

Nondestructive Cooling of an Atomic Quantum Register via State-Insensitive Rydberg Interactions

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(Received 3 September 2019; published 20 November 2019)

We propose a protocol for sympathetically cooling neutral atoms without destroying the quantum information stored in their internal states. This is achieved by designing state-insensitive Rydberg interactions between the data-carrying atoms and cold auxiliary atoms. The resulting interactions give rise to an effective phonon coupling, which leads to the transfer of heat from the data atoms to the auxiliary atoms, where the latter can be cooled by conventional methods. This can be used to extend the lifetime of quantum storage based on neutral atoms and can have applications for long quantum computations. The protocol can also be modified to realize state-insensitive interactions between the data and the auxiliary atoms but tunable and nontrivial interactions among the data atoms, allowing one to simultaneously cool and simulate a quantum spin model.

DOI: [10.1103/PhysRevLett.123.213603](https://doi.org/10.1103/PhysRevLett.123.213603)

Introduction.—In recent years, neutral atoms stored in individual traps [1–7] have emerged as a powerful resource for quantum information and quantum technologies [8–13]. Considerable effort is currently being invested in developing neutral atom traps that are insensitive to the internal state of the atom [2,14–16]. These so-called magic traps attempt to achieve what is naturally available with trapped ions, since the trapping of the latter relies on the net charge of the ion, and hence is independent of its internal electronic state. The magic trapping of neutral atoms reduces heating and dephasing associated with the fact that different electronic states may have different trapping potentials. Nevertheless, even with such magic trapping conditions, heating of the motional degrees of freedom (d.o.f.) of the atoms can occur because of, for example, the shaking of the atomic array due to laser intensity noise [17], mechanical forces from Rydberg interactions [18–20], or incoherent light scattering [21].

Such heating of the atomic motion, when combined with state-dependent Rydberg mediated gates, generally leads to reduced fidelities and loss of coherence, which is particularly problematic for long quantum simulations or computations [22–24]. It is therefore desirable to develop schemes to cool the atomic motion without destroying the quantum information stored in the internal states. The conventional laser cooling techniques [25–27] are not suitable for this task since they involve optical pumping which, in general, destroys the quantum information.

Several approaches for this problem have already been proposed in the past, from immersing the atomic lattice in a superfluid [28] to using cavity-assisted cooling [29]. It has also been shown that alkaline-earth atoms can be laser-cooled without destroying the quantum information provided it is stored in the nuclear spin [30].

In this Letter, we introduce two schemes for achieving state-insensitive interactions between neutral atoms, another natural and useful tool of trapped ions. We further show how to use these interactions to realize a state-preserving cooling procedure, inspired by sympathetic cooling of trapped ions [31,32]. In contrast to the protocols in Refs. [28,29], ours requires only ingredients and capabilities that are already present in many neutral atom experiments: auxiliary atoms and Rydberg interactions.

The scenario we have in mind is the following: we assume one starts with a quantum data register composed of an array of N atoms, each in an individual trap, cooled to the vibrational ground state and optically pumped to a particular ground state. Each atom encodes a two-level system in its ground states. One then uses Rydberg interactions to perform a quantum computation or simulation, during which the atoms are heated. To cool the data register we introduce N auxiliary atoms, one for each data atom [see Fig. 1(a)], that have been precooled using any of the standard methods. The data and auxiliary atoms can then be coupled via Rydberg interactions, implementing a *phonon-swap* gate—a coherent exchange of vibrational quanta. A key requirement of

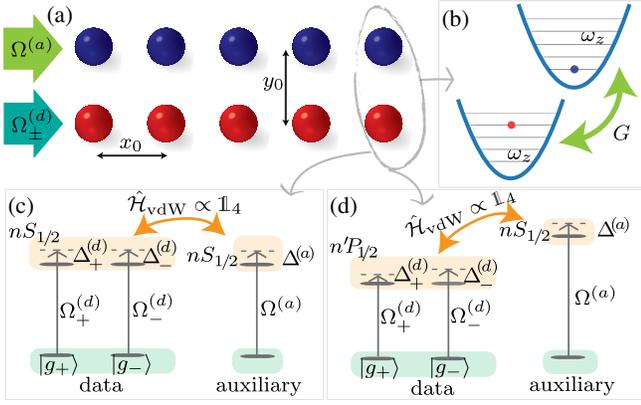


FIG. 1. Schematic of the phonon-swap protocol. (a) For each data atom (red, bottom) we place another auxiliary atom (blue, top) at an equal distance y_0 . We assume a 1D chain of data and auxiliary atoms, with a lattice spacing of x_0 . (b) The Rydberg interactions give rise to effective coupling G between the vibrational modes of the data and auxiliary atoms. (c),(d) Two schemes that lead to spin-insensitive interactions between the data and the auxiliary atoms are, in (c), the ground states of all atoms are weakly coupled to Rydberg $S_{1/2}$ states. In (d), the data atoms are coupled to $n'P_{1/2}$ states and the auxiliary to $nS_{1/2}$, where $|n - n'| \gg 1$.

this protocol is for the interactions between the auxiliary and data atoms to be insensitive to the internal state of the data atoms. Unlike the Coulomb interaction between ions which naturally satisfies this requirement, the Rydberg interactions between neutral atoms are inherently state dependent. As we show in this Letter, a careful choice of the Rydberg states can nevertheless satisfy this requirement.

Another requirement is that the phonon-swap interactions should not induce unwanted state-dependent couplings between the data atoms. In the first of our two schemes [see Figs. 1(c), 1(d)], the interactions between any pair of atoms (data-data and data-auxiliary) are independent of the internal state. This scheme consists of pausing the quantum computation or simulation, performing the phonon swap, and then resuming the computation or simulation. In the second scheme, the data and auxiliary atoms are addressed separately, which allows one to design state-insensitive data-auxiliary interactions but tunable data-data interactions. As an example, we show how this can be used to implement the phonon-swap while simultaneously performing a quantum simulation of a spin model on the data atoms. Finally, for both of the above schemes, one can laser cool the auxiliary atoms during the phonon swap. Because of the quantum Zeno effect [33], this has the additional advantage of preventing certain coherent heating mechanisms, such as those due to the Rydberg interactions, from taking place at all. We leave the detailed study of such a scheme for future work.

Phonon-swap for two atoms.—Let us first consider the case of two atoms: a two-level data atom “ d ” and a

single-level auxiliary atom “ a .” The two atoms are each trapped in a three-dimensional harmonic potential separated by a distance \mathbf{r} . In recent experiments [4,25–27,34,35], the confinement along two directions (x, y) is often much stronger than along the third (z); i.e., the trap frequencies satisfy $\omega_x, \omega_y \gg \omega_z$. For simplicity, we focus on cooling the weakest component perpendicular to the interatomic axis is a trivial generalization of this section. The third component [y direction in Fig. 1(a)] requires more care but can be cooled via an adiabatic protocol [36].

The Hamiltonian consisting of both the vibrational and the internal d.o.f. is ($\hbar = 1$) $\hat{\mathcal{H}} = \omega_z(\hat{d}^\dagger \hat{d} + \hat{a}^\dagger \hat{a}) + \hat{\mathcal{H}}_s + \hat{\mathcal{H}}_{\text{int}}(\mathbf{r})$, where $\hat{d}(\hat{a})$ is the phonon annihilation operator of the data (auxiliary) atom along the z direction; $\hat{\mathcal{H}}_s$ acts on the internal (spin) d.o.f. of the data atom, and $\hat{\mathcal{H}}_{\text{int}}(\mathbf{r})$ describes the interaction between the two atoms that, in principle, couples motion and spin. Since we want to preserve the spin state of the data atom, the phonon dynamics should be decoupled from the spin; i.e., we want $\hat{\mathcal{H}}_{\text{int}} = \mathbb{1}_{\text{internal}} \otimes V(r)$ to be an identity operator on the internal states. As we later show, by weakly laser dressing the ground states with Rydberg states, it is possible to obtain effective interactions of such form, where $V(r) = \mathcal{A}/(r^6 + R_c^6)$ for coupling \mathcal{A} and blockade radius R_c .

For now, let us assume these interactions and Taylor expand them to second order in the small quantum fluctuations on top of the macroscopic separation r_0 , which we assume to be along one of the strongly confined directions [Fig. 1(a)]. This gives rise to a quadratic Hamiltonian [36],

$$\hat{\mathcal{H}}_{\text{ph},2} = \omega_z(\hat{d}^\dagger \hat{d} + \hat{a}^\dagger \hat{a}) - \frac{G}{2} [(\hat{d} + \hat{d}^\dagger)^2 + (\hat{a} + \hat{a}^\dagger)^2] + G(\hat{d} + \hat{d}^\dagger)(\hat{a} + \hat{a}^\dagger), \quad (1)$$

where $G = (3\mathcal{A}/M\omega_z r_0^8) \{1/[1 + (R_c/r_0)^6]^2\}$ is the phonon coupling strength and M is the atomic mass. In the regime where $\omega_z \gg G$, only the number-conserving terms are relevant, giving a “beam splitter” interaction (in the rotating frame) $\hat{\mathcal{H}}_{\text{ph},2} = G(\hat{a}^\dagger \hat{d} + \hat{d}^\dagger \hat{a})$. This Hamiltonian effectuates a state-transfer between the two vibrational modes in a time of $t_s = \pi/2G$, swapping the phonons of the data atom with those of the auxiliary atom. This cools the data atom down to the initial phonon occupancy of the auxiliary atom.

Phonon-swap for 1D chain.—The discussed protocol can be easily generalized for an ensemble of atoms. We simply associate each data atom with a cold auxiliary atom. For concreteness, we consider a chain of data atoms with a lattice constant x_0 , separated by a distance of y_0 from a chain of cold auxiliary atoms [Fig. 1(a)]. The many-body Hamiltonian is quadratic with approximate power-law decaying hopping between the sites [36]

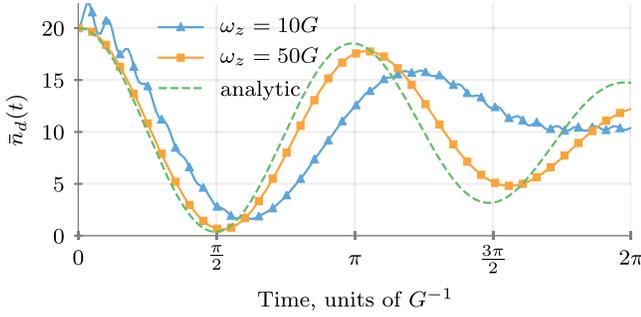


FIG. 2. The average number of phonons in the data atoms as a function of time computed numerically (solid lines) for different values of ω_z for two chains of 50 atoms, including the counter-rotating terms in Eq. (1), and using Eq. (3) (dashed line). Here, $\eta = 1$, $\bar{n}_d(0) = 20$, $\bar{n}_a(0) = 0$.

$$\hat{\mathcal{H}}_{\text{ph,1D}} = \sum_{i \neq j} \frac{G}{\eta^8 |i-j|^8} (\hat{a}_i^\dagger \hat{a}_j + \hat{d}_i^\dagger \hat{d}_j) + \sum_{ij} \frac{G}{[\eta^2(i-j)^2 + 1]^4} (\hat{a}_i \hat{d}_j^\dagger + \hat{a}_i^\dagger \hat{d}_j), \quad (2)$$

where $\eta \equiv x_0/y_0$. Here we defined G in terms of the smallest distance between a data atom and its auxiliary, i.e., y_0 (see Fig. 1). Clearly, as $\eta \rightarrow \infty$, it is sufficient to consider only the nearest-neighbor data-auxiliary interactions. In such a case, we recover the situation in the previous section: each data-auxiliary pair perfectly swaps their phonons after a time of $t_s = \pi/2G$. If we also take into account next-nearest-neighbor data-auxiliary interactions, we find [36] that the average phonon occupancy of the data atoms is

$$\bar{n}_d(t) = \frac{\bar{n}_a(0) + \bar{n}_d(0)}{2} - \frac{\bar{n}_a(0) - \bar{n}_d(0)}{2} \times J_0\left(\frac{4Gt}{(1+\eta^2)^4}\right) \cos(2Gt), \quad (3)$$

where $\bar{n}_d(t)$ [$\bar{n}_a(t)$] is the average occupancy of data (auxiliary) atoms at time t and $J_0(z)$ is a Bessel function of the first kind. Equation (3) is quantitatively accurate (see Fig. 2) at short timescales, when the effects of the long-range interactions are less important. As $\eta \rightarrow \infty$ we have $J_0 \rightarrow 1$ which reproduces the case of independent pairwise phonon swaps. Moreover, $t_s = \pi/2G$ is still the nearly optimal swap time (see Fig. 2) and even with $\eta = 1$ we can still achieve a high-efficiency swap. Assuming for simplicity that the auxiliary atoms are initially in the vibrational ground state, we obtain a swap efficiency of $1 - [\bar{n}_d(t_s)/\bar{n}_d(0)] = \frac{1}{2} + \frac{1}{2}J_0(\pi/8) \approx 98\%$. Furthermore, Eq. (3) remains qualitatively accurate even at longer timescales. As $t \rightarrow \infty$ we have $J_0 \rightarrow 0$ and we see that the mean phonon occupancy of all atoms is the average of the total initial number of phonons, as expected.

The above discussion concludes that to cool an atomic register consisting of many atoms in arbitrary geometries and dimensions, we simply perform the phonon swap as if all the data-auxiliary pairs are independent. The many-body interactions only lead to a small degradation in the swap efficiency.

State-insensitive Rydberg interactions.—We now turn to discuss how to obtain the spin-independent interactions by utilizing the van der Waals (vdW) couplings between Rydberg states. Specifically, we concentrate on alkali atoms and consider weakly laser admixing two hyperfine ground states (see Supplemental Material for an explicit example [36]) representing the spin-1/2, $|g_+\rangle, |g_-\rangle$, to Rydberg states $|r_+\rangle, |r_-\rangle$, depicting the magnetic sublevels of either $S_{1/2}$ or $P_{1/2}$ manifolds, as shown in Figs. 1(c), 1(d). The vdW couplings $\hat{\mathcal{H}}_{\text{vdW}}$ between the Rydberg states then give rise to effective interactions between the dressed ground states. The relevant Hamiltonian is $\hat{\mathcal{H}} = \sum_{i=1,2} (\hat{\mathcal{H}}_A^{(i)} + \hat{\mathcal{H}}_L^{(i)}) + \hat{\mathcal{H}}_{\text{vdW}}$ where $\hat{\mathcal{H}}_A^{(i)} = -\Delta_+^{(i)} |r_+^{(i)}\rangle \langle r_+^{(i)}| - \Delta_-^{(i)} |r_-^{(i)}\rangle \langle r_-^{(i)}|$ and $\hat{\mathcal{H}}_L^{(i)} = (\Omega_+^{(i)}/2) |g_+^{(i)}\rangle \langle r_+^{(i)}| + (\Omega_-^{(i)}/2) |g_-^{(i)}\rangle \langle r_-^{(i)}| + \text{H.c.}$ are the atomic and laser Hamiltonians, respectively, in the rotating frame within the rotating wave approximation. Here, $\Omega_\pm^{(i)}$ are Rabi frequencies and $\Delta_\pm^{(i)} \gg \Omega_\pm^{(i)}$ the laser detunings. Note that for the auxiliary atoms, it is sufficient to consider a single ground state and hence a single laser. However, we must take into account all the states in the Rydberg manifold because, in general, $\hat{\mathcal{H}}_{\text{vdW}}$ may contain both diagonal and off-diagonal matrix elements. This fact has been used previously to construct tunable spin-spin interactions [37,38]. A sufficient condition to obtain spin-independent interactions is for $\hat{\mathcal{H}}_{\text{vdW}}$ to be proportional to an identity, together with a suitable choice of the laser parameters. We show below two simple schemes using $S_{1/2}$ and $P_{1/2}$ states that satisfy well this requirement.

The vdW Hamiltonian between two atoms in either $S_{1/2} + S_{1/2}$, $S_{1/2} + P_{1/2}$ or $P_{1/2} + P_{1/2}$ in the Zeeman basis has the following form [36]:

$$\hat{\mathcal{H}}_{\text{vdW}} = \frac{C_6}{r^6} \mathbb{1}_4 - \frac{C_6^{(a)} + C_6^{(b)} - C_6^{(c)} - C_6^{(d)}}{r^6} \mathcal{D}_0(\theta, \phi), \quad (4)$$

$$C_6 \equiv \frac{2}{27} [C_6^{(a)} + 4C_6^{(b)} + 2(C_6^{(c)} + C_6^{(d)})], \quad (5)$$

where the $C_6^{(p)}$ coefficients correspond to the four different channels describing the possible (L, J) quantum numbers of the intermediate states and $\mathcal{D}_0(\theta, \phi)$ is a traceless matrix that depends on the angles between the interatomic and quantization axes. The channels for $S_{1/2} + S_{1/2}$ and $S_{1/2} + P_{1/2}$ are shown in Table I.

Phonon-swap with $S + S$ states.—The first scheme uses the fact that for a pair of atoms in $nS_{1/2}$ states, the second term in Eq. (4) approximately vanishes [37,39]. This can be

TABLE I. The four channels describing the dipole-allowed virtual processes $(L_1, J_1) + (L_2, J_2) \rightarrow (L'_1, J'_1) + (L'_2, J'_2)$ that lead to vdW interactions.

	$S_{1/2} + S_{1/2}$	$S_{1/2} + P_{1/2}$
(a)	$S_{1/2} + S_{1/2} \rightarrow P_{1/2} + P_{1/2}$	$S_{1/2} + P_{1/2} \rightarrow P_{1/2} + S_{1/2}$
(b)	$S_{1/2} + S_{1/2} \rightarrow P_{3/2} + P_{3/2}$	$S_{1/2} + P_{1/2} \rightarrow P_{3/2} + D_{3/2}$
(c)	$S_{1/2} + S_{1/2} \rightarrow P_{3/2} + P_{1/2}$	$S_{1/2} + P_{1/2} \rightarrow P_{3/2} + S_{1/2}$
(d)	$S_{1/2} + S_{1/2} \rightarrow P_{1/2} + P_{3/2}$	$S_{1/2} + P_{1/2} \rightarrow P_{1/2} + D_{3/2}$

seen from Table I, which shows that the difference between the four channels is only in the fine structure of the intermediate states. In the limit of vanishing fine structure, we have $C_6^{(a)} = C_6^{(b)} = C_6^{(c)} = C_6^{(d)}$. This can also be understood intuitively: the vdW interactions arise from second-order perturbation theory, where the two electrons undergo virtual transitions to intermediate states allowed by the selection rules. If we neglect the fine structure, we are free to use the uncoupled basis $(|L, m_L\rangle \otimes |S, m_S\rangle)$ for the intermediate levels. Since $S_{1/2}$ states are proportional to electronic spin states with definite m_S , i.e., $|S_{1/2}, m_J = \pm \frac{1}{2}\rangle = |L = 0, m_L = 0\rangle \otimes |S = \frac{1}{2}, m_S = \pm \frac{1}{2}\rangle$ and because the dipole-dipole interactions do not act on the electronic spin, the vdW couplings cannot mix states with different m_J . The correction to this scales as $\Delta_{\text{FS}}/\delta$, where Δ_{FS} is the fine structure splitting and δ the energy difference to the intermediate states.

Neglecting these small corrections, and to fourth order in the small parameter $\epsilon = \Omega/2\Delta$, the effective spin-spin interactions between any two data atoms are given by

$$\hat{\mathcal{H}}_{\text{int}}(r) = \text{diag}(\tilde{V}_{++} + \tilde{V}_{+-} - \tilde{V}_{-+} - \tilde{V}_{--}). \quad (6)$$

In the case of data-auxiliary interactions, we have a 2×2 version of Eq. (6). In both cases, the matrix elements are

$$\tilde{V}_{\mu\nu} = \left(\frac{\Omega_{\mu}^{(1)} \Omega_{\nu}^{(2)}}{4\Delta_{\mu}^{(1)} \Delta_{\nu}^{(2)}} \right)^2 \frac{C_6}{r^6 - \frac{C_6}{\Delta_{\mu}^{(1)} + \Delta_{\nu}^{(1)}}}, \quad (7)$$

which are spin independent (i.e., $\tilde{V}_{++} = \tilde{V}_{+-} = \tilde{V}_{-+} = \tilde{V}_{--}$) for a suitable choice of the laser parameters. A trivial example consists of the two laser fields being identical. The cooling protocol with this scheme would thus consist of stopping the quantum simulation or computation, weakly coupling the ground states of both the data and auxiliary atoms to $nS_{1/2}$ states, and waiting for a time of t_s . As an example, Rb atoms separated by $2.36 \mu\text{m}$, and weakly coupled to $60S_{1/2}$ ($C_6/2\pi \approx 138.5 \text{ GHz } \mu\text{m}^6$) with $\Omega/2\pi = 100 \text{ MHz}$ [40] and $\Delta/2\pi = 200 \text{ MHz}$ would experience a phonon coupling of $G/2\pi \approx 1.48 \text{ kHz}$ assuming a trap frequency of $\omega_z/2\pi = 15 \text{ kHz}$. G is about an order of magnitude smaller than the trap frequency and

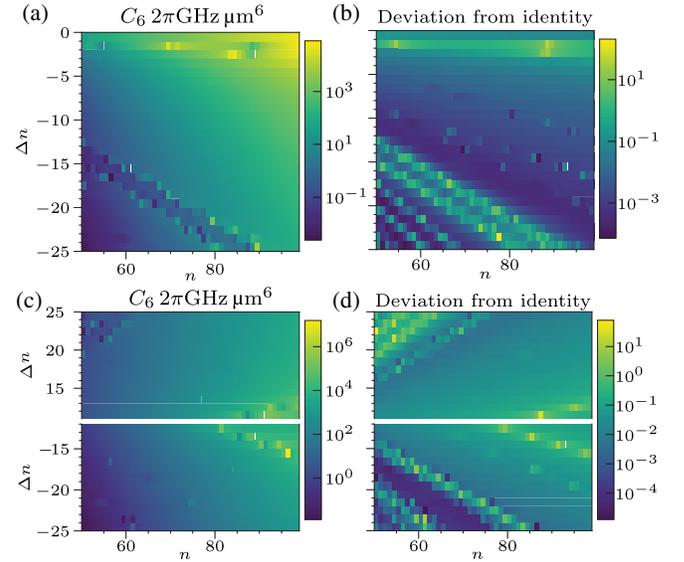


FIG. 3. (a),(c) The spin-insensitive interaction strength C_6 and (b),(d) deviation from identity for (top) $nS_{1/2} + n'S_{1/2}$ and (bottom) $nS_{1/2} + n'P_{1/2}$ as a function of n and $\Delta n = n' - n$ for Rb atoms. In the case of $nS_{1/2} + n'P_{1/2}$, we take $\min |\Delta n| = 10$.

about 2 orders of magnitude larger than the effective decay rate $\epsilon^2 \Gamma_{60S}/2\pi \approx 0.043 \text{ kHz}$, where Γ_{60S} is the decay rate of $60S_{1/2}$ states. The deviation of $\hat{\mathcal{H}}_{\text{vdW}}$ from identity, which we define by the ratio of the operator norms of the two terms in Eq. (4), is, in this case, ≈ 0.027 . This error can be reduced by driving the two atoms to different principal quantum numbers [Fig. 3(b)]. This generally reduces the C_6 coefficient [Fig. 3(a)], but it can nevertheless be sufficiently strong. For instance, $74S_{1/2} + 64S_{1/2}$ yields $C_6/2\pi \approx 29 \text{ GHz } \mu\text{m}^6$ (a factor of 5 smaller than for $60S_{1/2} + 60S_{1/2}$) with an error of ≈ 0.003 (an order of magnitude smaller).

Phonon-swap with S + P states.—This brings us to the second scheme, in which the auxiliary atoms are coupled to $nS_{1/2}$ states, while the data atoms to $n'P_{1/2}$ states, where $|\Delta n| = |n' - n| \gg 1$ in order to ensure that the dipolar interactions between them can be ignored [41]. Such a configuration not only gives spin-independent data-auxiliary interactions, as we will explain below, but also gives rise to tunable spin-spin interactions between the data atoms [37]. To see why $S_{1/2} + P_{1/2}$ gives rise to $\hat{\mathcal{H}}_{\text{vdW}} \propto \mathbb{1}$, note that channels (a, c) as well as (b, d) in Table I only differ by the fine structure in one of the terms. In the limit of vanishing fine structure, the four channels cancel each other pairwise, eliminating $\mathcal{D}_0(\theta, \phi)$ in Eq. (4).

Intuitively, the same argument as in the $S_{1/2} + S_{1/2}$ case shows that there cannot be any mixing between states involving different m_J of the $S_{1/2}$ atom. Hence, in the absence of fine structure in the intermediate manifold, the $S_{1/2}$ atom is effectively decoupled and $\hat{\mathcal{H}}_{\text{vdW}}$ must at least

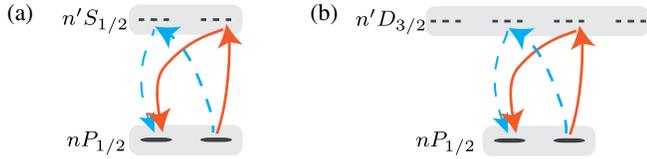


FIG. 4. The four virtual transitions that can couple the $m_J = 1/2$ magnetic state to $m_J = -1/2$, in the $P_{1/2}$ manifold.

be block diagonal. Within this approximation, we can understand why the remaining off-diagonal matrix elements also vanish by focusing solely on the $P_{1/2}$ atom. For each possible sub-channel of the $P_{1/2}$ atom [Fig. 4], there are exactly two processes that can couple its $m_J = +\frac{1}{2}$ and $m_J = -\frac{1}{2}$ states. These two processes, however, precisely destructively interfere. The resulting spin interactions between the data and auxiliary atoms have the same form as in Eqs. (6) and (7). The corresponding C_6 , and the error due to the spin-dependent couplings, are shown in Figs. 3(c), 3(d), respectively.

The data atoms, on the other hand, experience nontrivial spin-spin interactions due to the $P_{1/2} + P_{1/2}$ vdW couplings. For the configuration in Fig. 1 (quantization axis parallel to interatomic axis), the $\mathcal{D}_0(\theta, \phi)$ matrix reads

$$\mathcal{D}_0(0, \phi) = \frac{2}{81} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -4 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (8)$$

which gives rise to the following spin-1/2 Hamiltonian for the data atoms:

$$\hat{\mathcal{H}} = \sum_{ij} J_z^{ij} \hat{S}_z^{(i)} + J_{zz}^{ij} \hat{S}_z \hat{S}_z + (J_{+-}^{ij} \hat{S}_+^{(i)} \hat{S}_-^{(j)} + \text{H.c.}), \quad (9)$$

where $\hat{S}_\alpha^{(i)}$ are the spin-1/2 operators of atom i and $J_{\mu\nu}^{ij}$ are coefficients that depend on the geometry, laser parameters, and Rydberg interactions [37]. This approach can be extended to generate other spin-1/2 models, for instance, in two dimensions [37], with simultaneous cooling.

Summary and outlook.—We have presented a protocol for sympathetically cooling Rydberg atoms without destroying the quantum information stored in their internal states. This can have applications for future Rydberg-based quantum computers and simulators as well as other quantum technologies. Note that while we focused here on the weak coupling regime ($G \ll \omega_z$), which inevitably limits the phonon-swap time to $\sim 1/G \gg 1/\omega_z$, it is possible to speed it up by working in the strong coupling regime $G \sim \omega_z$ and employing optimal control techniques [42–44]. Furthermore, while we used vdW interactions, state-insensitive interactions can also be realized with dipole-dipole interactions and microwave dressing of

Rydberg states [45]. Finally, our state-insensitive interaction schemes could potentially be used in other contexts, such as generating nonclassical states [46] and novel phases of matter [47] combining motional and electronic d.o.f.

R. B., J. T. Y., P. B., Z. E., and A. V. G. acknowledge support by AFOSR, ARL CDQI, NSF PFC at JQI, the DOE ASCR Quantum Testbed Pathfinder Program (Award No. DE-SC0019040), DOE BES QIS program (Award No. DE-SC0019449), NSF PFCQC program, and ARO MURI. R. B. acknowledges in addition the support of NSERC and FRQNT. Z. E. is supported in part by the ARCS Foundation. A. M. K. acknowledges support by NIST. P. Z. was supported by PASQuanS EU Quantum Flagship.

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Supplemental Materials: Nondestructive cooling of an atomic quantum register via state-insensitive Rydberg interactions

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(Dated: October 18, 2019)

This supplemental material is organized as follows: in Sec. I, we derive the Hamiltonian for the van-der-Waals interactions between the Zeeman sublevels of two atoms in either $S_{1/2} + S_{1/2}$, $S_{1/2} + P_{1/2}$, or $P_{1/2} + P_{1/2}$. In Sec. II, we derive the Hamiltonian for the phonon interactions between two atoms and discuss how to implement the phonon-swap for up to two trap components. We also comment on the validity of the Taylor approximation. Then, in Sec. III, we present the adiabatic protocol for the phonon-swap and discuss how to perform 3D cooling by swapping all three trap components. In Sec. IV, we generalize the phonon-swap for a 1D chain of data and auxiliary atoms and derive the time-dependence of the average phonon number in each species. Finally, in Sec. V, we give an example of the spin-1/2 states and the choice of the laser polarizations for a ^{87}Rb atom.

I. VDW INTERACTIONS

In this section, we derive the van-der-Waals interactions (Eqs. (4), (5), and (8) in the main text) between the Zeeman sublevels of two atoms. In second order perturbation theory, this can be written as [S1]

$$\hat{\mathcal{H}}_{\text{vdW}} = \hat{P} \sum_{\alpha, \beta} \frac{\hat{V}_{dd} \hat{Q}_{\alpha, \beta} \hat{V}_{dd}}{\delta_{\alpha, \beta}} \hat{P}, \quad (\text{S1})$$

where $\hat{Q}_{\alpha, \beta} = |\alpha, \beta\rangle \langle \alpha, \beta|$ and $\hat{P} = \sum_{k, l} |k, l\rangle \langle k, l|$ are projectors onto the intermediate and initial states, respectively. The dipole-dipole operator, \hat{V}_{dd} , is given by

$$\hat{V}_{dd} = -\sqrt{\frac{24\pi}{5}} \frac{1}{r^3} \sum_{\mu, \nu} C_{\mu, \nu; \mu+\nu}^{1,1;2} Y_2^{\mu+\nu}(\theta, \phi)^* \hat{d}_{\mu}^{(1)} \hat{d}_{\nu}^{(2)}, \quad (\text{S2})$$

where $C_{m_1, m_2; M}^{J_1, J_2; J}$ is a Clebsch-Gordan coefficient and Y_l^m spherical harmonics. $\hat{d}_{\mu}^{(1)}$ and $\hat{d}_{\nu}^{(2)}$ are the spherical components of the dipole operators for the two atoms ($\mu, \nu \in \{-1, 0, 1\}$), whose matrix elements are

$$\langle n_a, L_a, J_a, m_a | \hat{d}_q | n_c, L_c, J_c, m_c \rangle \equiv \mathcal{R}_{n_a, L_a, J_a, n_c, L_c, J_c} J_{L_a, J_a, m_a, L_c, J_c, m_c}^q, \quad (\text{S3})$$

where

$$\mathcal{R}_{n_a, L_a, J_a, n_c, L_c, J_c} = \int R_{n_a, L_a, J_a}(r) R_{n_c, L_c, J_c}(r) r^3 dr, \quad (\text{S4})$$

is the overlap of the radial wavefunctions $R_{n, L, J}(r)$ and

$$J_{L_a, J_a, m_a, L_c, J_c, m_c}^q = (-1)^{2J_c+1/2+m_a} \sqrt{(2J_a+1)(2J_c+1)(2L_a+1)(2L_c+1)} \\ \begin{pmatrix} J_c & 1 & J_a \\ m_c & q & -m_a \end{pmatrix} \begin{pmatrix} L_a & 1 & L_c \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} J_a & 1 & J_c \\ L_c & 1/2 & L_a \end{Bmatrix}. \quad (\text{S5})$$

We can write the sum over the intermediate states α, β in Eq. (S1) as follows

$$\sum_{\alpha, \beta} = \sum_{\text{channels}} \sum_{n_{\alpha}, n_{\beta}} \sum_{m_{\alpha}, m_{\beta}}, \quad (\text{S6})$$

	$S_{1/2} + S_{1/2}$	$S_{1/2} + P_{1/2}$	$P_{1/2} + P_{1/2}$
(a)	$S_{1/2} + S_{1/2} \rightarrow P_{1/2} + P_{1/2}$	$S_{1/2} + P_{1/2} \rightarrow P_{1/2} + S_{1/2}$	$P_{1/2} + P_{1/2} \rightarrow S_{1/2} + S_{1/2}$
(b)	$S_{1/2} + S_{1/2} \rightarrow P_{3/2} + P_{3/2}$	$S_{1/2} + P_{1/2} \rightarrow P_{3/2} + D_{3/2}$	$P_{1/2} + P_{1/2} \rightarrow D_{3/2} + D_{3/2}$
(c)	$S_{1/2} + S_{1/2} \rightarrow P_{3/2} + P_{1/2}$	$S_{1/2} + P_{1/2} \rightarrow P_{3/2} + S_{1/2}$	$P_{1/2} + P_{1/2} \rightarrow S_{1/2} + D_{3/2}$
(d)	$S_{1/2} + S_{1/2} \rightarrow P_{1/2} + P_{3/2}$	$S_{1/2} + P_{1/2} \rightarrow P_{1/2} + D_{3/2}$	$P_{1/2} + P_{1/2} \rightarrow D_{3/2} + S_{1/2}$

TABLE S1. The four channels describing the dipole-allowed virtual processes $(L_1, J_1) + (L_2, J_2) \rightarrow (L'_1, J'_1) + (L'_2, J'_2)$ that lead to vdW interactions, in the case of both atoms in $S_{1/2}$ states (left), one atom in $S_{1/2}$ and the other in $P_{1/2}$ (middle), both atoms in $P_{1/2}$ states (right).

where the channels for $S_{1/2} + S_{1/2}$, $S_{1/2} + P_{1/2}$ and $P_{1/2} + P_{1/2}$ are given in Table S1.

Note that specifying the channel specifies both the L and J quantum numbers (the principal quantum numbers n are also implicitly specified for the left-hand-side of the channel).

Using Eqs. (S2), (S3) and (S6) we can rewrite Eq. (S1) in the following form

$$\hat{\mathcal{H}}_{\text{vdW}} = \frac{1}{r^6} \sum_{\text{channels}} \left(\sum_{n_\alpha, n_\beta} \frac{(\mathcal{R}_{\alpha,1})^2 (\mathcal{R}_{\beta,2})^2}{\delta_{\alpha\beta}} \right) \times \sum_{\substack{m_k, m_l \\ m'_k, m'_l}} \left[\frac{24\pi}{5} \sum_{m_\alpha, m_\beta} \left(\sum_{\substack{\mu, \nu \\ \mu', \nu'}} C_{\mu, \nu; \mu + \nu}^{1,1;2} Y_2^{\mu + \nu * J_{1k, \alpha}^\mu J_{2l, \beta}^\nu C_{\mu', \nu'; \mu' + \nu'}^{1,1;2} Y_2^{\mu' + \nu' * J_{\alpha, 1k'}^{\mu'} J_{\beta, 2l'}^{\nu'}} \right) |m_k, m_l\rangle \langle m_{k'}, m_{l'}| \right]. \quad (\text{S7})$$

In Eq. (S7), each term in the parentheses on the first line only depends on the intermediate n_α, n_β values, for a given channel. The label α in $\mathcal{R}_{\alpha,1}$ is short for $n_\alpha, L_\alpha, J_\alpha$ where L_α, J_α are specified by the channel. Similarly, the label 1 (2) is specifying the n, L, J values of the first (second) term in the channel.

The quantity in the second line of Eq. (S7) is a 4×4 matrix in the subspace of the magnetic sublevels $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$. For a given channel, the matrix elements are found by summing over the m_α, m_β values and are independent of n .

Thus, for a given channel, p , we can define a $C_6^{(p)}$ coefficient and a matrix $\mathcal{D}^{(p)}$

$$C_6^{(p)} = \sum_{n_\alpha, n_\beta} \frac{(\mathcal{R}_{\alpha,1})^2 (\mathcal{R}_{\beta,2})^2}{\delta_{\alpha\beta}}, \quad (\text{S8})$$

$$\mathcal{D}_{kl, k'l'}^{(p)} = \frac{24\pi}{5} \sum_{m_\alpha, m_\beta} \left(\sum_{\substack{\mu, \nu \\ \mu', \nu'}} C_{\mu, \nu; \mu + \nu}^{1,1;2} Y_2^{\mu + \nu * J_{1k, \alpha}^\mu J_{2l, \beta}^\nu C_{\mu', \nu'; \mu' + \nu'}^{1,1;2} Y_2^{\mu' + \nu' * J_{\alpha, 1k'}^{\mu'} J_{\beta, 2l'}^{\nu'}} \right).$$

With these, the van-der-Waals Hamiltonian takes the simple form

$$\hat{\mathcal{H}}_{\text{vdW}} = \frac{1}{r^6} \sum_p C_6^{(p)} \mathcal{D}^{(p)}. \quad (\text{S9})$$

For the channels in Table S1 (same results for all three cases) we find (different definition than in [S1])

$$\begin{aligned} \mathcal{D}^{(a)} &= \frac{2}{27} \mathbf{1} - \mathcal{D}_0, \\ \mathcal{D}^{(b)} &= \frac{8}{27} \mathbf{1} - \mathcal{D}_0, \\ \mathcal{D}^{(c)} &= \frac{4}{27} \mathbf{1} + \mathcal{D}_0, \\ \mathcal{D}^{(d)} &= \frac{4}{27} \mathbf{1} + \mathcal{D}_0, \end{aligned} \quad (\text{S10})$$

where

$$\mathcal{D}_0(\theta, \phi) = \begin{pmatrix} \frac{1}{81}(3 \cos(2\theta) - 1) & \frac{2}{27}e^{-i\phi} \cos(\theta) \sin(\theta) & \frac{2}{27}e^{-i\phi} \cos(\theta) \sin(\theta) & \frac{2}{27}e^{-2i\phi} \sin^2(\theta) \\ \frac{1}{27}e^{i\phi} \sin(2\theta) & \frac{1}{81}(1 - 3 \cos(2\theta)) & \frac{1}{81}(-3 \cos(2\theta) - 5) & \frac{1}{27}(-2)e^{-i\phi} \cos(\theta) \sin(\theta) \\ \frac{1}{27}e^{i\phi} \sin(2\theta) & \frac{1}{81}(-3 \cos(2\theta) - 5) & \frac{1}{81}(1 - 3 \cos(2\theta)) & \frac{1}{27}(-2)e^{-i\phi} \cos(\theta) \sin(\theta) \\ \frac{2}{27}e^{2i\phi} \sin^2(\theta) & -\frac{1}{27}e^{i\phi} \sin(2\theta) & -\frac{1}{27}e^{i\phi} \sin(2\theta) & \frac{1}{81}(3 \cos(2\theta) - 1) \end{pmatrix}, \quad (\text{S11})$$

is a traceless matrix.

Finally, the vdW Hamiltonian is thus given by Eq. (4) in the main text.

II. PHONON INTERACTIONS

In this section, we derive the effective phonon interactions (Eq. (1) in the main text) between two atoms in harmonic traps, separated by a macroscopic distance r . We assume that the interactions are independent of the internal state and are given by

$$\hat{\mathcal{H}}_{\text{int}}(r) = \frac{\mathcal{A}}{r^6 + R_c^6}, \quad (\text{S12})$$

where R_c is a blockade radius and \mathcal{A} depends on the vdW interaction strength. We further assume that the position of each atom can be decomposed into quantum fluctuations on top of a coherent (classical) part: $\mathbf{r}_i \rightarrow \mathbf{r}_i + \hat{\mathbf{r}}_i$. Without loss of generality, we assume that the macroscopic separation $r_0 = |\mathbf{r}_1 - \mathbf{r}_2|$ is along the y direction. In this case, to second order in the small quantum fluctuations we get

$$\hat{\mathcal{H}}_{\text{int}} \approx \hat{\mathcal{H}}_x + \hat{\mathcal{H}}_y + \hat{\mathcal{H}}_z + \text{Constants}, \quad (\text{S13})$$

where

$$\begin{aligned} \hat{\mathcal{H}}_x &= -3\mathcal{A} \frac{(\hat{x}_1 - \hat{x}_2)^2}{r_0^8 [1 + (R_c/r_0)^6]^2}, \\ \hat{\mathcal{H}}_y &= -6\mathcal{A} \frac{(\hat{y}_1 - \hat{y}_2)}{r_0^7 [1 + (R_c/r_0)^6]^2} + 3\mathcal{A} \frac{(\hat{y}_1 - \hat{y}_2)^2 [7 - 5(R_c/r_0)^6]}{r_0^8 [1 + (R_c/r_0)^6]^3}, \\ \hat{\mathcal{H}}_z &= -3\mathcal{A} \frac{(\hat{z}_1 - \hat{z}_2)^2}{r_0^8 [1 + (R_c/r_0)^6]^2}. \end{aligned} \quad (\text{S14})$$

The full Hamiltonian of the motional degrees of freedom is

$$\hat{\mathcal{H}} = \sum_{\alpha=x,y,z} \left[\sum_{i=1,2} \left(\frac{\hat{P}_{i,\alpha}^2}{2M} + \frac{1}{2} M \omega_\alpha^2 \hat{\alpha}_i^2 \right) + \hat{\mathcal{H}}_\alpha \right]. \quad (\text{S15})$$

The full Hamiltonian is therefore a sum of three independent, commuting Hamiltonians for the three directions, which means we can analyze each direction separately. Note that $\hat{\mathcal{H}}_x$ and $\hat{\mathcal{H}}_z$ have the same form while $\hat{\mathcal{H}}_y$ contains a linear term. This linear term, which represents the force between the two atoms, is inherently larger than the quadratic term which gives rise to the phonon-swap terms. This fact prevents an efficient cooling of the y direction. In Sec. III we show how one can overcome this and nevertheless cool all three directions using an adiabatic protocol. Here we assume that the confinement along y is sufficiently strong and hence focus on the x and z directions.

The Hamiltonian for the z direction in terms of bosonic creation and annihilation operators $\hat{z}_1 = \frac{1}{\sqrt{2M\omega_z}}(\hat{a}_z + \hat{a}_z^\dagger)$, $\hat{P}_{1,z} = -i\frac{\sqrt{M\omega_z}}{2}(\hat{a}_z - \hat{a}_z^\dagger)$ and $\hat{z}_2 = \frac{1}{\sqrt{2M\omega_z}}(\hat{d}_z + \hat{d}_z^\dagger)$, $\hat{P}_{2,z} = -i\frac{\sqrt{M\omega_z}}{2}(\hat{d}_z - \hat{d}_z^\dagger)$ is given by (the Hamiltonian for x is the same with $z \rightarrow x$)

$$\hat{\mathcal{H}}_{\text{ph},z} = \omega_z(\hat{d}_z^\dagger \hat{d}_z + \hat{a}_z^\dagger \hat{a}_z) - \frac{G_z}{2} [(\hat{d}_z + \hat{d}_z^\dagger)^2 + (\hat{a}_z + \hat{a}_z^\dagger)^2] + G_z(\hat{d}_z + \hat{d}_z^\dagger)(\hat{a}_z + \hat{a}_z^\dagger), \quad (\text{S16})$$

where the phonon-coupling G_z is

$$G_z = \frac{3\mathcal{A}}{M\omega_z r_0^8} \frac{1}{[1 + (R_c/r_0)^6]^2}. \quad (\text{S17})$$

which is given under Eq. (1) of the main text, where we dropped the z label. Assuming that $\omega_z \gg G_z$ and making the rotating wave approximation we have

$$\hat{\mathcal{H}}_{\text{ph},z} \approx \omega_z(\hat{d}_z^\dagger \hat{d}_z + \hat{a}_z^\dagger \hat{a}_z) + G_z(\hat{d}_z \hat{a}_z^\dagger + \hat{d}_z^\dagger \hat{a}_z), \quad (\text{S18})$$

or in the rotating frame simply $G_z(\hat{d}_z \hat{a}_z^\dagger + \hat{d}_z^\dagger \hat{a}_z)$.

This Hamiltonian effectuates a state-transfer between the two modes \hat{a}_z, \hat{d}_z , which can be seen from the solution to the Heisenberg equations of motion $(\hat{d}_z(t) = i[G_z(\hat{d}_z \hat{a}_z^\dagger + \hat{d}_z^\dagger \hat{a}_z), \hat{d}_z], \hat{a}_z(t) = i[G_z(\hat{d}_z \hat{a}_z^\dagger + \hat{d}_z^\dagger \hat{a}_z), \hat{a}_z])$

$$\begin{aligned} \hat{a}_z(t) &= \cos(G_z t) \hat{a}_z(0) - i \sin(G_z t) \hat{d}_z(0), \\ \hat{d}_z(t) &= -i \sin(G_z t) \hat{a}_z(0) + \cos(G_z t) \hat{d}_z(0). \end{aligned} \quad (\text{S19})$$

After a time of $t_s = \frac{\pi}{2G_z}$, the states of the two modes, and hence the phonon occupations, are swapped. If, in addition, we have that $\omega_z = \omega_x$ (and accordingly $G_z = G_x$) then the same swap process would cool both the x and z directions.

Finally, let us comment on the higher-order terms that we neglect in the Taylor expansion. Each term in the expansion of Eq. (S12) is smaller than the precedent by the dimensionless factor $\sim \frac{1}{r_0} \sqrt{\frac{1}{M\omega_\alpha}}$ ($\alpha = x, y, z$). For Rubidium atoms separated by $3 \mu\text{m}$ in $\omega_z/2\pi = 15 \text{ kHz}$ traps (assuming $\omega_z < \omega_{x,y}$) this factor is ~ 0.03 . Moreover, if we work in the regime where the rotating-wave approximation is valid, i.e $\omega_z \gg G_z$, all the terms that do not conserve the total number of excitations, and in particular all the odd powers in the expansion, can be neglected. Thus, in that regime, the leading order correction to Eq. (S16) is smaller by the factor $\sim \left(\frac{1}{r_0} \sqrt{\frac{1}{M\omega_z}}\right)^2 \sim 9 \times 10^{-4}$.

III. ADIABATIC PHONON-SWAP

In this section, we present an adiabatic protocol for performing the phonon-swap. As we have discussed in the previous section, the repulsive force between a pair of atoms prevents the simple phonon-swap from taking place for the trap component parallel to the inter-atomic axis. This manifests itself in the presence of the linear term in the y component of Eq. (S14). We show here how this can be mitigated by implementing a smooth, slowly varying $\pi/2$ pulse.

This adiabatic protocol can be intuitively understood as follows: we imagine slowly turning on and off the interactions $[\mathcal{A} \rightarrow \mathcal{A}(t)]$ such that the atoms adiabatically follow the new equilibrium positions, determined by the total potential, which is the sum of the trap potentials and the interactions. During this time, the phonon-swap can take place, swapping the phonon excitations while the displacements slowly change.

We assume the same setup as in the previous section, where the two atoms, data and auxiliary, are initially (at time $t = 0$) separated by some distance $r_0 \equiv r(0) = y_2^{eq}(0) - y_1^{eq}(0)$ (determined by the trap separation). As we slowly increase $\mathcal{A}(t)$, the equilibrium positions (which at $t = 0$ are $y_1^{eq}(0) = 0, y_2^{eq}(0) = r_0$) slowly change as well. These equilibrium positions are found by minimizing the full potential at each time t , and are given by the solutions to the following equations

$$\begin{aligned} M\omega_y^2 y_1^{eq}(t) - \frac{6\mathcal{A}(t)[y_1^{eq}(t) - y_2^{eq}(t)]^5}{[R_c^6 + (y_1^{eq}(t) - y_2^{eq}(t))^6]^2} &= 0, \\ \frac{6\mathcal{A}(t)[y_1^{eq}(t) - y_2^{eq}(t)]^5}{[R_c^6 + (y_1^{eq}(t) - y_2^{eq}(t))^6]^2} + M\omega_y^2 (y_2^{eq}(t) - r_0) &= 0. \end{aligned} \quad (\text{S20})$$

Taylor expanding the potential about those equilibrium positions gives (up to constants)

$$\hat{\mathcal{H}}_{\text{ph},y} = \frac{\hat{P}_{1,y}^2 + \hat{P}_{2,y}^2}{2M} + \frac{1}{2}M\omega_y^2 (\hat{y}_1 - y_1^{eq}(t))^2 + \frac{1}{2}M\omega_y^2 (\hat{y}_2 - y_2^{eq}(t))^2 - M\omega_y G_y(t) [\hat{y}_1 - y_1^{eq}(t) - (\hat{y}_2 - y_2^{eq}(t))]^2, \quad (\text{S21})$$

where

$$G_y(t) = -\frac{3\mathcal{A}(t)}{M\omega_y} \frac{7 - 5(R_b/r(t))^6}{r(t)^8 [1 + (R_b/r(t))^6]^3}, \quad r(t) = y_2^{eq}(t) - y_1^{eq}(t). \quad (\text{S22})$$

The x and z Hamiltonians are still given by Eq. (S14) and Eq. (S16) with the only difference being that G_z, G_x are now time-dependent. In the following we therefore first focus on the y component. Note that by expanding about

the equilibrium positions, we have implicitly assumed that the process is adiabatic. This assumption can be justified self-consistently, as we show later in this section.

We now transform to the bosonic creation and annihilation operators $\hat{y}_1 = \frac{1}{\sqrt{2M\omega_y}}(\hat{a}_y + \hat{a}_y^\dagger)$, $\hat{y}_2 = \frac{1}{\sqrt{2M\omega_y}}(\hat{d}_y + \hat{d}_y^\dagger)$, $\hat{P}_{1,y} = i\sqrt{\frac{M\omega_y}{2}}(\hat{a}_y^\dagger - \hat{a}_y)$, $\hat{P}_{2,y} = i\sqrt{\frac{M\omega_y}{2}}(\hat{d}_y^\dagger - \hat{d}_y)$ which gives

$$\hat{\mathcal{H}}_{\text{ph},y} = \omega_y(\hat{a}_y^\dagger \hat{a}_y + \hat{d}_y^\dagger \hat{d}_y) - \omega_y \alpha_1(t)(\hat{a}_y + \hat{a}_y^\dagger) - \omega_y \alpha_2(t)(\hat{d}_y + \hat{d}_y^\dagger) - \frac{G_y(t)}{2}[\hat{a}_y + \hat{a}_y^\dagger - 2\alpha_1(t) - (\hat{d}_y + \hat{d}_y^\dagger - 2\alpha_2(t))]^2, \quad (\text{S23})$$

where $\alpha_i(t) = \sqrt{\frac{M\omega_y}{2}} y_i^{eq}(t)$. Moving to the adiabatic frame with the displacement operator $\hat{D}(t) = \exp[-\alpha_1(t)(\hat{a}_y^\dagger - \hat{a}_y) - \alpha_2(t)(\hat{d}_y^\dagger - \hat{d}_y)]$ yields

$$\begin{aligned} \hat{\mathcal{H}}_{\text{ph},y,\text{ad}} &= \hat{D}(t)\hat{\mathcal{H}}_{\text{ph},y}\hat{D}^\dagger(t) + i\dot{D}(t)D^\dagger(t) \\ &= \omega_y(\hat{a}_y^\dagger \hat{a}_y + \hat{d}_y^\dagger \hat{d}_y) - \frac{G_y(t)}{2}[(\hat{a}_y^\dagger + \hat{a}_y)^2 + (\hat{d}_y^\dagger + \hat{d}_y)^2] + G_y(t)(\hat{a}_y^\dagger + \hat{a}_y)(\hat{d}_y^\dagger + \hat{d}_y) + i\dot{\alpha}_1(\hat{a}_y - \hat{a}_y^\dagger) + i\dot{\alpha}_2(\hat{d}_y - \hat{d}_y^\dagger). \end{aligned} \quad (\text{S24})$$

From Eq. (S24) we can see that the adiabatic Hamiltonian for the y component has a similar structure as the x and z Hamiltonians in the previous section in Eq. (S16), with additional non-adiabatic corrections proportional to $\dot{\alpha}_1$ and $\dot{\alpha}_2$. The adiabaticity condition is therefore $\omega_y \gg \dot{\alpha}_1, \dot{\alpha}_2$, which together with the condition $\omega_y \gg G_y$ allows us to make the rotating wave approximation, giving

$$\hat{\mathcal{H}}_{\text{ph},y,\text{ad}} \approx \omega_y(\hat{a}_y^\dagger \hat{a}_y + \hat{d}_y^\dagger \hat{d}_y) + G_y(t)(\hat{a}_y^\dagger \hat{d}_y + \hat{a}_y \hat{d}_y^\dagger). \quad (\text{S25})$$

If $\omega_y \gg \dot{\alpha}_1, \dot{\alpha}_2$, then the atoms follow adiabatically the equilibria of the total potential. This is in fact the justification for the self-consistent assumption mentioned at the beginning of this section.

Equation (S25) effectuates a state-transfer between the two modes, exactly as the time-independent version in the previous section [see Eq. (S18)]. The solution of the Heisenberg equations (in the rotating frame) are in this case (with equivalent expressions for the x, z components)

$$\begin{aligned} \hat{a}_y(t) &= \cos\left(\int G_y(t)dt\right)\hat{a}_y(0) - i\sin\left(\int G_y(t)dt\right)\hat{d}_y(0), \\ \hat{d}_y(t) &= -i\sin\left(\int G_y(t)dt\right)\hat{a}_y(0) + \cos\left(\int G_y(t)dt\right)\hat{d}_y(0). \end{aligned} \quad (\text{S26})$$

For a full phonon-swap of the y phonons to take place we require $\int G_y(t)dt = \frac{\pi}{2}$. However, since the phonon interaction strength for the y direction is different than for the x and z directions [i.e., $G_y(t) \neq G_{x,z}(t)$ as one can see in Fig. S1(a)] a $\pi/2$ pulse for y is not necessarily a $\pi/2$ pulse for the other two directions. Nevertheless, in typical scenarios the traps are not isotropic, and one can utilize this fact together with the different interaction curves to compensate and optimize a pulse that is as close as possible to $\pi/2$ for all three directions.

As a simple example, we take a Gaussian pulse for $\mathcal{A}(t) = \mathcal{A}_{\text{max}}\left[\exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right) - c\right]$ where c is a constant chosen such that $\mathcal{A}(0) = 0$. Using similar parameters as in the main text (see caption of Fig. S1), assuming $\omega_x/2\pi = \omega_z/2\pi = 15$ kHz and $\omega_y/2\pi = 50$ kHz we can find pulse parameters that yield $\int G_y(t)dt/(\pi/2) \approx \int G_{x,z}(t)dt/(\pi/2) \approx 1$. Furthermore, even if for a given parameter regime it is not possible to perform the 3D phonon-swap with a single pulse, one can always perform the cooling in three steps: one can first apply a $\pi/2$ pulse to swap the phonons in the x and z components. This would only partially swap the y phonons. One then has to cool the auxiliary atoms before applying another $\pi/2$ pulse, this time designed to fully swap the y phonons.

Finally, in Fig. S1(b) we show the ratios of $\dot{\alpha}_{1,2}$ to the trap frequency ω_y for the Gaussian pulse described above. As one can see, the adiabaticity constraint is satisfied well at all times.

IV. PHONON-SWAP FOR 1D CHAIN

Here we generalize the results of the previous sections to the more realistic case of an atomic register consisting of many atoms. We derive Eq. (3) from the main text.

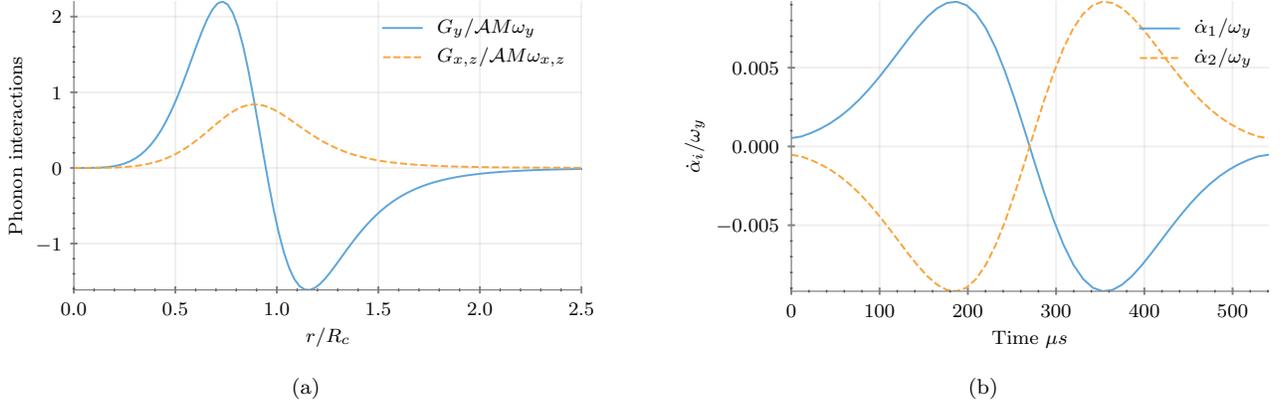


FIG. S1. (a) Normalized phonon interactions as a function of distance. (b) Non-adiabatic corrections normalized to the trap frequency $\omega_y/2\pi = 50$ kHz as a function of time for a Gaussian pulse with $\mathcal{A}_{max}/2\pi = 34.4$ MHz μm^6 , $\sigma = 6.7$ μs , $t_0 = 215.9$ μs for a trap separation of $r_0 = 1.93$ μm .

For concreteness we consider the setup in Fig. 1(a) of the main text where a 1D chain of N auxiliary atoms (lattice constant x_0) is brought to a distance of y_0 from an identical 1D chain of N data atoms. For simplicity we only consider a single trap direction (z) and the time-independent swap protocol. The generalizations to two or three directions and the adiabatic swap are straightforward. The Hamiltonian for the vibrational modes in the z direction is (we drop the z labels from here on)

$$\hat{\mathcal{H}} = \sum_{i=1}^N \omega_z (\hat{a}_i^\dagger \hat{a}_i + \hat{d}_i^\dagger \hat{d}_i) - \frac{1}{2} \sum_{i=1, j=1, i \neq j}^N G_{ij} [(\hat{z}_i^a - \hat{z}_j^a)^2 + (\hat{z}_i^d - \hat{z}_j^d)^2] - \sum_{i=1, j=1}^N F_{ij} (\hat{z}_i^a - \hat{z}_j^d)^2, \quad (\text{S27})$$

where by \hat{a}_i (\hat{d}_i) we denote the phonon-annihilation operator for an auxiliary (data) atom i and by $z_i^a = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger)$ ($z_i^d = \frac{1}{\sqrt{2}}(\hat{d} + \hat{d}^\dagger)$) the z coordinate of an auxiliary (data) atom (we have absorbed $M\omega_z$ into the definition of G_{ij} and F_{ij}). The $\frac{1}{2}$ is to avoid double-counting and the coefficients are given by

$$G_{ij} = \frac{G}{\eta^8 |i - j|^8}, \quad (\text{S28})$$

$$F_{ij} = \frac{G}{[\eta^2 (i - j)^2 + 1]^4},$$

where G is defined in Eq. (S17) with $r = y_0$ and $\eta \equiv \frac{x_0}{y_0}$. To be consistent with the two-atom case, we have defined G with the nearest neighbor separation between data-auxiliary atoms (y_0). We have also assumed that pairs of atoms that are farther apart than the nearest-neighbor separation (i.e next nearest-neighbors and so on) experience power-law interactions. In other words, we assumed that the separation between next-nearest neighbors is significantly larger than the blockade radius. In terms of bosonic operators Eq. (S27) is

$$\hat{\mathcal{H}} = \sum_{i=1}^N \left[\tilde{\omega}_{z,i} (\hat{a}_i^\dagger \hat{a}_i + \hat{d}_i^\dagger \hat{d}_i) - \frac{1}{2} (\tilde{G}_i + \tilde{F}_i) (\hat{a}_i^2 + \hat{d}_i^2 + \text{H.c.}) \right] + \frac{1}{2} \sum_{i \neq j} G_{ij} (\hat{a}_i \hat{a}_j + \hat{a}_i^\dagger \hat{a}_j^\dagger + \hat{d}_i \hat{d}_j + \hat{d}_i^\dagger \hat{d}_j^\dagger + \text{H.c.}) + \sum_{ij} F_{ij} (\hat{a}_i \hat{d}_j + \hat{a}_i^\dagger \hat{d}_j^\dagger + \text{H.c.}), \quad (\text{S29})$$

where

$$\tilde{\omega}_{z,i} = \omega_z + \tilde{G}_i + \tilde{F}_i, \quad (\text{S30})$$

$$\tilde{G}_i = \sum_{j \neq i} G_{ij},$$

$$\tilde{F}_i = \sum_j F_{ij}.$$

We now assume that the system is translationally invariant, i.e., $\tilde{\omega}_{z,i} \approx \tilde{\omega}_z$, $\tilde{G}_i \approx \tilde{G}$, $\tilde{F}_i \approx \tilde{F}$ for all i . This is a good approximation for the “bulk” of the atoms, away from the edges, in the limit where $N \rightarrow \infty$, or for a system with periodic boundary conditions. We also assume that $\tilde{\omega}_z \gg \tilde{G}, \tilde{F}$ which allows us to drop terms that do not conserve the total number of excitations. With these assumptions, the Hamiltonian in the rotating frame is given by

$$\hat{\mathcal{H}} = \sum_{i \neq j} G_{ij} (\hat{a}_i^\dagger \hat{a}_j + \hat{d}_i^\dagger \hat{d}_j) + \sum_{ij} F_{ij} (\hat{a}_i \hat{d}_j^\dagger + \hat{a}_i^\dagger \hat{d}_j). \quad (\text{S31})$$

Taking the Fourier transform with

$$\hat{a}_n = \frac{1}{\sqrt{N}} \sum_k \hat{a}_k e^{ikn x_0}, \quad \hat{a}_k = \frac{1}{\sqrt{N}} \sum_n \hat{a}_n e^{-ikn x_0}, \quad (\text{S32})$$

and using the fact that G_{ij} and F_{ij} are translationally invariant, i.e. depend on $|i - j|$, we get

$$\hat{\mathcal{H}} = \sum_k \left[G_k (\hat{a}_k^\dagger \hat{a}_k + \hat{d}_k^\dagger \hat{d}_k) + F_k (\hat{a}_k \hat{d}_k^\dagger + \hat{a}_k^\dagger \hat{d}_k) \right], \quad (\text{S33})$$

where we used $\sum_n e^{i(k-k')n x_0} = N \delta_{k,k'}$ and the following definitions

$$\begin{aligned} G_k &= \sum_{n=-N, n \neq 0}^N G_n e^{-ikn x_0} = 2 \sum_{n=1}^N G_n \cos(kn x_0), \\ F_k &= \sum_{n=-N}^N G_n e^{-ikn x_0} = F_{n=0} + 2 \sum_{n=1}^N F_n \cos(kn x_0). \end{aligned} \quad (\text{S34})$$

Equation (S33) can be diagonalized with the transformation

$$\hat{c}_k = \frac{\hat{a}_k + \hat{d}_k}{\sqrt{2}}, \quad \hat{b}_k = \frac{\hat{a}_k - \hat{d}_k}{\sqrt{2}}, \quad (\text{S35})$$

which gives

$$\hat{\mathcal{H}} = \sum_k \left[(G_k + F_k) \hat{c}_k^\dagger \hat{c}_k + (G_k - F_k) \hat{b}_k^\dagger \hat{b}_k \right]. \quad (\text{S36})$$

Using this, we can now compute the average excitation number in the auxiliary and data atoms, given by

$$\begin{aligned} \bar{n}_a(t) &= \frac{1}{N} \sum_n \langle \hat{a}_n^\dagger(t) \hat{a}_n(t) \rangle, \\ \bar{n}_d(t) &= \frac{1}{N} \sum_n \langle \hat{d}_n^\dagger(t) \hat{d}_n(t) \rangle. \end{aligned} \quad (\text{S37})$$

Below, we first compute $\bar{n}_a(t)$:

$$\begin{aligned} \bar{n}_a(t) &= \frac{1}{N} \sum_n \langle \hat{a}_n^\dagger(t) \hat{a}_n(t) \rangle \\ &= \frac{1}{4N^2} \sum_k \sum_{nm} e^{ikx_0(n-m)} \left\langle 4 \cos^2(F_k t) \hat{a}_n^\dagger \hat{a}_m + 4 \sin^2(F_k t) \hat{d}_n^\dagger \hat{d}_m + 2i \sin(2F_k t) \hat{a}_n^\dagger \hat{d}_m - 2i \sin(2F_k t) \hat{d}_n^\dagger \hat{a}_m \right\rangle. \end{aligned} \quad (\text{S38})$$

For simplicity, we now assume that the initial state is a product state and also that $\langle \hat{a}_i \rangle = \langle \hat{a}_i^2 \rangle = \langle \hat{d}_i \rangle = \langle \hat{d}_i^2 \rangle = 0$ for all i . This would be the case, for example, if every atom starts at a pure Fock state or a thermal state. With this assumption, only the diagonal terms in Eq. (S38) contribute, yielding

$$\bar{n}_a(t) = \frac{1}{N} \left[\sum_k \cos^2(F_k t) \right] \bar{n}_a(0) + \frac{1}{N} \left[\sum_k \sin^2(F_k t) \right] \bar{n}_d(0). \quad (\text{S39})$$

Taking the continuum limit $\frac{1}{N} \sum_k \rightarrow \frac{x_0}{2\pi} \int_{-\pi/x_0}^{\pi/x_0} dk$ and changing variables $kx_0 \rightarrow k$ gives

$$\bar{n}_a(t) = \bar{n}_a(0) \int_{-\pi}^{\pi} \frac{dk}{2\pi} \cos^2(F_k t) + \bar{n}_d(0) \int_{-\pi}^{\pi} \frac{dk}{2\pi} \sin^2(F_k t). \quad (\text{S40})$$

To obtain a closed form expression, we approximate the sum in F_k by the first term $n = 1$ which corresponds to only keeping up to next nearest-neighbors interactions between auxiliary and data atoms. This gives rise to

$$\begin{aligned} \bar{n}_a(t) &= \frac{\bar{n}_a(0) + \bar{n}_d(0)}{2} + \frac{\bar{n}_a(0) - \bar{n}_d(0)}{2} J_0 \left[\frac{4Gt}{(1 + \eta^2)^4} \right] \cos(2Gt), \\ \bar{n}_d(t) &= \frac{\bar{n}_a(0) + \bar{n}_d(0)}{2} - \frac{\bar{n}_a(0) - \bar{n}_d(0)}{2} J_0 \left[\frac{4Gt}{(1 + \eta^2)^4} \right] \cos(2Gt), \end{aligned} \quad (\text{S41})$$

where $J_0(z)$ is a Bessel function of the first kind.

V. LASER EXCITATION FROM GROUND STATES

In this section we give an example level structure and laser polarization choice for ^{87}Rb atoms for some of the schemes we presented in the main text. One choice for the spin-1/2 states of the data atoms are the following two hyperfine ground states

$$\begin{aligned} |g_-\rangle &\equiv |5^2S_{1/2}, F = 1, m_F = 1\rangle, \\ |g_+\rangle &\equiv |5^2S_{1/2}, F = 2, m_F = 2\rangle. \end{aligned} \quad (\text{S42})$$

To excite to $S_{1/2}$ states, we need to use an intermediate P state. Using σ_+ , σ_- , and σ_0 polarized light, one can for example use the following ladder scheme

$$\begin{aligned} |g_-\rangle &\xrightarrow{\sigma_0} |5P_{3/2}, F = 1, m_F = 1\rangle \xrightarrow{\sigma_+} |nS_{1/2}, m_J = +\frac{1}{2}\rangle, \\ |g_+\rangle &\xrightarrow{\sigma_0} |5P_{3/2}, F = 2, m_F = 2\rangle \xrightarrow{\sigma_-} |nS_{1/2}, m_J = -\frac{1}{2}\rangle. \end{aligned} \quad (\text{S43})$$

For the auxiliary atoms, a single state out of the two is sufficient. For exciting to $P_{1/2}$ states, one choice is the following

$$\begin{aligned} |g_-\rangle &\xrightarrow{\sigma_+} |nP_{1/2}, m_J = +\frac{1}{2}\rangle, \\ |g_+\rangle &\xrightarrow{\sigma_-} |nP_{1/2}, m_J = -\frac{1}{2}\rangle. \end{aligned} \quad (\text{S44})$$

[S1] A. W. Glaetzle, M. Dalmonte, R. Nath, C. Gross, I. Bloch, and P. Zoller, [Phys. Rev. Lett. **114**, 173002 \(2015\)](#).