Effect of Amplitude Mismatch on Entanglement Visibility in Photon-Pair Sources

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Abstract: Entangled photons produced by parametric down-conversion effectively have two down-conversion paths. Ideally, amplitudes of the two paths are matched. We show that the entanglement visibility is, to first order, insensitive to amplitude mismatch.

Entangled photon-pair sources are typically based on spontaneous parametric down-conversion (SPDC). In a type-II SPDC polarization-entangled source, the generated entangled state is

$$|\psi\rangle = \alpha|HV\rangle + \sqrt{1-\alpha^2}|VH\rangle, \tag{1}$$

where α is the amplitude ratio parameter, $|HV\rangle$ refers to horizontally polarized signal and vertically polarized idler, and $|VH\rangle$ refers to vertically polarized signal and horizontally polarized idler. The two terms in Eq. 1 correspond to two different biphotons that can be produced by two different physical paths [1], two different crystals [2], two different quasi-phasematching periods [3], or two different down-conversion paths in an aperiodically poled crystal [4,5].

In an ideal polarization-entangled state, the amplitudes of the two terms are equal. When measuring coincidence counts in the diagonal/anti-diagonal bases, in order to get zero coincidences (required for high entanglement visibility), there must be perfect destructive interference of the $|HV\rangle$ and $|VH\rangle$ photon paths (assuming no background noise). If the amplitudes in Eq. 1 are unequal, then this destructive interference is not possible. In several of the SPDC techniques mentioned above, the amplitude ratios are fixed during fabrication and it seems that good entanglement visibility can still be obtained even if the amplitudes are unbalanced [3–5].

High entanglement visibility requires high indistinguishability between the two possible down-conversion paths. We can quantify this indistinguishability by considering Hong-Ou-Mandel (HOM) interference between the two down-conversion paths. Imagine interfering the two down-conversion paths (or two biphotons) on a beam splitter. If the paths are indistinguishable, then no coincidences will be observed between the two outputs of the beam splitter, resulting in the well-known HOM dip. The closer the HOM dip is to zero, the better the indistinguishability and the better entanglement visibility we expect.

We can calculate the depth of the HOM dip by considering the joint spectral intensity distribution of two photons exiting opposite ports of a beam splitter, which is given by [6,7]

$$I(\omega_1,\omega_2) \propto \frac{1}{2}\left|C(\omega_1,\omega_2)e^{i(\omega_1 t_1+\omega_2 t_2)} - C(\omega_2,\omega_1)e^{i(\omega_2 t_1+\omega_1 t_2)}\right|^2, \tag{2}$$

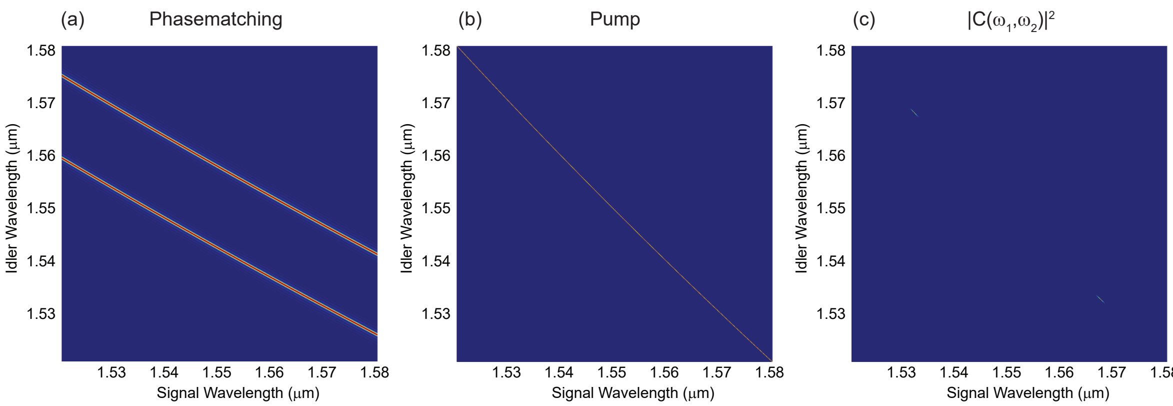

Fig. 1. The (a) phasematching and (b) pump functions are multiplied to produce (c) $|C(\omega_1,\omega_2)|^2$. The case of equal amplitude is shown.

where $C(\omega_1, \omega_2)$ is the joint spectral amplitude of the two photon wave-function incident on the beam splitter, ω_1 and ω_2 are the frequency of the two photons, and t_1 and t_2 are their corresponding arrival times. The coincidence detection rate for obtaining counts in both output ports of the beam splitter, R_c, is [6]

$$R_c \propto \iint d\omega_1 d\omega_2 I(\omega_1, \omega_2). \tag{3}$$

When the photons arrive at the same time ($t_1 = t_2$) and $C(\omega_1, \omega_2)$ is symmetric (i. e., $C(\omega_1, \omega_2) = C(\omega_2, \omega_1)$), then $R_c = 0$ and we expect perfect indistinguishability of the two down-conversion paths.

As a specific example, we studied the down-conversion process presented in Ref. [5]. A domain-engineered, lithium niobate crystal is designed to simultaneously phasematch both $|HV\rangle$ and $|VH\rangle$ processes for the wavelengths 775 nm $\longrightarrow$ 1533 nm + 1568 nm. We used a narrowband pump that constrains the idler frequency, ω_2, to be related to the pump, ω_p, and signal, ω_1, frequencies by $\omega_2 = \omega_p - \omega_1$. The $C(\omega_1, \omega_2)$ function is the product of the phasematching and pump functions, as shown in Fig. 1. The phasematching function has two lines for the two simultaneous down-conversion processes.

Using Eq. 3, we calculated the coincidence rate as a function of the mismatch in amplitudes the two down-conversion paths. Figure 2 plots R_c as a function of peak ratio, which is equal to $(1 - \alpha^2)/\alpha^2$. R_c is normalized to 1 as the peak ratio goes to zero. We see that when the peak ratio is near 1, R_c is near zero and is to first order independent of the peak ratio. This indicates that the two down-conversion paths are nearly indistinguishable and that entanglement visibility is not sensitive to small mismatches in amplitudes of the two down-conversion processes.

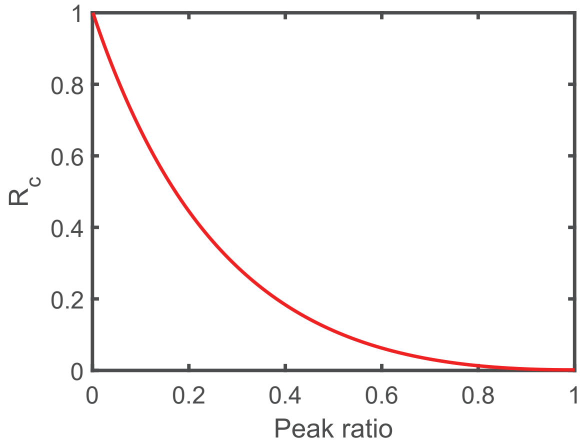

Fig. 2. Coincidence rate as a function of peak ratio.

In conclusion, we quantify the indistinguishability between two down-conversion paths by considering an analogy to HOM interference. We observed that mismatches in amplitudes between the $|HV\rangle$ and $|VH\rangle$ states do not significantly affect R_c and in turn, do not significantly degrade the polarization entanglement visibility. This observation explains how SPDC sources whose properties are fixed during fabrication can still have high polarization entanglement visibility even in the presence of fabrication imperfections.

References

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Introduction

Entangled photon pairs are a key resource for quantum information systems. Entangled photon pairs are typically produced by spontaneous parametric down-conversion (SPDC). High entanglement visibilities are obtained for many SPDC sources even when there are non-ideal fabrication or experimental conditions. In this work, we consider how amplitude mismatch in the SPDC process can still lead to high entanglement visibility.

Entangled Photon Pair Generation

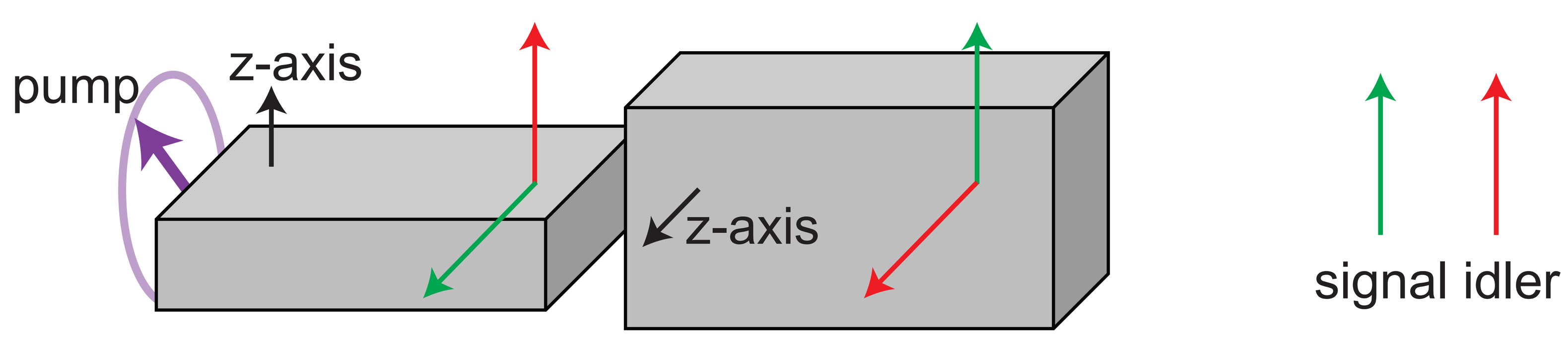
- In a Type-II, polarization-entangled SPDC source, the generated wavefunction is

$$|\psi\rangle = \alpha|HV\rangle + \sqrt{1-\alpha^2}\,|VH\rangle \qquad (1)$$

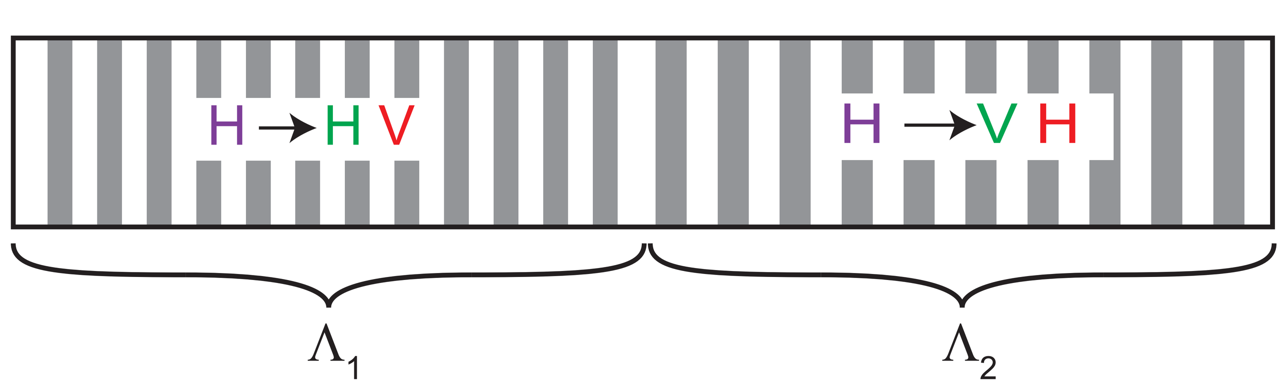
where $|HV\rangle$ refers to horizontally (H) polarized signal and vertically (V) polarized idler, $|VH\rangle$ refers to vertically polarized signal and horizontally polarized idler, and α is the amplitude parameter.

- When $\alpha^2 \neq 1/2$, the amplitudes are mismatched
- A state such as shown in (1) can be produced many ways including:

Two co-rotated SPDC crystals



Two quasi-phasematching (QPM) periods

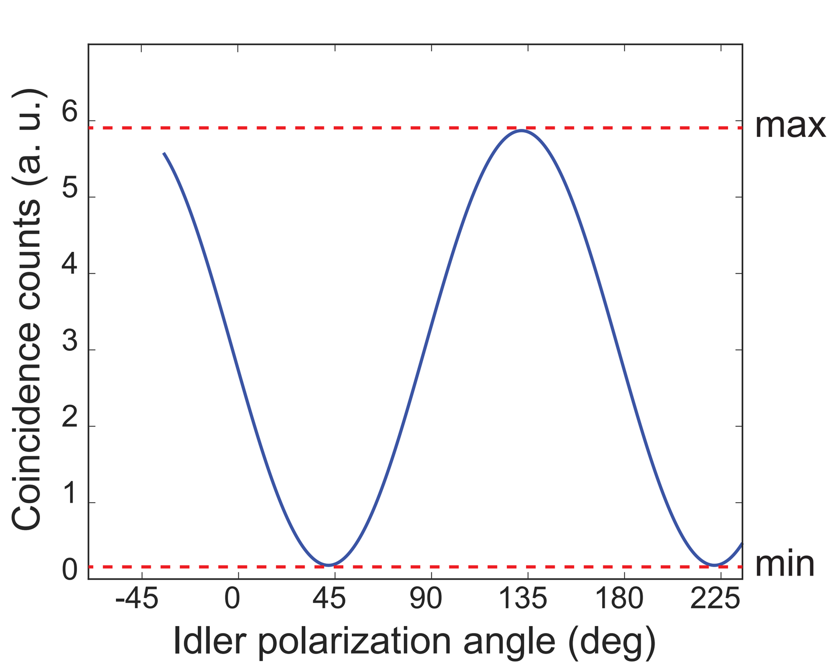


or using dual-periodically-poled or domain-engineered SPDC; see

C. Sun, et al., Opt. Lett. **44**, 5598 (2019)
P. S. Kuo, et al., OSA Continuum **3**, 295 (2020)

- The two terms in Eq. (1) correspond to two biphotons that take two different down-conversion paths (through two crystals, two QPM periods, etc.)
- In several schemes to generate Eq. (1), α is fixed during fabrication. In these cases, experiments have shown good entanglement visibility even when $\alpha^2 \neq 1/2$.

Entanglement Visibility and the HOM Interference Dip



- In a typical polarization entanglement measurement, coincidence counts are recorded while rotating the idler polarization and holding the signal polarization fixed
- The visibility, V, is calculated by

$$V = \frac{\max - \min}{\max + \min} \qquad (2)$$

- Maximum visibility ($V = 1$) is obtained when min = 0

- Neglecting noise, in Type-II SPDC, when the polarization analyzers for signal and idler are both H, there are no coincidences (min = 0)
- To get min = 0 when the signal is diagonally polarized, there must be perfect destructive interference between the two idler down-conversion paths, akin to perfect Hong-Ou-Mandel (HOM) interference

→ Examine the HOM interference dip to understand visibility

Spectrally Resolved HOM Interference

- HOM interference is a measure of indistinguishability. The HOM dip allows quantification of the effect of mismatch
- The joint spectral intensity distribution of two photons exiting opposite ports of a beam splitter is[1]

$$I(\omega_1, \omega_2) \propto \frac{1}{2}\left|C(\omega_1, \omega_2)\exp[i(\omega_1 t_1+\omega_2 t_2)] - C(\omega_2, \omega_1)\exp[i(\omega_2 t_1+\omega_1 t_2)]\right|^2 \qquad (3)$$

$C(\omega_1, \omega_2)$ = Joint spectral amplitude of the two-photon wave-function incident on the beam splitter
ω_1, ω_2 = Frequencies of the two photons
t_1, t_2 = Corresponding arrival times of the two photons

- The coincidence detection rate, R_c, for obtaining counts in both output ports of the beam splitter is[1]

$$R_c \propto \iint I(\omega_1, \omega_2)\, d\omega_1 d\omega_2 \qquad (4)$$

[1]T. Gerrits, et al., Phys. Rev. Lett. **91**, 013830 (2015)

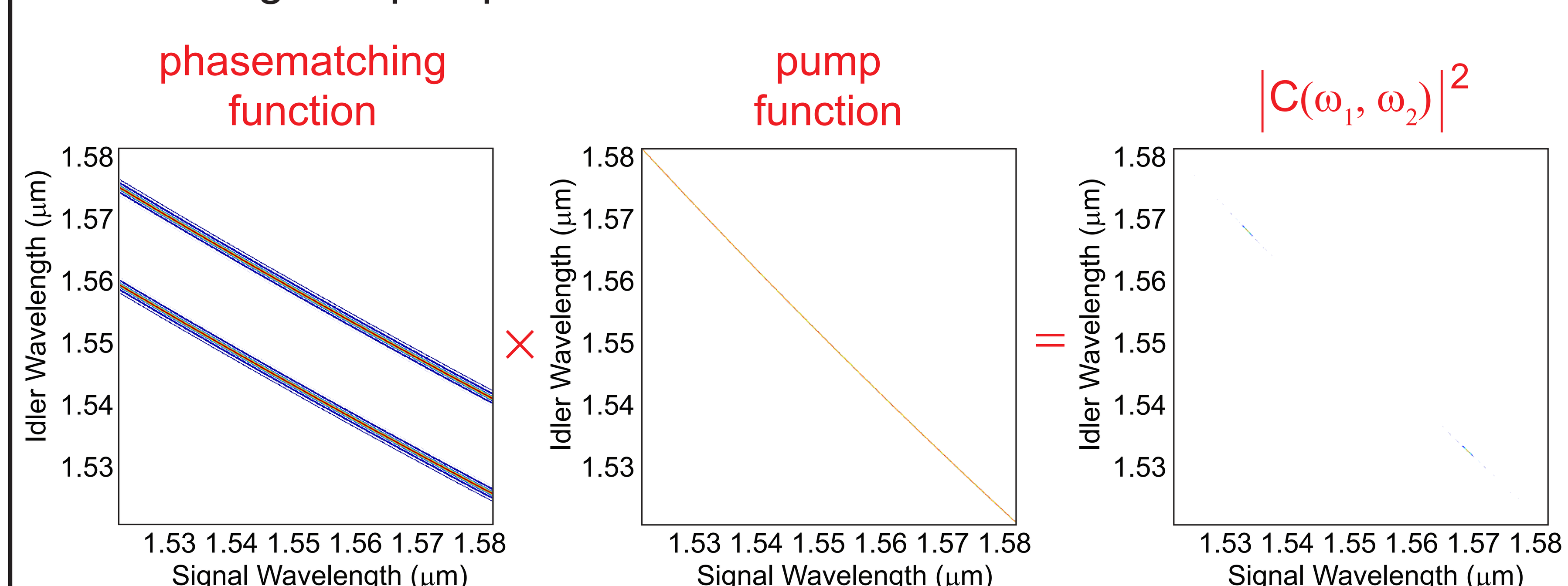
- Plotting R_c vs. $t_1 - t_2$ traces out the HOM dip

Joint Spectral Amplitude

- We modeled Type-II SPDC in periodically poled lithium niobate having two QPM periods and a narrowband pump

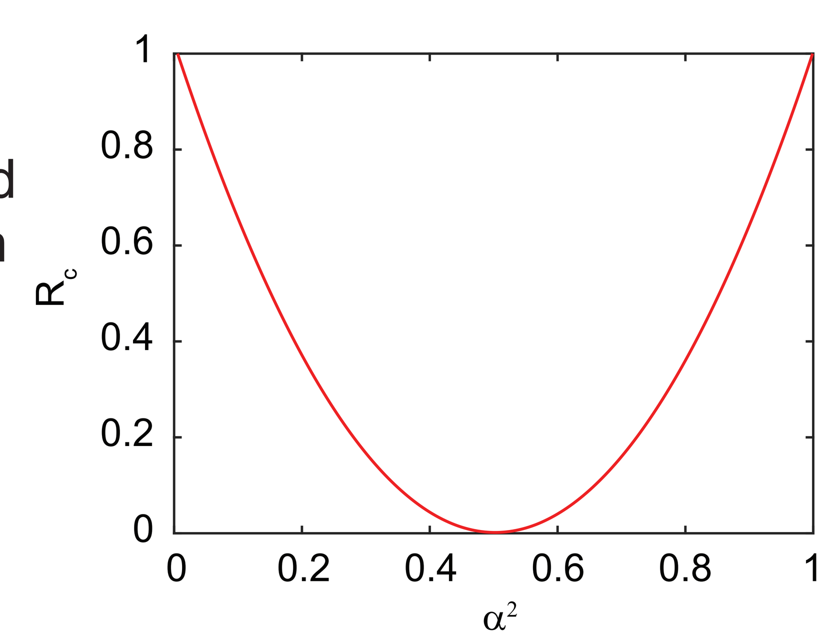
775 nm ⟶ 1533 nm + 1568 nm

- The joint spectral amplitude, $C(\omega_1, \omega_2)$, is the product of the phase-matching and pump distributions



HOM Dip when $\alpha^2 \neq 1/2$

- Using Eq. (3) and (4), we calculated the minimum of the HOM dip (given by R_c with t_1=t_2) when α^2 is varied
- At $\alpha^2 = 1/2$, there is perfect destructive HOM interference and $R_c = 0$



- Near $\alpha^2 = 1/2$, R_c varies quadratically with α^2, which means that to first order, the HOM dip minimum is independent of changes in α^2, i.e. amplitude mismatch

→ By the previous arguments, the entanglement visibility is to first order independent of the amplitude mismatch

Conclusions

- We quantify the indistinguishability between the two down-conversion paths by considering the analogy to HOM interference
- The HOM dip and the entanglement visibility are robust to small amounts of amplitude mismatch (deviations from equal amplitudes)
- Small mismatches in amplitudes, such as those caused by non-ideal fabrication or experimental conditions, do not seriously degrade the entanglement visibility