Enhancing LAA Co-existence Using MIMO Under Imperfect Sensing

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***Abstract*—To meet the ever growing wireless network demands, in terms of subscribers and data throughput, operating long term evolution (LTE) in unlicensed bands, such as license assisted access (LAA), is introduced as a promising solution. However, the LAA network must co-exist with incumbent IEEE 802.11 systems. In this paper, we use MIMO operation to boost LAA-WLAN coexistence. We take the impact of imperfect sensing into account and analyze key performance indicators (KPIs), such as throughput, probabilities of successful transmissions, and collisions. Moreover, we characterize the optimal LAA contention window size to maximize the LAA sum rate while assuring Wi-Fi throughput above a predetermined threshold. Numerical results show the efficiency of the introduced algorithm and that if the parameters are appropriately selected, the throughput of both systems increases.**

***Index Terms*—Beamforming, coexistence, contention window, imperfect sensing, LTE-LAA, MAC layer, PHY layer, WLAN.**

I. Introduction

The big data era is being shaped with the ongoing growth of commercial data services, with mobile wireless networks constituting a major source of data. Wireless communication is becoming tightly integrated in our daily lives; especially with the global spread of laptops, tablets, smartphones, video streaming, and online social networking applications. This globalization has paved the way to dramatically increase wireless network dimensions in terms of subscribers and amount of data traffic. Cisco Systems forecasts that the number of mobile-connected devices per capita will reach 3.6 by 2022 and global mobile data traffic will increase sevenfold between 2017 and 2022 [1]. Licensed spectrum bands are high-priced and the theoretical potential of the physical layer is almost achieved. However, there are still some licensed bands that are under utilized. As a consequence, sharing the under utilized licensed spectrum among networks and using the unlicensed spectrum bands are inescapable.

Formerly, the unlicensed bands were dominated by Wi-Fi traffic, and occasionally used by commercial cellular carriers for offloading data that would otherwise have been sent via long term evolution (LTE) in licensed spectrum. Lately, mobile network operators have paid close attention to operate LTE in unlicensed bands, such as license-assisted access (LAA), in addition to data offloading. However, this coexistence may have an enormous influence on Wi-Fi operation and create a number of challenges for both Wi-Fi and LTE networks to constructively share the spectrum. Having a good comprehension of these challenges demands a deep dive into the operations of both networks in the MAC and physical layers.

In this paper we boost LAA-Wi-Fi coexistence by employing multiple-input multiple-output (MIMO) operation and reducing the interference between LAA and Wi-Fi networks. We take the impact of imperfect sensing (the networks' cross-technology probability of detection and false alarm) into account and obtain the LAA and Wi-Fi throughput as a function of the sensing errors. Moreover, by maximizing the LAA sum rate while assuring Wi-Fi throughput above a predetermined threshold, we find the optimum LAA contention window size.

The remainder of this paper is organized as follows. Section II describes the system model and assumptions required for our analysis. Section III presents the problem formulation and analysis. Section IV explains the impact of the sensing threshold and imperfect spectrum sensing in a coexistence scenario. Numerical results are shown in Section V. Finally, in Section VI, a summary of the results and some concluding remarks are presented.

Notation: Throughout this paper, normal letters are used for scalars. Boldface capital and lower case letters denote matrices and vectors, respectively. The Hermitian of a complex vector $\mathbf{a}$ is denoted by $\mathbf{a}^H$. $\mathcal{CN}(0,\sigma^2)$ represents a circularly symmetric complex Gaussian random variable and $\mathbb{E}[\cdot]$ denotes the expectation operator. See Table I for major notations and symbols used in this paper.

II. System Model and Assumptions

We consider a downlink multi-cell coexistence scenario where $n_L \in \mathcal{L} \triangleq \{n_\ell|\ell = 1,2,\ldots,L\}$ LTE-LAA eNodeBs and $n_W \in \mathcal{W} \triangleq \{n_w|w = 1,2,\ldots,W\}$ Wi-Fi access points (APs) share the same unlicensed band $k \in \mathcal{K} \triangleq \{1,2,\ldots,K\}$ in an industrial, scientific, and medical (ISM) radio band[1].

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[1]Note that the focus of this study is the operation of cellular base stations in an unlicensed band. However, these base stations may have permission to utilize a licensed band as well.

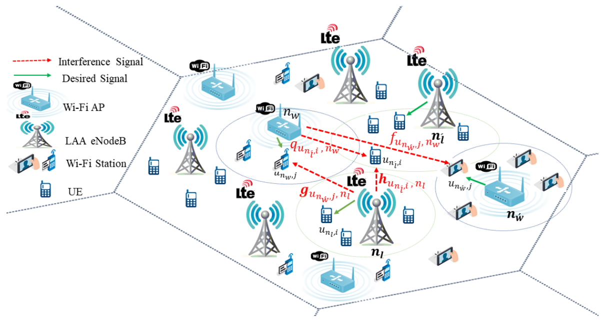

Fig. 1: Multi-cell system model in a coexistence scenario

We assume a coexistence scenario in which both Wi-Fi and LAA are in the saturated traffic condition. We also assume that LAA uses orthogonal frequency-division multiple access (OFDMA). Hence, we equally divide the unlicensed band k into N_k sub-channels, i.e., $\mathcal{N}_k = \{1, 2, \ldots, N_k\}$, where each sub-channel has a bandwidth of B_k. Moreover, we assume that each unlicensed sub-channel can be shared between the UEs and Wi-Fi stations in a time-sharing fashion.

A. Network Topology

We also assume that the eNodeBs and APs are randomly distributed over each cell, while Wi-Fi clients (LTE user equipment (UEs)) are distributed around each AP (eNodeB) independently and uniformly, as depicted in Figure 1. The eNodeB n_ℓ is equipped with N_t transmit antennas and simultaneously serves a set of $|\mathcal{U}_{n_\ell}|$ single antenna UEs on the unlicensed band, where $\mathcal{U}_{n_\ell} = \{u_{n_\ell,1}, u_{n_\ell,2}, \ldots, u_{n_\ell,|\mathcal{U}_{n_\ell}|}\}$. Since multiple eNodeBs and UEs are operating on the same unlicensed band as Wi-Fi devices, this coexistence may have an enormous influence on the Wi-Fi operation, and vice versa. Hence, multiple transmit antennas at each eNodeB can provide beamforming and interference-nulling opportunities for the associated UEs and the Wi-Fi nodes, respectively. We assume that the eNodeB n_ℓ transmits with power p_{n_ℓ} and the user association is based on the received power (*i.e.,* each UE will be assigned to the eNodeB that provides it with the highest power).

The n_w-th AP is equipped with a single antenna and serves a set of $|\mathcal{U}_{n_w}|$ single antenna Wi-Fi stations, one station at a time, where $\mathcal{U}_{n_w} = \{u_{n_w,1}, u_{n_w,2}, \ldots, u_{n_w,|\mathcal{U}_{n_w}|}\}$. Similar to the LAA network, each Wi-Fi station will be associated with the Wi-Fi AP that delivers the largest received power to it. We also assume that the Wi-Fi AP n_w transmits with power p_{n_w}.

B. Channel Model

The channel (propagation) coefficients between the n_ℓ eNodeB and the $u_{n_{\ell'},i}$ UE form the channel matrix $\mathbf{h}_{u_{n_{\ell'},i,n_\ell}} = \sqrt{\beta_{u_{n_{\ell'},i,n_\ell}}}\tilde{\mathbf{h}}_{u_{n_{\ell'},i,n_\ell}} \in \mathbb{C}^{N_t}$ where $\beta_{u_{n_{\ell'},i,n_\ell}}$ is a large-scale fading coefficient that depends upon the shadowing and distance between the corresponding UE and eNodeB. The large-scale fading coefficient is denoted by $\beta_{u_{n_{\ell'},i,n_\ell}} = \psi_{u_{n_{\ell'},i,n_\ell}} d^{-\alpha}_{u_{n_{\ell'},i,n_\ell}}$, where $d_{u_{n_{\ell'},i,n_\ell}}$ is the distance between the $u_{n_{\ell'},i}$ UE and the n_ℓ eNodeB, α is the path-loss exponent,

TABLE I: Nomenclatures and Notations Used

Notation	Description
n_ℓ	The ℓ-th eNodeB
n_w	The w-th AP
$u_{n_\ell,i}$	The i-th UE associated to the n_ℓ-th eNodeB
$u_{n_w,j}$	The j-th Wi-Fi client associated to the n_w-th AP
$\beta_{u_{n_{\acute{\ell}},i,n_\ell}}$	Large-scale fading coefficient between $u_{n_{\acute{\ell}},i}$ and n_ℓ
$d_{u_{n_{\acute{\ell}},i,n_\ell}}$	The distance between $u_{n_{\acute{\ell}},i}$ and n_ℓ
$\mathbf{h}_{u_{n_{\acute{\ell}},i,n_\ell}}$	The channel coefficient between $u_{n_{\acute{\ell}},i}$ and n_ℓ
$\mathbf{g}_{u_{n_{\acute{w}},j,n_\ell}}$	The channel coefficient between $u_{n_{\acute{w}},j}$ and n_ℓ
$q_{u_{n_{\acute{\ell}},i,n_w}}$	The channel coefficient between $u_{n_{\acute{\ell}},i}$ and n_w
$f_{u_{n_{\acute{w}},j,n_w}}$	The channel coefficient between $u_{n_{\acute{w}},j}$ and n_w
$\mathbf{x}^{(m,k)}_{n_\ell}$	Precoded symbol vector from n_ℓ to its associated UEs
$\tilde{p}^{(m,k)}_{u_{n_\ell,i,n_\ell}}$	Transmit power allocated to $u_{n_\ell,i}$ from n_ℓ at (m,k)
$\mathbf{v}^{(m,k)}_{u_{n_\ell,i,n_\ell}}$	Beamformer n_ℓ uses to transmit to the $u_{n_\ell,i}$ at (m,k)
$s^{(m,k)}_{u_{n_\ell,i,n_\ell}}$	Transmitted signal from n_ℓ to the $u_{n_\ell,i}$ at (m,k)
$\tilde{s}^{(k)}_{u_{n_w,j,n_w}}$	Transmitted signal from n_w to the $u_{n_w,j}$ at the sub-channel k
$y^{(m,k)}_{u_{n_\ell,i,n_\ell}}$	Received signal at the $u_{n_\ell,i}$ at (m,k)
$W^{(k)}_{\text{min},i}$	Min. CW size of the i-th transmitter on the k-th channel
$m^{(k)}_i$	Max. back-off stage of the i-th transmitter
$p^{(k)}_{tr}$	The prob. of occupation on the k-th unlicensed channel
$p^{(k)}_{tr,i}$	The prob. of transmitting a packet by the i-th transmitter
$p^{(k)}_{c,i}$	The prob. of collision experienced by the i-th transmitter
$p^{(k)}_{s,i}$	The prob. of successful transmission of the i-th transmitter
$p^{(k)}_{s,\mathcal{W}}$	The prob. of successful transmission of Wi-Fi network
$p^{(k)}_{s,\mathcal{L}}(\gamma_{k,n_\ell})$	The prob. of successful transmission of LAA network
$p^{(k)}_{c,\mathcal{W}}$	The prob. of collision among Wi-Fi transmissions
$p^{(k)}_{c,\mathcal{L}}$	The prob. of collision among LAA transmissions
$p^{(k)}_{c,\mathcal{W},\mathcal{L}}$	The prob. of collision between LAA and Wi-Fi transmissions
$p^{(k)}_{tr,\mathcal{L}}$	The prob. of transmission of LAA network
$p^{(k)}_{tr,\mathcal{W}}$	The prob. of transmission of Wi-Fi network
$\lambda_{u_{n_\ell,i}}$	The priority of user $u_{n_\ell,i}$ in the LAA network
$\lambda_{u_{n_w,j}}$	The priority of user $u_{n_w,j}$ in the Wi-Fi network

and $\psi_{u_{n_{\ell'},i,n_\ell}}$ is a log-normal random variable (*i.e.*, the quantity $10\log_{10}(\psi_{u_{n_{\ell'},i,n_\ell}})$ is distributed as zero-mean Gaussian with a standard deviation of $\sigma_{\text{shadowing}}$). The small-scale fading coefficients (*i.e.*, elements of $\tilde{\mathbf{h}}_{u_{n_{\ell'},i,n_\ell}}$) are modeled as i.i.d. complex Gaussian variables with zero-mean and unit-variance. We further assume a block fading model, where small-scale channels are constant over a few time slots with respect to channel estimation and channel state information (CSI) feedback procedures. Similarly, we assume that large-scale fading coefficients $\beta_{u_{n_{\ell'},i,n_\ell}}$ stay constant during large-scale coherence blocks. The small-scale and large-scale fading coefficients in different coherence blocks are assumed to be independent.

Similarly, the channel matrix $\mathbf{g}_{u_{n_{w'},j,n_\ell}} = \sqrt{\beta_{u_{n_{w'},j,n_\ell}}}\tilde{\mathbf{g}}_{u_{n_{w'},j,n_\ell}} \in \mathbb{C}^{N_t}$ denotes the channel coefficient between the n_ℓ eNodeB and the Wi-Fi station $u_{n_{w'},j}$. Moreover, the channel coefficient between Wi-Fi AP n_w and UE $u_{n_{\ell'},i}$ and Wi-Fi station $u_{n_{w'},j}$ form the channel metrics $q_{u_{n_{\ell'},i,n_w}} = \sqrt{\beta_{u_{n_{\ell'},i,n_w}}}\tilde{q}_{u_{n_{\ell'},i,n_w}} \in \mathbb{C}$ and $f_{u_{n_{w'},j,n_w}} = \sqrt{\beta_{u_{n_{w'},j,n_w}}}\tilde{f}_{u_{n_{w'},j,n_w}} \in \mathbb{C}$, respectively.

C. Received Signals

Prior to transmitting on the sub-channel $m \in \mathcal{N}_k$ of the unlicensed channel k, the n_ℓ eNodeB linearly precodes its symbol vector

$$\mathbf{x}_{n_\ell}^{(k,m)} = \sum_{u_{n_\ell,i} \in \mathcal{U}_{n_\ell}} \sqrt{\tilde{p}_{u_{n_\ell,i},n_\ell}^{(k,m)}} \mathbf{v}_{u_{n_\ell,i},n_\ell}^{(k,m)} s_{u_{n_\ell,i},n_\ell}^{(k,m)}$$

where $\tilde{p}_{u_{n_\ell,i},n_\ell}^{(k,m)}$ stands for the transmit power allocated to user $u_{n_\ell,i}$ from the n_ℓ eNodeB on the sub-channel m of the unlicensed band k, and $\mathbf{v}_{u_{n_\ell,i},n_\ell}^{(k,m)}$ denotes the unit-norm beamformer that eNodeB n_ℓ uses to transmit the signal $s_{u_{n_\ell,i},n_\ell}^{(k,m)}$ to receiver $u_{n_\ell,i}$ on the m-th sub-channel of the unlicensed band channel k [2], [3]. Each eNodeB n_ℓ on the unlicensed channel k is under a transmit power constraint of $P_{n_\ell}^{k,\max}$ due to the regulations on the unlicensed channels [4]. Under our assumptions, the received signal at the UE $u_{n_\ell,i}$ on the (k,m) unlicensed band can be written as

$$\begin{aligned} y_{u_{n_\ell,i}}^{(k,m)} &= \underbrace{\sqrt{\tilde{p}_{u_{n_\ell,i},n_\ell}^{(k,m)}} \mathbf{h}_{u_{n_\ell,i},n_\ell}^{(k,m)^H} \mathbf{v}_{u_{n_\ell,i},n_\ell}^{(k,m)} s_{u_{n_\ell,i},n_\ell}^{(k,m)}}_{\text{Desired signal}} \\ &+ \underbrace{\sum_{\substack{\acute{i}=1 \\ \acute{i}\neq i}}^{|\mathcal{U}_{n_\ell}|} \sqrt{\tilde{p}_{u_{n_\ell,\acute{i}},n_\ell}^{(k,m)}} \mathbf{h}_{u_{n_\ell,i},n_\ell}^{(k,m)^H} \mathbf{v}_{u_{n_\ell,\acute{i}},n_\ell}^{(k,m)} s_{u_{n_\ell,\acute{i}},n_\ell}^{(k,m)}}_{\text{Intra-cell interference from the serving eNodeB}} \\ &+ \underbrace{\sum_{\substack{\acute{\ell}=1 \\ \acute{\ell}\neq \ell}}^{L} \sum_{\acute{i}=1}^{|\mathcal{U}_{n_{\acute{\ell}}}|} \sqrt{\tilde{p}_{u_{n_{\acute{\ell}},\acute{i}},n_{\acute{\ell}}}^{(k,m)}} \mathbf{h}_{u_{n_\ell,i},n_{\acute{\ell}}}^{(k,m)^H} \mathbf{v}_{u_{n_{\acute{\ell}},\acute{i}},n_{\acute{\ell}}}^{(k,m)} s_{u_{n_{\acute{\ell}},\acute{i}},n_{\acute{\ell}}}^{(k,m)}}_{\text{Inter-cell interference from other eNodeBs}} \\ &+ \underbrace{\sum_{w=1}^{W} \sqrt{p_{n_w}^{(k)}} q_{u_{n_\ell,i},n_w}^{(k)} \tilde{s}_{u_{n_w,j},n_w}^{(k)}}_{\text{Interference from Wi-Fi APs}} + \underbrace{\epsilon_{u_{n_\ell,i}}^{(k,m)}}_{\text{Noise}}, \end{aligned} \tag{1}$$

where $\tilde{s}_{u_{n_w,j},n_w}^{(k)}$ is the transmitted signal from the AP n_w to the Wi-Fi station $u_{n_w,j}$ on the channel k, and $\epsilon_{u_{n_\ell,i}}^{(k,m)} \sim \mathcal{CN}(0, \sigma_{u_{n_\ell,i}}^{(k,m)^2})$.

Recall that the signal-to-interference-plus-noise ratio (SINR) is defined as the ratio of the received signal power at the desired user to the interference plus noise power. The SINR for serving user $u_{n_\ell,i}$ on the (k,m) unlicensed band can be expressed as (2). Assuming each sub-channel m can be shared between the UEs associated with n_ℓ eNodeB in a time sharing fashion, the transmission rate for the UE $u_{n_\ell,i}$ on the (k,m) unlicensed band can be expressed as

$$R_{u_{n_\ell,i}}^{(k,m)} = \gamma_{k,n_\ell} c_{u_{n_\ell,i}}^{(k,m)} B_{k,m} \log_2(1 + \chi_{u_{n_\ell,i}}^{(k,m)} \cdot \text{SINR}_{u_{n_\ell,i}}^{(k,m)}),$$

where γ_{k,n_ℓ} represents the fraction of time when the n_ℓ-th eNodeB is active on the unlicensed channels k, $0 \leq c_{u_{n_\ell,i}}^{(k,m)} \leq 1$ denotes the time sharing component of the UE $u_{n_\ell,i}$ on the (k,m) unlicensed band that fulfills $0 \leq \sum_{n_\ell \in \mathcal{N}_L} \sum_{u_{n_\ell,i} \in \mathcal{U}_{n_\ell}} c_{u_{n_\ell,i}}^{(k,m)} \leq 1$, and $\chi_{u_{n_\ell,i}}^{(k,m)}$ is the channel access indicator which is 1 if the eNodeB n_ℓ serves the UE $u_{n_\ell,i}$ on the (k,m), and is 0 otherwise.

It is worth noting that the LAA transmission on the unlicensed channel k will happen if (i) there is no Wi-Fi transmission in the network and the LAA eNodeB's channel observations do not generate any false alarms or (ii) the Wi-Fi transmission is occurring but the LAA network does not become aware of that. While the latter happens with probability $(1 - P_{d,\mathcal{L}}^{(k)}) p_{tr,\mathcal{W}}^{(k)}$, the probability of the former is $(1 - P_{fa,\mathcal{L}}^{(k)})(1 - p_{tr,\mathcal{W}}^{(k)})$, where $P_{d,\mathcal{L}}^{(k)}$, $P_{fa,\mathcal{L}}^{(k)}$, and $p_{tr,\mathcal{W}}^{(k)}$ denotes the LAA's cross-technology probability of detection, probability of false alarm, and Wi-Fi's transmission probability on the unlicensed channel k (which will be discussed in Section IV), respectively. Hence, the achievable data rate of the UE $u_{n_\ell,i}$ can be expressed as

$$\begin{aligned} R_{u_{n_\ell,i}} = \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{N}_k} \Big(&(1 - p_{tr,\mathcal{W}}^{(k)})(1 - P_{fa,\mathcal{L}}^{(k)}(\eta_{\mathcal{L}})) R_{u_{n_\ell,i}}^{(k,m)^{\mathcal{H}_0}} \\ &+ p_{tr,\mathcal{W}}^{(k)}(1 - P_{d,\mathcal{L}}^{(k)}(\eta_{\mathcal{L}})) \cdot R_{u_{n_\ell,i}}^{(k,m)^{\mathcal{H}_1}} \Big) \end{aligned}$$

where $\eta_{\mathcal{L}}$ indicates the LAA's carrier sense threshold level (function of SINR), $R_{u_{n_\ell,i}}^{(k,m)^{\mathcal{H}_1}}$ and $R_{u_{n_\ell,i}}^{(k,m)^{\mathcal{H}_0}}$ denote the achievable data rate of the UE $u_{n_\ell,i}$ on the (k,m) unlicensed band when there is and is not Wi-Fi transmission, respectively.

Similarly, the received signal at the Wi-Fi station $u_{n_w,j}$ on the k-th unlicensed band can be written as

$$\begin{aligned} y_{u_{n_w,j}}^{(k)} &= \underbrace{\sqrt{p_{n_w}^{(k)}} f_{u_{n_w,j},n_w}^{(k)} \tilde{s}_{u_{n_w,j},n_w}^{(k)}}_{\text{Desired signal}} + \underbrace{\epsilon_{u_{n_w,j}}^{(k)}}_{\text{Noise}} \\ &+ \underbrace{\sum_{\substack{\acute{w}=1 \\ \acute{w}\neq w}}^{W} \sqrt{p_{n_{\acute{w}}}^{(k)}} f_{u_{n_w,j},n_{\acute{w}}}^{(k)} \tilde{s}_{u_{n_{\acute{w}},\acute{j}},n_{\acute{w}}}^{(k)}}_{\text{Interference from other Wi-Fi APs}} \\ &+ \underbrace{\sum_{m=1}^{N_k} \sum_{\ell=1}^{L} \sum_{i=1}^{|\mathcal{U}_{n_\ell}|} \sqrt{\tilde{p}_{u_{n_\ell,i},n_\ell}^{(k,m)}} \mathbf{g}_{u_{n_w,j},n_\ell}^{(k,m)^H} \mathbf{v}_{u_{n_\ell,i},n_\ell}^{(k,m)} s_{u_{n_\ell,i},n_\ell}^{(k,m)}}_{\text{Interference from LTE-LAA eNodeBs}}, \end{aligned} \tag{4}$$

where $\epsilon_{u_{n_w,j}}^{(k)} \sim \mathcal{CN}(0, \sigma_{u_{n_w,j}}^{(k)^2})$. Accordingly, the SINR for serving the Wi-Fi station $u_{n_w,j}$ on the k-th unlicensed band can be given by (3) and the transmission rate for the Wi-Fi station $u_{n_w,j}$, on the k-th unlicensed band can be expressed in general form as

$$R_{u_{n_w,j}}^{(k)} = \tilde{\gamma}_{k,n_w} B_k \log_2(1 + \chi_{u_{n_w,j}}^{(k)} \text{SINR}_{u_{n_w,j}}^{(k)}),$$

where $0 \leq \tilde{\gamma}_{k,n_w} \leq 1$ represents the time fraction that the Wi-Fi AP n_w occupied the unlicensed channel k, B_k denotes the bandwidth of the k-th unlicensed band, and $\chi_{u_{n_w,j}}^{(k)} = 1$ if the Wi-Fi AP n_w serves the Wi-Fi station $u_{n_w,j}$ on the unlicensed channel k, and is 0 otherwise. Following the same notation, the achievable data rate of the Wi-Fi station $u_{n_w,j}$ can be written as

$$\begin{aligned} R_{u_{n_w,j}} = \sum_{k \in \mathcal{K}} \Big(&(1 - p_{tr,\mathcal{L}}^{(k)})(1 - P_{fa,\mathcal{W}}^{(k)}(\eta_{\mathcal{W}})) R_{u_{n_w,j}}^{(k)^{\mathcal{H}_0}} \\ &+ p_{tr,\mathcal{L}}^{(k)}(1 - P_{d,\mathcal{W}}^{(k)}(\eta_{\mathcal{W}})) R_{u_{n_w,j}}^{(k)^{\mathcal{H}_1}} \Big), \end{aligned}$$

$$\mathrm{SINR}_{u_{n_\ell,i}}^{(k,m)} = \frac{\tilde{p}_{u_{n_\ell,i},n_\ell}^{(k,m)} |\mathbf{h}_{u_{n_\ell,i},n_\ell}^{(k,m)^H} \mathbf{v}_{u_{n_\ell,i},n_\ell}^{(k,m)}|^2}{\sum_{\substack{\acute{i}=1\\ \acute{i}\neq i}}^{|\mathcal{U}_{n_\ell}|} \tilde{p}_{u_{n_\ell,\acute{i}},n_\ell}^{(k,m)} |\mathbf{h}_{u_{n_\ell,i},n_\ell}^{(k,m)^H} \mathbf{v}_{u_{n_\ell,\acute{i}},n_\ell}^{(k,m)}|^2 + \sum_{\substack{\acute{\ell}=1\\ \acute{\ell}\neq \ell}}^{L} \sum_{\acute{i}=1}^{|\mathcal{U}_{n_{\acute{\ell}}}|} \tilde{p}_{u_{n_{\acute{\ell}},\acute{i}},n_{\acute{\ell}}}^{(k,m)} |\mathbf{h}_{u_{n_\ell,i},n_{\acute{\ell}}}^{(k,m)^H} \mathbf{v}_{u_{n_{\acute{\ell}},\acute{i}},n_{\acute{\ell}}}^{(k,m)}|^2 + \sum_{w=1}^{W} p_{n_w}^{(k)} |q_{u_{n_\ell,i},n_w}^{(k)}|^2 + \sigma_{u_{n_\ell,i}}^{(k,m)^2}}. \quad (2)$$

$$\mathrm{SINR}_{u_{n_w,j}}^{(k)^{\mathcal{H}_1}} = \frac{p_{n_w}^{(k)} |f_{u_{n_w,j},n_w}^{(k)}|^2}{\sum_{\substack{\acute{w}=1\\ \acute{w}\neq w}}^{W} p_{n_{\acute{w}}}^{(k)} |f_{u_{n_w,j},n_{\acute{w}}}^{(k)}|^2 + \sum_{m=1}^{N_k} \sum_{\ell=1}^{L} \sum_{i=1}^{|\mathcal{U}_{n_\ell}|} \tilde{p}_{u_{n_\ell,i},n_\ell}^{(k,m)} |\mathbf{g}_{u_{n_w,j},n_\ell}^{(k,m)^H} \mathbf{v}_{u_{n_\ell,i},n_\ell}^{(k,m)}|^2 + \sigma_{u_{n_w,j}}^{(k)^2}}. \quad (3)$$

where $R_{u_{n_w,j}}^{(k)\ \mathcal{H}_1}$ and $R_{u_{n_w,j}}^{(k)\ \mathcal{H}_0}$ denote the achievable data rate of the Wi-Fi station $u_{n_w,j}$ on the k-th unlicensed band when there is and is not LAA transmission, respectively.

Considering this system model, in the next section, we will propose an efficient spectrum sharing method between Wi-Fi and LAA networks on the unlicensed bands. In this scheme, each LAA eNodeB adjusts its contention window size on an unlicensed band in such a way that guarantees the Wi-Fi's network throughput while maximizing its achievable rate.

III. Problem Formulation and Analysis

In this section, the problem of interest is to maximize the LTE-LAA network throughput on an unlicensed band while Wi-Fi network performance is assured above a predetermined threshold, taking into account the contribution of both MAC-layer parameters (like contention window size) and PHY-layer characteristics (such as carrier sense threshold, transmit power, beamforming). To be specific, the problem of maximizing the weighted sum rate of the LTE-LAA network with respect to the fraction of time that LAA is active is formulated subject to the Wi-Fi throughput constraint as follows

$$\max_{\boldsymbol{\gamma}} \quad \sum_{\ell=1}^{L} \sum_{i=1}^{|\mathcal{U}_{n_\ell}|} \lambda_{u_{n_\ell,i}} R_{u_{n_\ell,i}} \quad (5a)$$

$$\text{s.t.} \quad \sum_{w=1}^{W} \sum_{j=1}^{|\mathcal{U}_{n_w}|} \lambda_{u_{n_w,j}} R_{u_{n_w,j}} \geq \hat{r}_w, \quad (5b)$$

$$0 \leq \gamma_{k,n_\ell} \leq 1, \quad (5c)$$

where $\hat{r}_w$ denotes the required data rate by the Wi-Fi network, $\lambda_{u_{n_\ell,i}}$ ($\lambda_{u_{n_w,j}}$) indicates the priority of user $u_{n_\ell,i}$ ($u_{n_w,j}$) in the LAA (Wi-Fi) network, and (5b) guarantees the Wi-Fi network's throughput in the aforementioned coexistence scenario.

Proposition 1. *The problem of maximizing the weighted sum rate of the LAA network with respect to $\boldsymbol{\gamma}$ is convex.*

Proof. The second derivative of the twice differentiable function $R_{u_{n_\ell,i}}^{(k,m)}$ with respect to γ_{k,n_ℓ} is negative and its Hessian is symmetric negative definite. Hence, the objective function is concave in the vector of eNodeBs' occupying unlicensed channel factor. Moreover, the constraint set is composed of linear constraints. These render the optimization problem (5) convex. Hence, the optimization problem (5) can be efficiently solved using numerical iterative algorithms [5]. ■

Remark 1. *Depending on the time occupation of the unlicensed band k by the eNodeB n_ℓ, i.e. γ_{k,n_ℓ} ,the eNodeB n_ℓ is able to adaptively alter its minimum backoff contention window size $CW_{\min,n_\ell}^{(k)}$ at the beginning of each LTE frame on the k-th unlicensed channel. The reason is, in a coexistence scenario, the n_L eNodeBs compete with the n_W APs to access the unlicensed band. This rivalry is based on the Wi-Fi time slot duration. As the LTE frame duration exceeds the length of Wi-Fi time slot, the probability of a successful transmission by the eNodeB $n_\ell \in \mathcal{L}$ can be interpreted as the time fraction in which the eNodeB n_ℓ occupies the k-th unlicensed channel, i.e. $0 \leq \gamma_{k,n_\ell} = p_{s,n_\ell}^{(k)} \leq 1$ [6].*

In the next section, we will calculate the impact of imperfect sensing in Wi-Fi's and LAA's transmission and collision probabilities.

IV. Imperfect Spectrum sensing and Impact of sensing threshold

A. MAC Layer Schemes

Wi-Fi systems count on a contention-based medium access with a random back off process, a.k.a. Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) [7], [8]. LTE uses a listen before talk (LBT) channel access mechanism to maintain fair coexistence with the Wi-Fi. Among different LAA-LBT schemes, in this paper, we consider the Category 4 (Cat 4 LBT). Since it is based on the same Wi-Fi CSMA/CA scheme, it is well-suited in a coexistence scenario, as recommended by [9].

According to the analytical model based on Markov chain in [10], the probability of transmitting a packet by a transmitter $i \in \mathcal{I} = \{n_w, n_\ell\}$ in a following idle time slot on the unlicensed channel k can be expressed by

$$p_{\mathrm{tr},i}^{(k)} = \frac{2(1-2p_{c,i}^{(k)})}{(1-2p_{c,i}^{(k)})(1+\mathrm{CW}_{\min,i}^{(k)}) + p_{c,i}^{(k)}\mathrm{CW}_{\min,i}^{(k)}A}, \quad (6)$$

where $p_{c,i}^{(k)}$ is the probability of collision experienced by the i-th transmitting node on the k-th unlicensed channel, $A = (1-(2p_{c,i}^{(k)})^{m_i^{(k)}})$, and $\mathrm{CW}_{\min,i}^{(k)}$ and $m_i^{(k)}$ are the minimum contention window size and the maximum back-off stage of the transmitting node i on the unlicensed band k, respectively.

Therefore, in a coexistence scenario, the probability of occupation of the k-th unlicensed channel either by Wi-Fi or LAA can be expressed as

$$p_{\text{tr}}^{(k)} = 1 - \prod_{w=1}^{W}(1 - p_{\text{tr},n_w}^{(k)}) \prod_{\ell=1}^{L}(1 - p_{\text{tr},n_\ell}^{(k)}), \tag{7}$$

and the collision probability of the transmitting node $i \in \{n_w, n_\ell\}$ on a shared unlicensed band k can be indicated as

$$p_{c,i}^{(k)} = 1 - \prod_{\acute{i}\neq i}(1 - p_{\text{tr},\acute{i}}^{(k)}) \prod_{\mathcal{I}/i}(1 - p_{\text{tr},-i}^{(k)}), \tag{8}$$

where $\acute{i} \in \mathcal{W}(\mathcal{L})$, if $i = n_w(n_\ell)$ and $p_{\text{tr},-i}^{(k)} \triangleq \left(p_{\text{tr},j}^{(k)}\right)_{j\neq i}$. The probability of a successful transmission for the transmitting node i on the unlicensed band k can be calculated as

$$p_{s,i}^{(k)} = p_{\text{tr},i}^{(k)} \prod_{\acute{i}\neq i}(1 - p_{\text{tr},\acute{i}}^{(k)}) \prod_{\mathcal{I}/i}(1 - p_{\text{tr},-i}^{(k)}). \tag{9}$$

Accordingly, the successful transmission probability of the whole Wi-Fi and LAA networks on the k-th unlicensed channel can be given as $p_{s,\mathcal{W}}^{(k)} = \sum_w p_{s,n_w}^{(k)}$ and $p_{s,\mathcal{L}}^{(k)} = \sum_\ell p_{s,n_\ell}^{(k)}$, respectively. Moreover, the collision probability can be divided into three probabilities, i.e., the collision probability among the Wi-Fi transmissions which is given by

$$p_{c,\mathcal{W}}^{(k)} = (1 - p_{tr,\mathcal{L}}^{(k)})\left[p_{tr,\mathcal{W}}^{(k)} - p_{s,\mathcal{W}_0}^{(k)}\right], \tag{10}$$

the collision probability among the LAA transmissions, i.e.,

$$p_{c,\mathcal{L}}^{(k)} = (1 - p_{tr,\mathcal{W}}^{(k)})\left[p_{tr,\mathcal{L}}^{(k)} - p_{s,\mathcal{L}_0}^{(k)}\right], \tag{11}$$

and the collision probability between the Wi-Fi and the LAA transmission that can be calculated as

$$p_{c,\mathcal{W},\mathcal{L}}^{(k)} = p_{tr,\mathcal{L}}^{(k)} \cdot p_{tr,\mathcal{W}}^{(k)}, \tag{12}$$

where, $p_{tr,\mathcal{L}}^{(k)} = 1 - \prod_{\ell=1}^{L}(1 - p_{\text{tr},n_\ell}^{(k)})$ denotes the LAA's probability of transmission on the k-th unlicensed channel and implies that at least one of the n_L eNodeBs transmits a packet, and $p_{tr,\mathcal{W}}^{(k)} = 1 - \prod_{w=1}^{W}(1 - p_{\text{tr},n_w}^{(k)})$ represents the Wi-Fi's transmission probability on the unlicensed channel k. The probability of a successful transmission for the LAA eNodeB n_ℓ (Wi-Fi AP n_w) in an LAA-only (a Wi-Fi only) network is indicated and given by $p_{s,\mathcal{L}_0}^{(k)} = \sum_\ell p_{\text{tr},n_\ell}^{(k)} \prod_{\acute{\ell}\neq\ell}(1 - p_{\text{tr},n_{\acute{\ell}}}^{(k)})$ ($p_{s,\mathcal{W}_0}^{(k)} = \sum_w p_{\text{tr},n_w}^{(k)} \prod_{\acute{w}\neq w}(1 - p_{\text{tr},n_{\acute{w}}}^{(k)})$). Therefore, the average duration to support one successful transmission in the unlicensed channel k can be calculated by following the procedures incorporated by T_{ave} in [11, Eq. (19)]:

$$\begin{aligned} T_{\text{avg}}^{(k)} = (1 - p_{tr}^{(k)})\mathbb{E}\{T_{\text{idle}}^{(k)}\} + p_{s,\mathcal{W}}^{(k)}\mathbb{E}\{T_{s,\mathcal{W}}^{(k)}\} + p_{s,\mathcal{L}}^{(k)}\mathbb{E}\{T_{s,\mathcal{L}}^{(k)}\} \\ + p_{c,\mathcal{W}}^{(k)}\mathbb{E}\{T_{c,\mathcal{W}}^{(k)}\} + p_{c,\mathcal{L}}^{(k)}\mathbb{E}\{T_{c,\mathcal{L}}^{(k)}\} + p_{c,\mathcal{W},\mathcal{L}}^{(k)}\mathbb{E}\{T_{c,\mathcal{W},\mathcal{L}}^{(k)}\}, \end{aligned}$$

where $T_{s,\mathcal{L}}^{(k)}$, $T_{s,\mathcal{W}}^{(k)}$, $T_{c,\mathcal{W}}^{(k)}$, $T_{c,\mathcal{L}}^{(k)}$, and $T_{c,\mathcal{W},\mathcal{L}}^{(k)}$ indicate the time that the k-th channel is being occupied by: an LAA successful transmission, a Wi-Fi successful transmission, a collision among the Wi-Fi transmissions, a collision among the LAA transmissions, and a collision between the Wi-Fi and the LAA transmissions, respectively.

B. Impact of sensing threshold

In a coexistence scenario, the main problem of interest of each transmitting node is to figure out whether or not the unlicensed band is equipped with other active transmitters. Therefore, the detection problem can be formulated based on the following hypothesis tests

$$\begin{cases} \mathcal{H}_0: & \text{Channel is idle} \\ \mathcal{H}_1: & \text{Channel is busy.} \end{cases} \tag{13}$$

In the sensing mechanism, the probabilities of interest include the probability of detection ($P_d = P(T(y) > \eta|\mathcal{H}_1)$) and the probability of false alarm ($P_{fa} = P(T(y) > \eta|\mathcal{H}_0)$), where y is the received signal, $T(y)$ denotes the test statistic of the energy detector, and η states the carrier sense threshold level that may vary among different technologies.

In order to maximize both networks' spectral efficiency, selecting the carrier sense threshold level of both systems plays an important role. In a coexistence scenario, the threshold in each system is not only based on the cross-technology false alarm, but also is based on the cross-technology probability of miss-detection since miss-detection leads to the supplementary collisions on the unlicensed band.

Proposition 2. *Taking the impact of the sensing thresholds into account, the collision and the successful transmission probabilities of the Wi-Fi AP n_w (LAA eNodeB n_ℓ) on the shared unlicensed band k can be respectively given as*

$$\begin{aligned} p_{c,n_w}^{(k)} &= p_{c,n_w,\mathcal{W}_0}^{(k)} + p_{tr,\mathcal{L}}^{(k)}(1 - P_{d,\mathcal{W}}(\eta_\mathcal{W}))(1 - p_{c,n_w,\mathcal{W}_0}^{(k)}), \\ &= 1 - \left(1 - p_{tr,\mathcal{L}}^{(k)} P_{md,\mathcal{W}}(\eta_\mathcal{W})\right) \prod_{\acute{w}\neq w}(1 - p_{tr,n_{\acute{w}}}^{(k)}) \\ p_{c,n_\ell}^{(k)} &= p_{c,n_\ell,\mathcal{L}_0}^{(k)} + p_{tr,\mathcal{W}}^{(k)}(1 - P_{d,\mathcal{L}}(\eta_\mathcal{L}))(1 - p_{c,n_\ell,\mathcal{L}_0}^{(k)}) \\ &= 1 - \left(1 - p_{tr,\mathcal{W}}^{(k)} P_{md,\mathcal{L}}(\eta_\mathcal{L})\right) \prod_{\acute{\ell}\neq\ell}(1 - p_{tr,n_{\acute{\ell}}}^{(k)}) \\ p_{s,n_w}^{(k)} &= \left(1 - p_{tr,\mathcal{L}}^{(k)} P_{d,\mathcal{W}}(\eta_\mathcal{W})\right) p_{tr,n_w}^{(k)} \prod_{\acute{w}\neq w}(1 - p_{tr,n_{\acute{w}}}^{(k)}), \\ p_{s,n_\ell}^{(k)} &= \left(1 - p_{tr,\mathcal{W}}^{(k)} P_{d,\mathcal{L}}(\eta_\mathcal{L})\right) p_{tr,n_\ell}^{(k)} \prod_{\acute{\ell}\neq\ell}(1 - p_{tr,n_{\acute{\ell}}}^{(k)}) \end{aligned}$$

where $p_{c,n_w,\mathcal{W}_0}^{(k)}$ ($p_{c,n_\ell,\mathcal{L}_0}^{(k)}$) states the collision probability of the Wi-Fi AP n_w (LTE-LAA eNodeB n_ℓ) in a Wi-Fi (LTE-LAA) only network, and $\eta_\mathcal{W}$ ($\eta_\mathcal{L}$) indicates the Wi-Fi (LAA) carrier sensing threshold level. This modification is due to the fact that the inaccuracy in detecting the Wi-Fi APs by the LAA eNodeBs ensues the supplementary collisions on the unlicensed band, and vice versa.

Proof. In order to capture the effect of miss-detection probability (in sensing the LAA users) on the performance of the Wi-Fi network, we show that the collision probability consists of the sensing threshold level, by rewriting (8) as follows

$$\begin{aligned} p_{c,n_w}^{(k)} &= p_{tr,\mathcal{L}}^{(k)} \prod_{\acute{w}\neq w}(1 - p_{\text{tr},n_{\acute{w}}}^{(k)}) + 1 - \prod_{\acute{w}\neq w}(1 - p_{\text{tr},n_{\acute{w}}}^{(k)}) \\ &= p_{tr,\mathcal{L}}^{(k)}(1 - p_{c,n_w,\mathcal{W}_0}^{(k)}) + p_{c,n_w,\mathcal{W}_0}^{(k)} \end{aligned}$$

since the inaccuracy in detecting the LAA eNodeBs by the Wi-Fi APs ensues the supplementary collisions on the unlicensed band, we multiply $p_{tr,\mathcal{L}}^{(k)}$ with the probability of cross-technology miss-detection $P_{md,\mathcal{W}} = 1 - P_{d,\mathcal{W}}$ to capture the impact of the sensing threshold in calculating $p_{c,n_w}^{(k)}$. Following the same approach, the other probabilities can be calculated. ∎

Proposition 3. *Taking the impact of imperfect sensing on the MAC layer performance into account, the collision probability discerned by the Wi-Fi AP n_w (caused by the LAA transmission) in the k-th unlicensed band is a function of cross-technology probability of detection, Wi-Fi's transmission probability, and LAA's transmission probability, and can be revised as follows*

$$\begin{aligned} p_{c,n_w}^{(k)} &= (1 - P_{d,\mathcal{L}}^{(k)})Pr\{\phi_k = 1 | \psi_k = 0, \theta_k = 1\} \\ &\quad + P_{d,\mathcal{L}}^{(k)} Pr\{\phi_k = 1 | \psi_k = 1, \theta_k = 1\} \end{aligned}$$

where ϕ_k, ψ_k, and θ_k denote the channel access decision (0: no access and 1: access), the sensing outcome (0: idle and 1: busy), and the channel status (0: idle and 1: busy) of the k-th unlicensed band.

Proof. The collision probability discerned by the Wi-Fi AP n_w in the k-th unlicensed band can be determined as

$$\begin{aligned} p_{c,n_w}^{(k)} &= \Pr\{\phi_k = 1 | \theta_k = 1\} \\ &= \Pr\{\psi_k = 0 | \theta_k = 1\}\Pr\{\phi_k = 1 | \psi_k = 0, \theta_k = 1\} \\ &\quad + \Pr\{\psi_k = 1 | \theta_k = 1\}\Pr\{\phi_k = 1 | \psi_k = 1, \theta_k = 1\}, \end{aligned}$$

where $\Pr\{\phi_k = 1 | \psi_k = 0, \theta_k = 1\} = \big(p_{tr,\mathcal{L}}/(1 - P_{d,\mathcal{L}}) \cdot p_{tr,\mathcal{W}}\big) \cdot \Pr\{\psi_k = 0, \theta_k = 1 | \phi_k = 1\}$. ∎

V. Simulation Evaluations

In this preliminary numerical evaluation, we evaluate the performance of the proposed scheme in a coexistence scenario. The setup of parameters is given in Table II, as in [6] and [11].

Fig. 2 shows the normalized throughput of both LAA and Wi-Fi networks for the different numbers of APs and eNodeBs. As the number of transmitters in each network gets bigger, the overall throughput of that network increases. However, there is a tradeoff between increasing the number of transmitters in one network and maintaining the desired throughput of the other network. Specifically, the throughput of LAA will decrease if the number of APs increases in the network. The Wi-Fi throughput follows the same trend.

The normalized throughput of LAA versus the minimum contention window size of LAA and Wi-Fi are demonstrated in Figures 3 and 4. It is observed that the larger the contention window size of the Wi-Fi, the better throughput can be obtained at the LAA network. However, by increasing the contention window size of the LAA, the LAA throughput increases until some point and then starts decreasing. It can be interpreted as follows: as CW increases the channel is less crowded and the chances of collision are low, however, setting

TABLE II: PHY/MAC Layer Parameters

Parameter	value
LAA's packet payload duration	2 ms
Wi-Fi's packet payload duration	1 ms
Mac header	272 bits
PHY header	128 bits
ACK	112 bits + PHY header
SIFS	16 μs
DIFS	34 μs
Slot time	9 μs
eNodeB transmit power	30 dBm
AP transmit power	24 dBm
Noise figure	9 dB
Thermal noise	-174dBm/Hz
Path loss model	COST-231 Hata model [12]
Shadowing standard deviation	8 dB

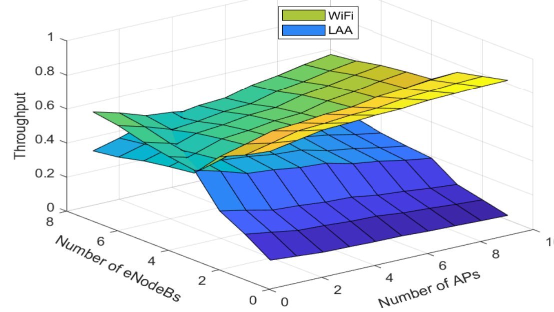

Fig. 2: Normalized Throughput of both LAA and Wi-Fi networks.

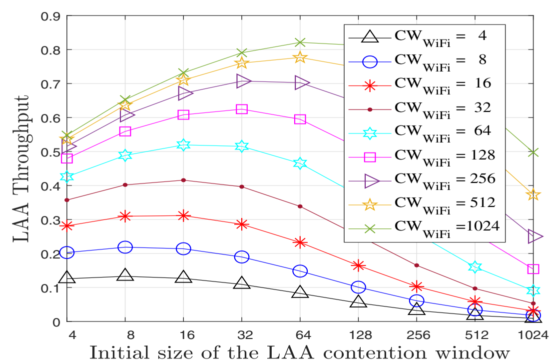

Fig. 3: LAA throughput versus CW$_{LAA}$.

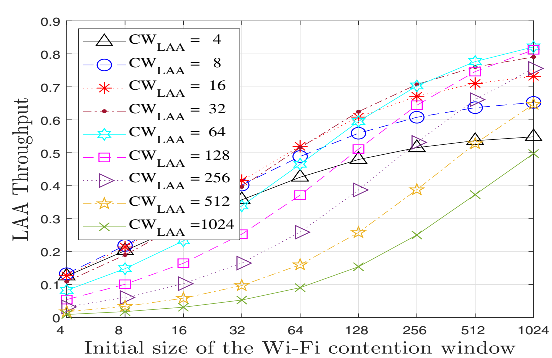

Fig. 4: LAA throughput versus CW$_{Wi-Fi}$.

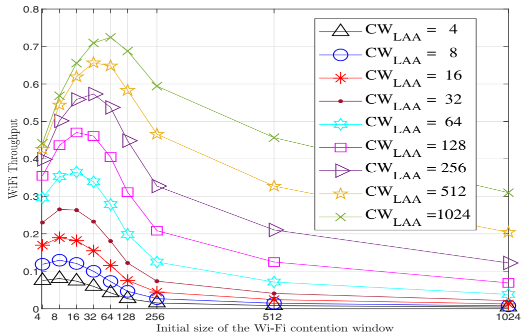

Fig. 5: Wi-Fi throughput versus CW$_{Wi-Fi}$.

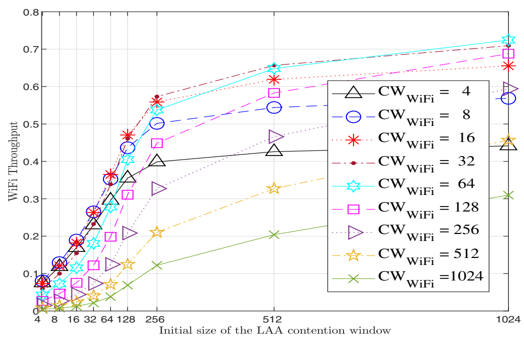

Fig. 6: Wi-Fi throughput versus CW$_{LAA}$.

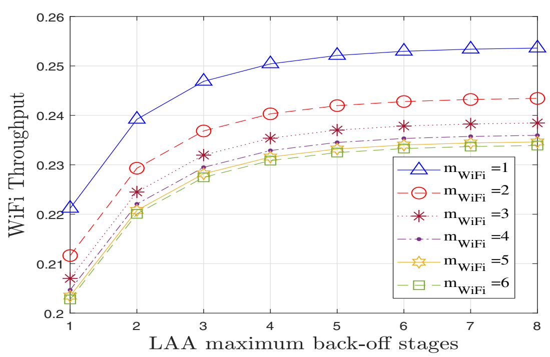

Fig. 7: Wi-Fi throughput versus m_{LAA}.

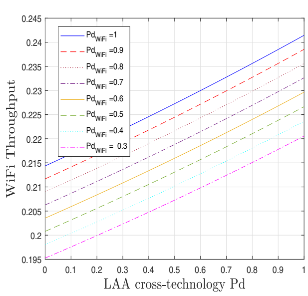

Fig. 8: Wi-Fi throughput versus $P_{d,\mathcal{L}}$.

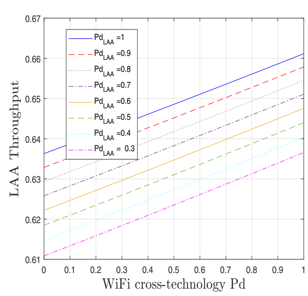

Fig. 9: LAA throughput versus $P_{d,\mathcal{W}}$.

up the CW to a large value minimizes the channel access probability and reduces the network throughput. Figures 5 and 6 plot the Wi-Fi throughput versus the Wi-Fi and the LAA minimum contention window size and follows the same trend as the LAA throughput.

The effect of the LAA maximum backoff stage on the Wi-Fi throughput is investigated in Figure 7. As shown, the larger the LAA maximum backoff stage, the larger Wi-Fi throughput can be achieved.

Figures 8 and 9 demonstrate the impact of the cross-technology probability of detection on the network throughput. The results can be interpreted as follows: the probability of detection (miss-detection) plays a critical role in computing the network throughput. When P_d approaches zero, the performance decreases. However, increasing P_d significantly increases both networks' throughput.

VI. Conclusion

In this paper, we studied the MAC-layer performance of the LAA and Wi-Fi networks by considering the contribution of both MAC and PHY layers characteristics. By taking the impact of imperfect sensing into account, we analyzed each network's KPIs, such as throughput, and probabilities of succcessful transmissions and collisions. By maximizing the LAA throughput while assuring Wi-Fi throughput above a predetermined threshold, the optimum LAA contention size was found. Numerical results demonstrate that efficiently selecting the MAC-layer and PHY-layer parameters greatly influences the network throughput of both systems.

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