Retrieval of the Refractive Index Structure Parameter from Frequency Comb Timing Jitter Data

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Abstract: We measure optical timing jitter due to atmospheric turbulence using frequency comb lasers. Retrieving the refractive index structure parameter, C_n^2, from this data, we find windspeed-dependent agreement to C_n^2 values derived from in situ instruments. Work of the U.S. government, not subject to copyright.

1. Introduction

As light propagates through the turbulent atmosphere, variations in the refractive index of air cause distortions in both the amplitude and phase. While technologies such as free-space optical communication are heavily impacted by amplitude scintillation, phase noise poses an additional concern for coherent LIDAR, long-baseline interferometry, and free-space time transfer [1]. Here we measure the phase noise, which manifests as frequency comb pulse timing jitter, over a 280 meter round-trip path. We simultaneously measure mean values and turbulent fluctuations of pressure, temperature, and the 3D wind vector with a quartz-crystal barometer and ultrasonic anemometer-thermometers ("sonics") mounted on a meteorological tower. From the resulting timing jitter data we extract the refractive index structure parameter, C_n^2, and compare it to C_n^2 derived independently from the in situ measurements. We find that for windspeeds greater than 1.5 m/s there is good agreement between these C_n^2 estimates.

2. Experimental Configuration

We used two fiber frequency comb lasers centered at 1560 nm to measure the impact of turbulence on pulse timing jitter. The pulses from one comb were sent through the atmosphere and combined with pulses from the other comb for heterodyne detection using linear optical sampling following [2], resulting in a time-series of interferogram arrival times that map directly to pulse timing. The meteorological tower was positioned approximately 2 m from the beam path, and 40 m from the laser terminal. Measurements were performed above flat ground covered with low-growing grasses and brush. During the experiment, conditions were sunny and winds were light and variable.

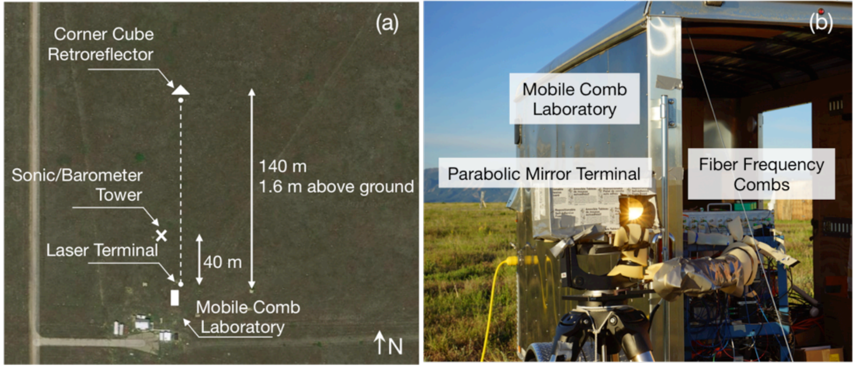

Fig. 1: (a) Aerial schematic of experimental setup located at Table Mountain north of Boulder, Colorado. We used frequency comb lasers housed in a mobile comb laboratory to measure the path-integrated timing jitter due to turbulence across the 280 m round-trip path. (b) Photograph of the frequency combs in the mobile comb trailer. Also shown is the parabolic mirror terminal used to launch laser light from the fiber into free-space.

3. Results

We gathered frequency comb data over several days in June of 2019 while the in situ sensors ran continuously. On the first and last day, we took measurements over a short ($L < 1$ m) free-space path to establish a system noise floor. The power spectral density (PSD) of the timing jitter from June 7th is shown in Figure 2a with the frequency axis normalized into wave numbers using $K=(2\pi f)/V$ where f is frequency and V is the absolute value of the wind velocity perpendicular to the beam path. We compare our data to three models of timing jitter PSDs: the Greenwood-Tarazano model [3], the von Kármán model [4], and a classical power law model [4]. All three models are scaled by the sonic-derived windspeed and C_n^2, yet vary in the filter functions used to account for the outer scale at low wave numbers. Despite having no free parameters, we see good agreement between all three models and our log-averaged timing jitter data in the region of 0.5 rad/m < K < 50 rad/m. At lower wave numbers, the Greenwood-Tarazano model having an outer scale of L_0=1.6 m, chosen to match the beam path height above ground [5], agrees well with the data while the von Kármán model underestimates the power spectral density. The -8/3 power law has no rolloff because it does not account for outer scale turbulence effects. At wave numbers above K = 50 rad/m we begin to see noise due to data fading and comb measurement noise.

To compare the turbulence seen across the integrated laser path to the point measurements taken at the meteorological tower, we fit PSDs of one-minute intervals of data to the -8/3 power law to retrieve C_n^2. The model for the timing jitter PSD is given by:

$$S_\tau(K) = (4)(2)(0.0163)n_g^2 c^{-2} L\, C_n^2 \left(\frac{KV}{2\pi}\right)^{-8/3} V^{5/3} \qquad (1)$$

c is the speed of light, L is the one-way path length, and n_g is the group refractive index approximated to 1 in our fitting. This model is consistent with equation 35 in section 4.52 of [4] for the phase spectrum, with the substitution $KV/(2\pi) = f$ and an additional factor of 4 used to account for the perfect correlation between turbulence on the paths to and from the retro and single terminal used for both transmitting and receiving. As described above, the classical -8/3 law fits the data for intermediate wave numbers, so we fit between K = 0.63 rad/m and K = 63 rad/m, corresponding to length scales between 10 cm and 10 m. We fix the windspeed parameter using the time-averaged windspeed from the sonics and float C_n^2. The optical C_n^2 estimates are shown in Figure 2b (black dots) along with C_n^2 estimates based on the in situ measurements (blue line). As windspeed values decreases towards 0 m/s, the singularity in the model causes the fit value of C_n^2 to artificially inflate to maintain a good fit with the data causing a systematic over-estimation of C_n^2 for low windspeed values. For moderate windspeeds (0.5 m/s < V < 1.5 m/s), the 1-min estimates of C_n^2 agree within half an order of magnitude, and for higher windspeeds (> 1.5 m/s) they agree within about a factor of 2.

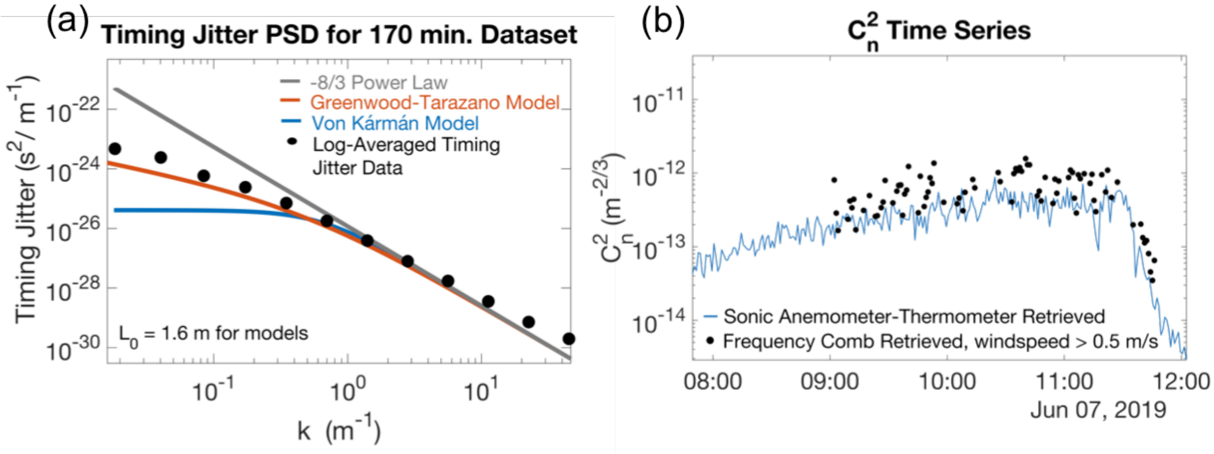

Fig. 2: (a) Power spectral density of the log-averaged timing jitter data. All three models shown were created using the average windspeed $\langle v \rangle$=0.63 m/s and $\langle C_n^2 \rangle$=3.2*10^{-13} $m^{-2/3}$ derived from collocated sonic and barometer measurements. The -8/3 power law model with C_n^2 and windspeed as the free parameters is derived from Ref. [4], and is the same model used to estimate C_n^2 from the the comb timing jitter data. Also shown are two models that account for the outer scale, the Greenwood-Tarazano model [3] and the von Kármán model [4]. For both models we set the outer scale to L_0=1.6 m. (b) Time series of C_n^2 retrieval using the sonic data (blue line) and comb timing jitter data (black dots). Comb timing results for wind speeds < 0.5 m/s were not plotted.

In summary, we have presented multiple hours of phase-continuous, atmospherically-induced timing jitter measurements taken with optical frequency combs across a path characterized by meteorological instruments. We compared the low frequency behavior of two models, finding the Greenwood-Tarazano model agrees well with the data while the von Kármán model underestimates the low-frequency power for the same outer scale. Finally, we compared C_n^2 values retrieved from comb timing jitter to those gathered from sonics, finding agreement of within a factor of 2 for windspeeds higher than 1.5 m/s.

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