A Robust Algorithm for PAM4 Eye-Diagram Analysis

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***Abstract*— We propose an approach for analyzing PAM4 (pulse amplitude modulation 4-level) eye diagrams that always provides a unique solution by making use of a K-Means algorithm in conjunction with a robust, shortest interval location estimator. Our motivation for developing this technique is to create an independent, nonproprietary method that can be used to compare and verify existing algorithms. Utilizing our approach, we calculate various metrics that have been developed for PAM4 standards, including the time midpoint of the middle eye, eye amplitudes, inner eye widths, and inner eye heights. We compare our results to those of a commercial channel simulator and obtain excellent agreement.**

Keywords*— algorithm, comparison, eye diagram, K-Means clustering, pulse amplitude modulation 4-level, shortest interval location estimator.

I. Introduction

Pulse Amplitude Modulation 4-level (PAM4) is a modulation scheme used to transmit electrical or optical signals and has become increasingly popular as data rates continue to increase. In fact, PAM4 has been specified in many recent standards being developed by the IEEE 802.3 Ethernet Working Group and the Optical Internetworking Forum (OIF) [1-5]. Each of the four levels (typically referred to as 0, 1, 2, and 3) in a PAM4 signal represents two bits of information per symbol, which have four possible combinations – 00, 01, 10, and 11. PAM4 requires half the bandwidth of traditional Non-Return-to-Zero (NRZ) modulation for the same data throughput since NRZ utilizes only two levels, or one bit of information per symbol (0 or 1) [6, 7]. PAM4 has also been shown to suffer less inter-symbol interference (ISI) than NRZ [7].

Eye diagrams for both PAM4 and NRZ modulations can be constructed for assessing the quality of their respective signals by applying a data waveform to the input of an oscilloscope and overlapping the combinations on the instrument's display to span three intervals. Although research is being done to address hardware deficiencies and calibration improvements [8, 9], we focus our attention on an algorithm for calculating metrics of PAM4 eye diagrams, such as the time midpoint of the middle eye, eye amplitudes, inner eye widths, and inner eye heights.

There are some disadvantages of PAM4 modulation compared to NRZ, such as requiring more complex transmitter and receiver designs and higher costs, as well as a higher susceptibility to noise due to reduced eye heights [3]. For a given signal amplitude, the eye heights for PAM4 are one-third that of NRZ resulting in a signal loss of approximately 9.5 dB. In practice, there can be further degradation in PAM4 signals due to nonlinearity [6]. The eye widths for PAM4 are comparable to those of NRZ, however, the top and bottom eyes in PAM4 are not identical to the middle eye since they include nonsymmetric transitions. There are only two distinct transitions in NRZ signals, while there are 12 in PAM4.

Since eye diagrams are aggregate representations of PAM4 signals, histogram analysis is usually used to derive fundamental eye parameters [9]. The problem with this approach is that solutions can vary with the chosen number of bins and bin sizes, and thus do not provide unique solutions. With histogram methods being sensitive to binning and data distributions and because oscilloscope manufacturers' algorithms are proprietary, we present an alternate method akin to reference [10] for NRZ modulation that always provides a unique solution by making use of a K-Means approach for clustering data into groups [11] and then utilizing the shorth estimator for determining "collective" modes for the data clusters [12].

Here, we propose an algorithm to calculate metrics that have been specifically developed for PAM4 standards [1, 2], including the time midpoint of the middle eye (T_{mid}), eye amplitudes (AV_{low}, AV_{mid}, and AV_{upp}), inner eye widths (H_{low}, H_{mid}, and H_{upp}), and inner eye heights (V_{low}, V_{mid}, and V_{upp}). These metrics are illustrated in Fig. 1. The asymmetry of the lower and upper eyes results in the widest portions of the eyes to be off-center with respect to the voltages. Thus, the inner eye widths are considerably smaller than the widest openings. Likewise, the inner eye heights, which are determined at T_{mid}, are smaller than the highest openings..

II. Proposed Algorithm

In this section, we describe our algorithm for analyzing a PAM4 eye diagram. We derive the fundamental parameters of the eye, and then using these calculated parameters, show how to calculate the time midpoint of the middle eye, eye amplitudes, inner eye widths, and inner eye heights.

A. Group Eye Diagram into Four Amplitude Sets

To determine levels 0, 1, 2, and 3, we begin by grouping the PAM4 eye diagram data based on amplitude (Y-axis) into four sets. This is done with a K-Means algorithm for clustering objects into groups [11]. The algorithm works by first randomly partitioning the data into k initial sets (in our case $k = 4$). Next, the centroid of each set is calculated. A new partition is constructed by associating each point with the closest centroid. Then, the centroids are recalculated for the new clusters, and the algorithm iterates until convergence occurs, which is accomplished when the points no longer switch clusters.

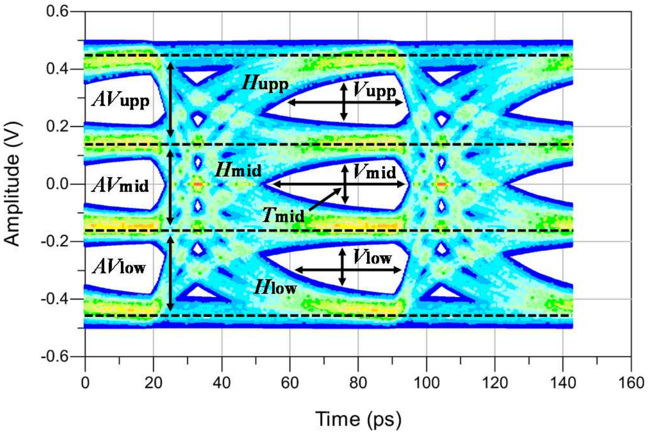

Fig. 1. Fundamental metrics of a PAM4 eye diagram.

B. Determine Approximate Values of Levels 0, 1, 2, and 3

Next, we determine the approximate values of levels 0, 1, 2, and 3 from the centers of their respective clusters' shortest intervals. For each level, the shortest interval that contains 50 % of the data is determined. Let $y_1, \ldots, y_n$ be the data in a level. The shortest interval is the one that produces the smallest of the following differences:

$$y_{(h)} - y_{(1)}, y_{(h+1)} - y_{(2)}, \ldots, y_{(n)} - y_{(n-h+1)} \tag{1}$$

where $h = \lfloor n/2 \rfloor + 1$, and $y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(n)}$ are the ordered observations. Reference [10] provides an example.

Instead of locating a single-number mode, we determine a concentration, or a "collective" mode, in the data. The arithmetic mean of this shortest interval is called the shorth estimator [12] and can be considered a mode estimator for approximating the value of the level. The shorth estimator can tolerate up to 50 % of contaminated data, thus making it robust against outliers. For each of the four levels, we determine their approximate values with this method and refer to them as $\bar{v}_{M0}, \bar{v}_{M1}, \bar{v}_{M2}$, and $\bar{v}_{M3}$, respectively.

C. Determine Time Midpoint of Middle Eye

With $\bar{v}_{M0}, \bar{v}_{M1}, \bar{v}_{M2}$, and $\bar{v}_{M3}$ determined, we compute the time midpoint of the middle eye. First, we select the middle eye data that falls within a small range of the central voltage values:

$$\frac{\bar{v}_{M1} + \bar{v}_{M2}}{2} \pm 0.01(\bar{v}_{M2} - \bar{v}_{M1}). \tag{2}$$

Next, we group this data based on time (X-axis) into left and right halves using the K-means algorithm. We determine the time midpoint by taking the average of the minimum value of the right-hand side ($t_{r,\min}$) and the maximum value of the left-hand side ($t_{l,\max}$):

$$T_{\mathrm{mid}} = (t_{r,\min} + t_{l,\max})/2. \tag{3}$$

D. Compute Levels 0, 1, 2, and 3 at the Centers of the Eyes

Established PAM4 standards [1, 2] specify levels are computed within the central 5% of the eye time midpoint. To stay consistent, we define the middle eye time midpoint as:

$$T_{\mathrm{mid}} \pm 0.025(t_{r,\min} - t_{l,\max}). \tag{4}$$

Next, the data located in the central 5 % of the eye diagram is separated into four clustered states. This is done by using the values $(\bar{v}_{\mathrm{M0}} + \bar{v}_{\mathrm{M1}})/2$, $(\bar{v}_{\mathrm{M1}} + \bar{v}_{\mathrm{M2}})/2$, and $(\bar{v}_{\mathrm{M2}} + \bar{v}_{\mathrm{M3}})/2$ as the vertical separators. For each cluster, we calculate the means ($\bar{v}_0, \bar{v}_1, \bar{v}_2$, and $\bar{v}_3$).

E. Determine Eye Amplitudes

The three eye amplitudes are defined as follows:

$$AV_{\mathrm{low}} = \bar{v}_1 - \bar{v}_0, \tag{5}$$

$$AV_{\mathrm{mid}} = \bar{v}_2 - \bar{v}_1, \tag{6}$$

and

$$AV_{\mathrm{upp}} = \bar{v}_3 - \bar{v}_2. \tag{7}$$

F. Determine Inner Eye Heights

In established PAM4 standards [6], an inner eye height is usually defined as the vertical distance across a symbol error rate ($\mathrm{SER} = 10^{-6}$) contour determined from the voltage cumulative distribution function (CDF) in a ±0.025 unit-interval (UI) time window centered on T_{mid}. In our algorithm, we do not make use of contour plots, so we compute an inner eye height by taking the difference between the minimum value of the upper level and the maximum value of the lower level within the ± 0.025 UI time window centered on T_{mid}.

Using the four clusters of voltages ($\{\boldsymbol{y}_0\}$, $\{\boldsymbol{y}_1\}$, $\{\boldsymbol{y}_2\}$, and $\{\boldsymbol{y}_3\}$) taken within the central 5 % of the PAM4 eye in Step 4 of this section, we calculate the inner eye heights of the lower, middle, and upper eyes as follows:

$$V_{\mathrm{low}} = \min(\{\boldsymbol{y}_1\}) - \max(\{\boldsymbol{y}_0\}), \tag{8}$$

$$V_{\mathrm{mid}} = \min(\{\boldsymbol{y}_2\}) - \max(\{\boldsymbol{y}_1\}), \tag{9}$$

and

$$V_{\mathrm{upp}} = \min(\{\boldsymbol{y}_3\}) - \max(\{\boldsymbol{y}_2\}). \tag{10}$$

G. Determine Inner Eye Widths

In established PAM4 standards [6], an inner eye width is usually defined as the horizontal distance across an SER = 10^{-6} contour determined from the CDF along eye edges halfway between an $\mathrm{SER} = 10^{-6}$ contour determined from the voltage CDF of the eye. Since we do not make use of contour plots, we compute an inner eye width by taking the difference between the minimum value of the right-hand time crossing and the maximum value of the left-hand time crossing within the ± 0.01 interval centered on the midpoint between the two voltage levels.

With $\bar{v}_0$ and $\bar{v}_1$ determined, we compute the inner eye width of the lower eye. First, we select the lower eye data that falls within a small range of the central voltage values:

$$\frac{\bar{v}_0 + \bar{v}_1}{2} \pm 0.01(\bar{v}_1 - \bar{v}_0). \tag{11}$$

Next, we group this data based on time (X-axis) into left and right halves using the K-means algorithm. We determine the inner eye width of the lower eye (H_{low}) by taking the difference between the minimum value of the right-hand side ($t_{r,\min}$) and the maximum value of the left-hand side ($t_{l,\max}$):

$$H_{\text{low}} = \left(t_{r,\min} - t_{l,\max}\right). \qquad (12)$$

Similar calculations can be performed for the inner eye widths of the middle eye (H_{mid}) and upper eye (H_{upp}).

III. Simulation Results

We generated eye diagrams from a PAM4 channel using a commercial simulator and compared their computed values with those obtained from our algorithm. The signal consisted of a pseudorandom binary sequence with a symbol rate of 28 Gbps and an amplitude of 0.5 Vpp centered at 0 V.

The transmitter consisted of an industry-standard IBIS-AMI (Input/Output Buffer Information Specification – Algorithmic Modeling Interface) behavioral model that takes an input signal and outputs a modified waveform. The eye diagram of the transmitted signal is illustrated in Fig. 1. This signal travels through a channel, which attenuates the signal, and is terminated with an IBIS-AMI receiver model. The eye diagram of the received signal, shown in Fig. 2, is visibly noisier and more distorted than the transmitted signal.

We compared the computed PAM4 eye-diagram metrics obtained from the simulator to their respective values with those determined with our algorithm. The simulator's algorithm is identical to the one utilized by the manufacturer's oscilloscopes [13]. The results for the transmitted eye are listed in Table I and the results for the received eye are listed in Table II. We can see the values determined by both compare well.

IV. Conclusions

We presented an approach for analyzing PAM4 eye diagrams, which relies on a K-Means algorithm in conjunction with a shortest interval location estimator. The performance metrics calculated by use of our algorithm agree well with those obtained from a commercial simulator. Our motivation for developing this algorithm was to create an independent, benchmark method that can function as a comparison tool and is conceivably amenable to an uncertainty analysis. The main advantages of our approach are that it always provides a unique solution that is insensitive to outliers and enables us to perform calculations without relying on proprietary algorithms. Furthermore, the computation speed is high – we processed 300,000 points in 12 sec.

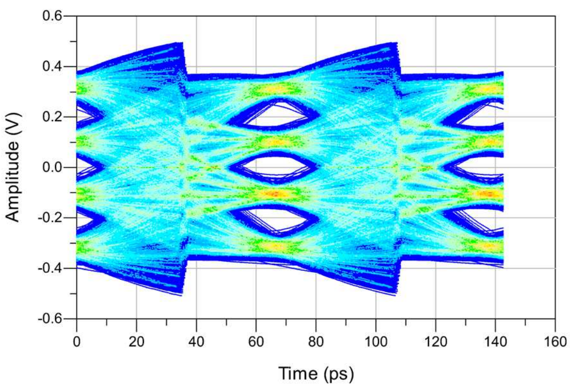

Fig. 2. PAM4 eye diagram of the received signal.

TABLE I.
COMPARISON OF SIMULATED METRICS
OF THE TRANSMITTED EYE

Metric	Commercial Simulator	Proposed Algorithm
T_{mid}	73.6 ps	73.5 ps
AV_{low}	0.282 V	0.285 V
AV_{mid}	0.281 V	0.285 V
AV_{upp}	0.274 V	0.286 V
H_{low}	37.2 ps	37.1 ps
H_{mid}	43.2 ps	43.2 ps
H_{upp}	37.5 ps	37.1 ps
V_{low}	0.145 V	0.145 V
V_{mid}	0.146 V	0.146 V
V_{upp}	0.146 V	0.146 V

TABLE II.
COMPARISON OF SIMULATED METRICS
OF THE RECEIVED EYE

Metric	Commercial Simulator	Proposed Algorithm
T_{mid}	65.7 ps	65.9 ps
AV_{low}	0.209 V	0.212 V
AV_{mid}	0.208 V	0.208 V
AV_{upp}	0.208 V	0.209 V
H_{low}	15.7 ps	15.0 ps
H_{mid}	21.8 ps	19.6 ps
H_{upp}	15.4 ps	14.3 ps
V_{low}	0.077 V	0.078 V
V_{mid}	0.072 V	0.073 V
V_{upp}	0.080 V	0.084 V

Acknowledgment

This work was supported by the U.S. Department of Commerce and is not subject to U.S. copyright. Jerome Cheron is also with the Department of Physics, University of Colorado, Boulder, CO USA.

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