Operational Measurement Uncertainty and Bayesian Probability Distribution

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Summary:
The JCGM documents have undermined the operational concept of uncertainty in measurement established by the GUM and restored the pre-GUM practice of stating possible error relative to the true value, supposedly to align with Bayesian interpretation. It is possible to revise the JCGM documents to agree with the operational view of uncertainty in measurement as well as align them with Bayesian thinking.

Keywords: Bayesian inference, Metrology, Probability, True value, Uncertainty in measurement

Introduction

The signal contribution of the 1993 Guide to the Expression of Uncertainty in Measurement (GUM) is the operational concept of measurement uncertainty [1]. It departs from the earlier views which were about stating possible error relative to the true value. The Joint Committee for Guides in Metrology (JCGM) documents have restored the pre-GUM view by introducing a coverage interval as the dominant expression of uncertainty, where a coverage interval is an interval containing the true value with a stated probability [2]. So, we reiterate the operational concept of uncertainty in measurement. The JCGM-101 states that a coverage interval corresponds to Bayesian interpretation [2]. The JCGM is developing a new GUM aligned with Bayesian interpretation [3]. We offer an alternative interpretation of a Bayesian probability distribution that corresponds to the operational measurement uncertainty. This short paper is based on references [4-7].

True value

The earliest attempts to quantify uncertainty in measurement were based on statistical estimation. A large part of statistical estimation is about predicting an outcome which could become known later. The object of prediction is called the true value. In metrology, it is difficult to define the idea of true value [8]. The JCGM-200 defines true value as *a quantity value consistent with the definition of a quantity* [9]. A quantity value is a known value which is assigned by definition or by measurement. Thus, a true value cannot be a quantity value. The JCGM-200 definition of true value is indefensible. In metrology, true value and error, however defined, are unknowable; therefore, they cannot be a basis for any decision or action.

Operational uncertainty in measurement

A measurand is a magnitude of a property of something (a phenomenon, body, or substance) that is intended to be measured. A result of measurement consists of the measured value (best assigned value) and its associated uncertainty. The essential GUM is the GUM excluding Annex G and its links with the rest. The essential GUM guides us to think of measurement not as estimating (determining) a true value but as assigning a result of measurement to describe (characterize) the measurand. Uncertainty in measurement is *a parameter, associated with a result of a measurement (measured value), that characterizes the dispersion of the values that could reasonably be attributed to the measurand* [1]. Here, the word ‘reasonably’ refers to the bases for the assigned result of measurement being reasonable. Uncertainty in measurement is an operational concept that does not refer to the idea of true value. The GUM states the following. *The focus of this Guide is on the measurement result and its evaluated uncertainty rather than on the unknowable quantities “true” value and error (see Annex D). By taking the operational views that the result of a measurement (best assigned* *value) is simply the value attributed to the measurand and that the uncertainty of that result is a measure of the dispersion of the values that could reasonably be attributed to the measurand, this Guide in effect uncouples the often-confusing connection between uncertainty and the unknowable quantities "true" value and error* [1]. Uncertainty in measurement is an evaluated expression. It does not include uncertainty from unrecognized components of uncertainty and from those components which are believed to have negligible contribution.

A measuring system is required for measurement of an unknown quantity. It compares the unknow quantity with an appropriate reference value provided by a measurement standard (etalon). The reference values are intended to remain constant over time and space and form a coherent system. In metrology, only an observed deviation of a measured value from a reference value is relevant. Measuring instruments and material measures that form the measuring system are maintained through a hierarchy of calibrations using measurement standards of progressively increasing metrological qualities such as reference standards, secondary standards, and primary standards. The higher-level measurement standards are calibrated with national and international measurement standards. National and international measurement standards and measuring techniques of highest metrological qualities are assessed by inter-comparison for metrological compatibility. Measured value that are traceable to the same reference values (through hierarchical chains of calibrations) are metrologically comparable in time and space. The worth of a result of measurement for a quantity is determined by metrological compatibility (lack of significant difference) with independent results for the same quantity without invoking true values. Every result of measurement should be supported with the measurement function and complete uncertainty budget, so incompatible results can be investigated [4].

Operational Bayesian probability distribution

Suppose Θ is a random variable with a probability distribution $\pi(\Theta)$ which expresses the state of knowledge about a quantity. The domain of $\pi(\Theta)$ is the range of possible value for that quantity. Suppose (θ_l, θ_h) is a result of measurement expressed as an interval for that quantity where θ_l and θ_h are any two possible values of Θ and $\theta_l < \theta_h$. Now suppose $\tau[\Theta]$ is a conceptual true value of that quantity. The theoretical Bayesian interpretation of $\pi(\Theta)$ is that it describes the probability that the true value $\tau[\Theta]$ lies within the interval (θ_l, θ_h). This interpretation agrees with the JCGM-101 idea of a coverage interval, but it disagrees with the operational concept of measurement uncertainty established by the GUM. An operational interpretation of $\pi(\Theta)$ is that it is a probability (degree of belief) distribution for the values that could be attributed to the quantity in view of the presently available information. Thus, $\pi(\Theta)$ describes the probability associated with a result of measurement expressed as the interval (θ_l, θ_h). The operational interpretation agrees with the essential GUM and aligns with Bayesian thinking.

References

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