our sample. Accordingly, we fit to the minimum transmission through the sample at $\Delta = 0$.

$$T = e^{-2n_2 kz} \tag{3}$$

where $\varepsilon = (n_1 + in_2)^2$, $z = 10$ mm is the length of the cyrstal, $k = 2\pi n_{\mathrm{bkg}}/\lambda$ is a wavevector with $n_{\mathrm{bkg}} = 1.80$ being the background index of refraction and λ the free-space wavelength. The observed value is $T = 0.0141$ equivalent to an attenuation of 18.5 dB.

Typical results for the dielectric function are shown in Fig. 1. Without the AFC, the inhomogeneous band gives rise to modulation on the scale of the inhomogeneous band width. The AFC is seen, in Fig. 1a to be a small portion of this band in our experiment. A blow-up is shown in Fig. 1b showing the rapid oscillations imposed by shifting the ground state population during by the AFC preparation. The dielectric function is seen to differ from the background value by only a few parts in 10^5.

The photon echoes are found by analyzing the input pulse into its Fourier components, applying the phase given by the dielectric function for a given frequency, then returning to time domain. Symbolically,

$$U_1(t) = \mathscr{F}^{-1}\left\{\mathscr{F}\left[U_0(t)\right]\exp\left[in(\Delta)kz\right]\right\} \tag{4}$$

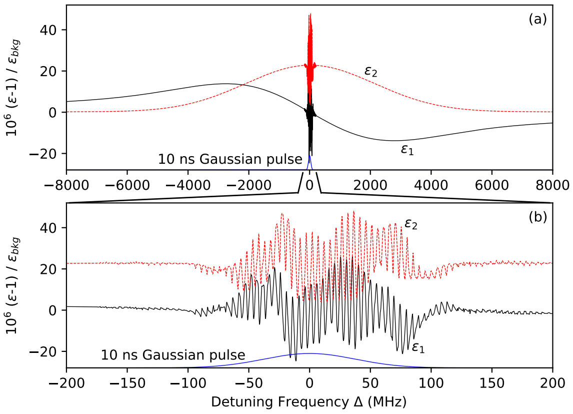

Fig. 1. Dielectric function for AFC in Pr:YSO, (a) across inhomogeneous bandwidth, and (b) in the active region of the AFC. A 10 ns FWHM pulse is shown in frequency domain as well.

where $n(\Delta) = [\varepsilon(\Delta)]^{1/2}$, $U_0(t)$ and $U_1(t)$ are the scalar wave function associated with the pulse at the entrance exit faces of the crystal, and $\mathscr{F}$ is the Fourier transform from time domain to the domain of detuned frequency Δ. The results are shown in Fig. 2 for a variety of AFCs, characterized by different repetition rates. The theory give a good account of many features observed in the photon echoes, including large gaps in echo generation when the driving frequency is near 1.9 MHz, 3.1 MHz, 3.7 MHz and 4.6 MHz. The first photon echoes are reasonably accounted for, although the second and third are underestimated in the theory.

Hopefully, the predictive model will play a role in creating useful quantum memories through optimization of the AFCs. Because the AFCs are created with lasers of a fair amount of power (milliwatts) it is likely that these will be in coherent states for the foreseeable future. On the other hand, the pulses will ultimately need to be nonclassical light. The dielectric modes calculated here can be used as starting point to create a second quantized description of the dielectric medium.

References

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2. A. N. Sharma, M. A. Ritter, K. H. Kagalwala, Z. H. Levine, E. J. Weissler, E. A. Goldschmidt, and A. L. Migdall, "Effect of hyperfine structure on atomic frequency combs in Pr:YSO," arXiv:2011.04086 [quant-ph] (2020).

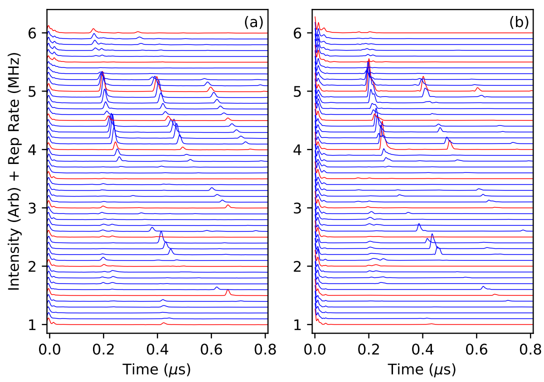

Fig. 2. Photon echoes from (a) experiment [2] and (b) theory for repetition rates in the range of 1 MHz to 6 MHz in steps of 0.1 MHz.