

# *Frequency Invariant Beampatterns for Wideband Synthetic Aperture Channel Sounders*

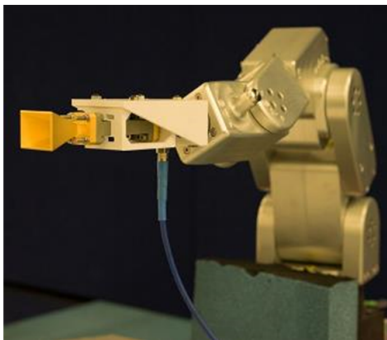
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18 May 2020

- Synthetic Apertures
- Spatial Fourier Transform Properties
- Wideband Regime
- True Time Delay Beamsteering and PADP Output
- Frequency Invariant Beamformer
- Measured Results
- Conclusion

# Wideband Synthetic Aperture



A robot arm moves the receive horn precisely along points on a sampling lattice

In spherical coordinates the beamformed output of the synthetic aperture is given by,

$$B(\theta, \phi) = E(\theta, \phi) \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} w_{mn} e^{jk(x_m \sin \theta \cos \phi + y_n \sin \theta \sin \phi)}$$

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This equation can be rewritten as a 2-D spatial Fourier Transform by using sine space coordinates  $u = \sin \theta \cos \phi$  and  $v = \sin \theta \sin \phi$

$$B(u, v) = E(u, v) \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} w_{mn} e^{jk(md_x u + nd_y v)}$$

## Fourier shift property

When the main beam is steered to a direction  $(u_0, v_0)$  the linear phase taper  $e^{-jk(md_x u_0 + nd_y v_0)}$  is applied across the aperture. The main beam will shift by an angular distance equal to the slope of the linear phase taper in the  $u$  and  $v$  directions.

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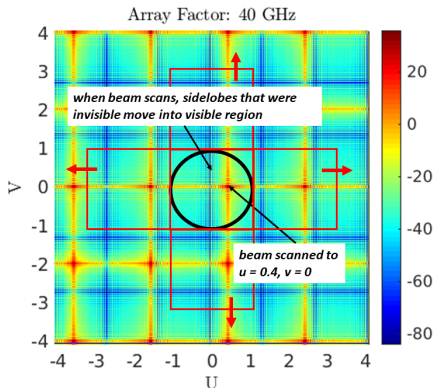
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## Fourier Scaling Property

The Fourier scaling property states that changing the sample spacing of a 2D discrete sequence  $h(m, n)$  expands or contracts the output of the Fourier transform  $H(u, v)$  according to

$$H\left(\frac{u}{a}, \frac{v}{b}\right) \Longleftrightarrow \frac{1}{|ab|} h(am, bn)$$

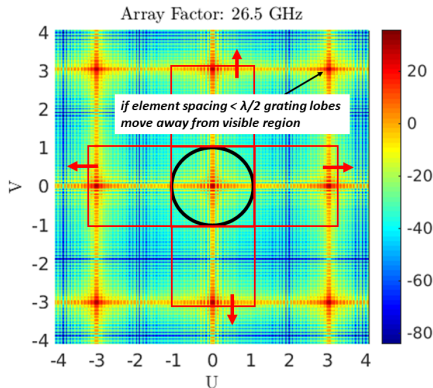
# Array Factor: 40 GHz



Spatial Fourier Transform and the array factor at 40 GHz for a 35-by-35 planar aperture with  $\lambda/2$  spacing at 40 GHz (3.7 mm)



# Array Factor: 26.5 GHz



At a lower frequency if the distance between spatial samples remains fixed the inter-element spacing  $< \lambda/2$

In wideband regimes where the beam steering phase taper is computed for a single center frequency but applied to the aperture over a wider bandwidth, the main beam will squint, or point to different directions as the operating frequency changes. Beam squint is an undesired effect that can be mitigated by using true time delay beam steering or a frequency invariant beamformer

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True time delay beam steering applies a frequency independent time delay or a frequency dependent linear phase shift between array elements to steer the beam and can be easily implemented on synthetic apertures to create a Wideband Power Angle Delay Profile (PADP)

Array output vector  $\mathbf{y}(\omega_k)$  at each frequency  $\omega_k$  and desired beam pointing direction  $(u_0, v_0)$

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Beamform the array output vector  $\mathbf{y}(\omega_k)$  at each frequency by forming the dot product  $b(\omega_k; u_0, v_0) = \mathbf{w}(\omega_k; u_0, v_0)^H \mathbf{y}(\omega_k)$

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Compute the Inverse Fourier Transform (temporal) to obtain the beam output (directional PDP),  $x(\tau_k; u_0, v_0) = IFT[b(\omega_k; u_0, v_0)]$

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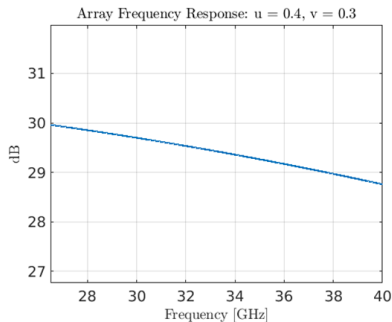
For a fixed delay,  $\tau = \tau_0$ ,  $x(\tau_0; u, v)$  is the spatial frequency spectrum of all signal sources impinging on the array and can be used to estimate angles of arrival



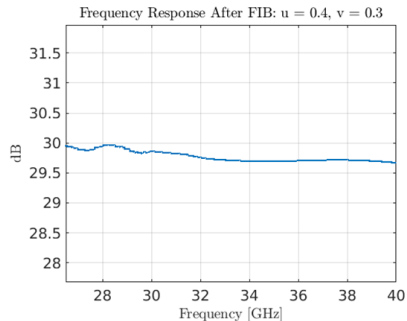
The frequency invariant beamformers proposed in this paper replace the phase steering vectors  $\mathbf{w}(\omega_k; u_0, v_0)$  with optimized weight vectors computed at every frequency for the desired steering direction

## Frequency Response

**Without FIB**



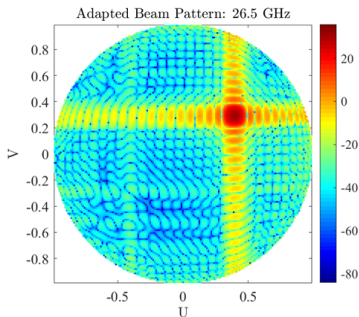
**With FIB**



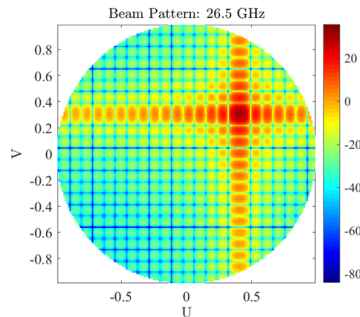
- *The array frequency response is equalized and more nearly constant with FIB*
- *Additional results show a much more dramatic effect on array sidelobes*

## Frequency 1

**With FIB**



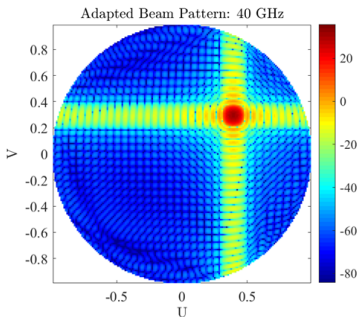
**Without FIB**



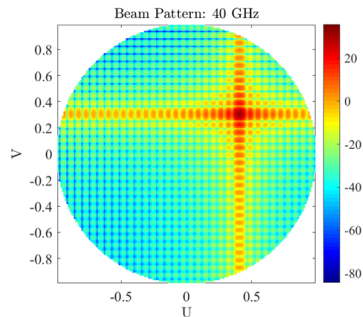
- *Want the width of the main beam to be constant versus frequency*
- *The natural tendency is for the beamwidth to decrease with frequency because the array gets electrically larger*

## Frequency 1351

**With FIB**



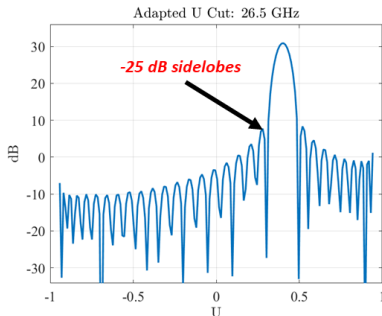
**Without FIB**



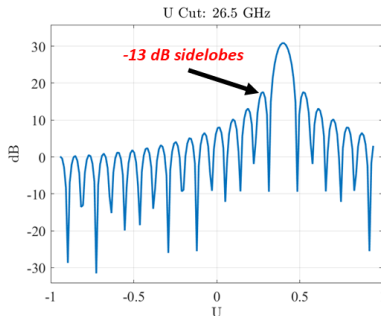
- *The beamformer output has maintained a constant beamwidth*

## U-Cut: Frequency 1

**With FIB**



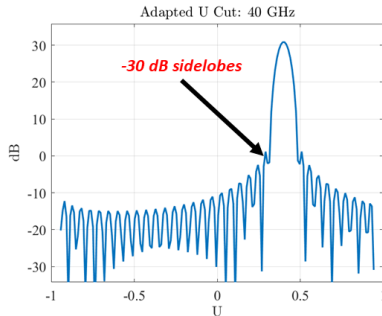
**Without FIB**



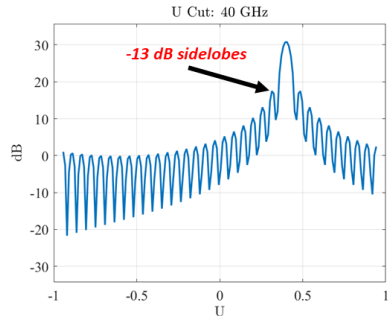
- *Peak of main beam is held constant but the FIB sidelobes are much lower because the beamwidth was allowed to increase slightly beyond the Rayleigh limit  $\sim \lambda/D$*
- *The lower spatial sidelobes will attenuate received energy due to multipath fading (which is incoming from many random directions outside the mainbeam)*

## U-Cut: Frequency 1351

**With FIB**

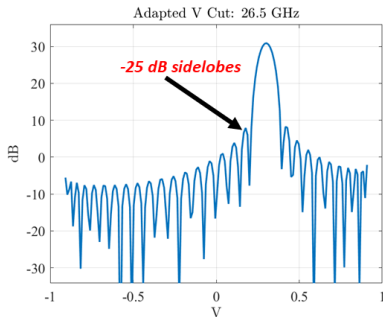


**Without FIB**

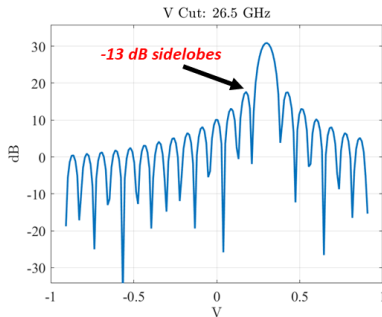


## V-Cut: Frequency 1

**With FIB**

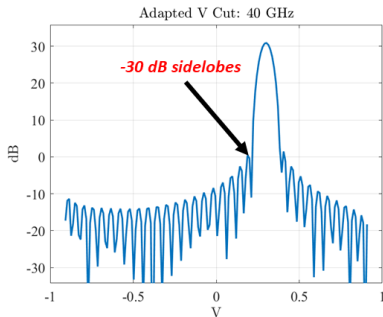


**Without FIB**

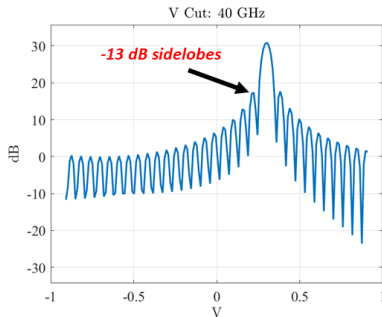


## V-Cut: Frequency 1351

**With FIB**

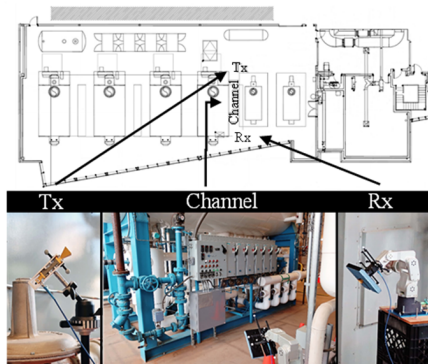


**Without FIB**



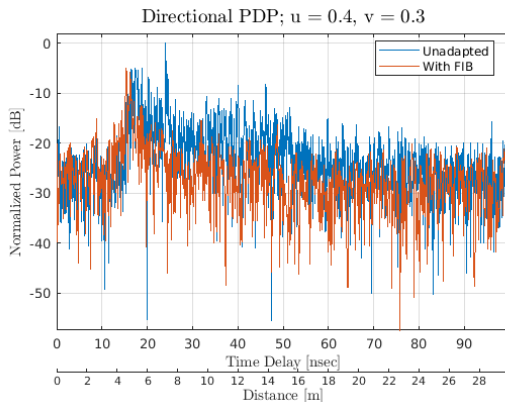


# *Dense Multipath Environment*



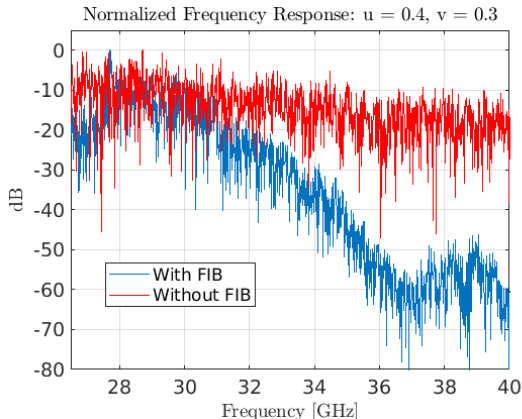
Central Utility Plant (CUP) is highly reflective environment

# *PDP Comparison*



With a FIB the PADP exhibits less diffuse multipath energy that enters through the array sidelobes

# Frequency Response Comparison



With a FIB the measured frequency response is more band-limited

- In the CUP data there is a clear multipath fading profile with a duration of approximately 35 nsec that corresponds to many superimposed multipath reflections arriving from random directions. This energy is primarily received through the sidelobes of the beampattern and is not confined to the angular width of the main beam. The multipath fading profile using a frequency invariant pencil beam is attenuated due to the reduced sidelobes of the beampattern.
- With an unadapted beampattern, the channel frequency response appears nearly flat and noise-like due to the contributions of the received diffuse multipath energy. After applying the frequency invariant pencil beampattern, the frequency response of the beam output is much more band limited since the long duration multipath fading has been attenuated by the lower sidelobes of the beamformer