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On Practice and Theory of Constructive Composite Geometry and Topology

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Abstract

This paper synthesizes a theory from industrial best practices codified in recent standards. Recent editions of ASME and ISO standards codify the evolving industrial best practices in defining and modeling the information about products made from fibrous composite materials. A theory of constructive composite geometry and topology is synthesized from these practices. Major features of this theory include (1) a constructive composite geometry tree that is equivalent to the ply/laminate tables of the standards, (2) an adjacency graph that captures a crucial aspect of the topology of the geometric cell complex structure of composite products, and (3) conformal mapping of ply surfaces using rosettes in the lay-up process. It also addresses the geometrical and topological structure of fiber arrangements inside the plies. The goal of the theory is to provide a scientific basis for standards that enable the digital transformation of composite product manufacturing.

1 Introduction

Products made of fibrous composite materials are everywhere, but a good theory for representing them in practice is notably absent. This has forced engineers to depend on drawings, cross-sectional views, notes, and tabular entries to define fibrous composite products and guide their manufacturing processes. Some solid and geometrical modeling tools have been improvised to aid and assist this cumbersome exercise, but these are viewed merely as workarounds in the absence of a good theory and computational tools that support the theory [1].

There are several reasons why it has been so hard to find a good theory in this field. Traditional metallic products can be defined by current computer-aided design (CAD) systems and manufactured by conventional processes; in fact, there are usually multiple ways to manufacture such traditional products. Therefore, such a product can be defined independent of the manufacturing process. But composite products are different because they are strongly dependent on the manufacturing processes used to produce them, and these manufacturing processes are subject to unceasing innovation and evolution. Since *the process defines the product* in these cases, traditional design theory and methodology (which hold that design can be decoupled from manufacturing) no longer apply. This is also the reason why traditional solid and geometrical modeling theories and methodologies implemented in current CAD systems are not sufficient to represent fibrous composite products. This problem is not restricted to composite products – there is a long list of other product categories for which current CAD systems do not provide adequate support [2].

Such deficiencies and drawbacks have not deterred engineers from using the current solid and geometrical modeling systems. They have augmented these systems with additional information, sometimes with ingenious informational artifacts, to carry out their mission. This has created a special breed of CAD systems that cater specifically to the composite product manufacturing domain. This is a bottom-up evolution, where pragmatic practical solutions are leading the development of engineering information systems and tools, without the benefit of a cohesive theory.

Such grass-roots developments have also resulted in major standardization efforts in ASME and ISO (International Organization for Standardization). Past and current editions of ASME Y14.37, ISO 10303-209, and ISO 10303-242 standards have codified the evolving industrial best practices in defining and modeling the information content of composite products [3-5]. These ASME and ISO standards are a combination of what the current solid and geometrical modeling systems can offer, and what additional information is required, to design and manufacture these composite products. What they lack is a comprehensive theory that explains, and expands upon, these sound industrial practices.

In a mathematical sense, composite products are cell complexes with geometrical and topological structures [2]. This paper synthesizes a theory of constructive composite geometry and topology from the industrial best practices codified in the ASME and ISO standards. It offers a procedural representation from which relevant information can be extracted and other representations can be computed. It bears some resemblance to the theory of constructive solid geometry [6, 7], but there are some significant differences due to the rich internal fibrous structures and the flexibility of constituents in composite products.

This paper is envisioned as a first attempt in postulating a theory that is close to practice for fibrous composite products, thereby initiating further discussion to strengthen the theory while still retaining strong relevance to practice. The major scientific and technical contributions of this paper include (1) the introduction of the concept of constructive composite geometry, with a tree representation for it, (2) the capture of some of the critical constructive composite topology information in the form of an adjacency graph, and (3) the use of conformal mapping of oriented surfaces in the ply lay-up. These notions systematize what is already practiced in industry and partially codified in recent standards from ASME and ISO.

The paper is organized as follows. Section 2 describes the constructive nature of composite product definition codified in the ASME standards. Then Section 3 offers a constructive composite geometry representation in the form of a hierarchical tree. Additional information is captured in Section 4 for constructive composite topology in the form of an adjacency graph. Section 5 addresses the geometry and topology of fibers embedded the plies. Section 6 concludes the paper with a summary and some directions for further research and standardization.

2 Constructive Nature of Composite Product Definition

This paper focuses on fibrous composite products, which are also known as fiber-reinforced plastics (FRP). In these composite products, thin fibers (of circular cross-section) made up of materials such as carbon, glass, or aramid (e.g., Kevlar) are embedded in a plastic resin medium called a matrix. Both fibers and resins play important roles in composite products. Fibers are the load bearing members and are the major contributors to the strength and stiffness. The resins transfer loads among the fibers and they provide much needed protection to the fibers from ambient environment (e.g., resistance to corrosion); they are also responsible for the ductility and toughness of the composite product. Thermosetting resins, which include epoxy and polyester, are currently very popular even though they are not recyclable. Thermoplastic resins are recyclable, but such resins are still under development for large-scale industrial use.

An important discrete module of a composite product is a *ply*, which is usually an arrangement (in potentially intricate geometric patterns) of reinforcement fibers in a resin matrix. The ply is often treated as a fabric; in fact, the technology and terminology of manufacturing a ply is strongly influenced by the textile industry. Fibers can be woven in special patterns called 'preforms,' which can be impregnated in resins to produce 'prepregs' [8, 9]. The ASME standard defines a ply as 'one discrete piece of manufactured material (e.g., fabric, tape, adhesive film)' [3]. Two or more plies can be stacked up to form a *laminate* (see Fig. 4 for an illustration).

Another important discrete module in a composite product is a *core*, which is a light weight component sandwiched between plies. The primary role of a core is to increase the 'section modulus' of thin-walled structures without increasing their weight considerably. A core can have hollow interiors that may be filled with air or special gases.

Composite products are produced by stacking up (or, laying up) plies, cores, and other items in a specified sequence, and then subjecting them to a curing process [8, 9]. Curing is a chemical process that enables polymers in the resins contained in the plies to cross-link, which produces a harder and more homogenous matrix within which the fiber reinforcements are firmly embedded. This leads naturally to a procedural representation, where the process for constructing a geometric and information model for the product mimics the manufacturing process that produces it. This notion is retained strongly in the current industrial practice for defining and modeling composite products.

This contrasts with how the geometry of many conventional metallic products are defined, say in a CAD system, using a set of geometrical construction commands. Such commands are intended to be better suited for the ease of constructing the *nominal* three-dimensional (3D) solid and geometric model of a part, ostensibly with little concern to the way in which the part is manufactured, say using traditional machining processes. This approach is strongly encouraged by other popular and long-standing ASME and ISO standards [10, 11] that enforce the dogma that product geometry definition should not be mixed with manufacturing process specification. The manufacturing concerns are, of course, taken into account during tolerancing these parts to ensure their manufacturability. While this good intention of separating design from manufacturing permits multiple ways to produce the same product – thereby enabling optimization on other metrics such as cost, quality, and time – this philosophy does not seem to apply to composite products.

The constructive nature of composite product definition has long been seized upon by engineers who came up with drafting and modeling schemes, while augmenting them with tables and specialized symbols. Such engineering schemes were initially standardized by ISO in 2001 for information exchange and later by ASME in 2012 for drawing practices [12, 13]. These standards have been updated recently in 2019 [3-5].

The standardization of composite product definition by ASME is best illustrated with an example. Figure 1 shows the plan view of a composite part. Following the 'dash number' convention, this part is denoted as -101 and it is constructed by stacking up several plies and a core as shown in a cross-sectional view in Fig. 2. It is clear from these two illustrations that this composite part is actually a bonded assembly of several plies and a core. In fact, it is common to refer to such a part as a 'bond assembly,' sometimes abbreviated as 'BOND ASSY.'

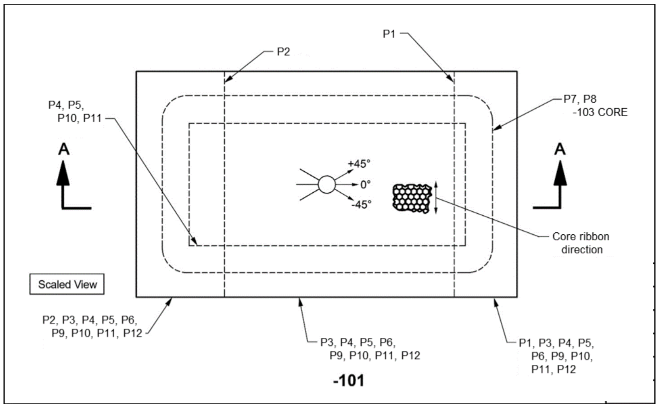


Fig. 1 Plan view of a composite part [3].

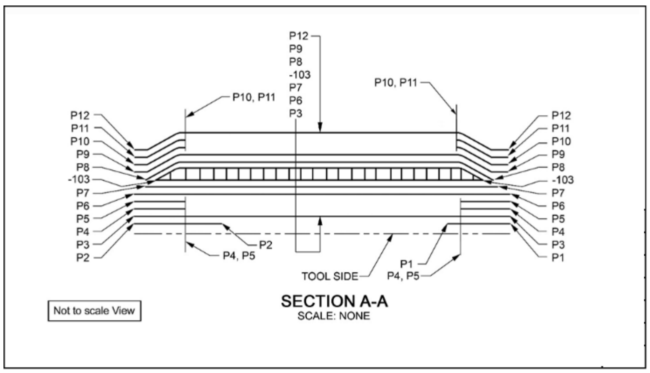


Fig. 2 Cross-sectional view of the composite part in Fig. 1 [3].

Table 1 Ply table for the composite part in Fig. 1 [3].

-101 BOND ASSEMBLY			
PLY LEVEL	PLY/ITEM	ORIENTATION	MATERIAL
1	P1	0°	10745
1	P2	0°	10745
2	P3	0°	10721
3	P4	45°	10721
4	P5	0°	10721
5	P6	45°	10721
6	P7	-45°	10679
7	-103 CORE		
8	P8	-45°	10679
9	P9	45°	10721
10	P10	0°	10721
11	P11	45°	10721
12	P12	0°	10721

The plan and (cross-sectional) elevation views in Figs. 1 and 2 are augmented by an important informational artifact called a *ply table* (also known as a *laminate table*) such as the one shown in Table 1. While these plan and elevation views follow the traditional drawing conventions for all industrial parts, the ply table is a special information that is associated only with composite products. In addition to the 'ply level' and 'ply/item' columns, the ply table in Table 1 has an 'orientation' column to indicate the fiber orientations that provide the necessary direction-dependent anisotropic properties. The last column in Table 1 identifies the material code for each ply, which can link to a much richer set of information specific to that ply. It is often useful to provide a 3D exploded view of the part as shown in Fig. 3 to accompany a ply table and sectional views.

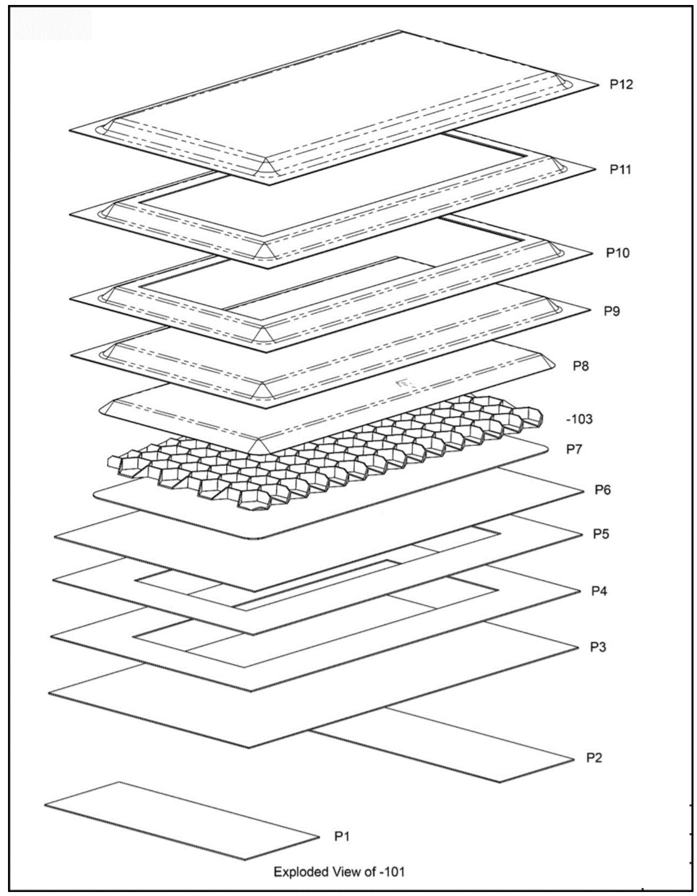


Fig. 3 A 3D exploded view of the composite part in Fig. 1 [3].

The figures and the ply table seen thus far provide a partial definition of a composite part in an *uncured* state. But this is not the final state of the product. Figure 4 explains how the ASME Y14.37 standard defines the uncured and cured states. In the uncured state on the left of Fig. 4, various plies are stacked up in a particular order to form a laminate. This laminate is then subjected to a curing process to produce the cured composite part on the right of Fig. 4. The ply interfaces disappear in this final product. It is interesting to note that much of the ASME and ISO standards are devoted to the definition and information modeling of the uncured state of the composite product and not to the 'net shape.' This important fact will be explored further in the rest of the paper.

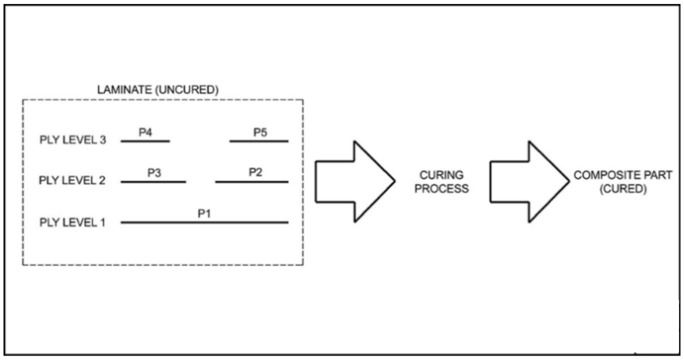


Fig. 4 Uncured and cured states in ASME standard [3].

The composites manufacturing process illustrated in these figures thus far may appear to be similar to semiconductor chip manufacturing. Both employ layered sequences of manufacturing processes, but there are important differences. One difference is that the semiconductor manufacturing uses subtractive processes (e.g., etching) as well as additive processes (e.g., sputtering). Another difference is that the composite manufacturing can produce complex 3D parts. Nevertheless, some of the ideas on geometrical and information modeling associated with semiconductor products may be useful for defining composite products.

While the logical design of a semiconductor chip can be carried out without invoking the manufacturing processes, the physical design of such a chip is strongly dictated by the sequence of process steps to which a semiconductor wafer is subjected. This notion will be carried further for composite products in rest of the paper. Section 3 will address the geometrical aspect, while Section 4 will consider the topological aspect of the constructive nature of composite product definition.

3 Constructive Composite Geometry

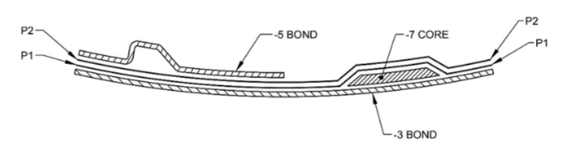
The example illustrated in Section 2 brings out some salient features of plies and ply tables in composite product definition. These include the following:

1. A ply is used both as a discrete physical artifact and as an information container. As a physical artifact, it contains literally the fibers and resins. As an information container it encapsulates complex information about the geometry, topology, and material of the fibers and resins in each instance of the ply. This seems to be the only practical way to handle the physical and information complexity of a composite product.

2. A ply table provides an ordered sequence (arranged from the top row to the bottom row) of the placement of the plies and cores. In a physical sense, this is part of the recipe for the manufacturing process that leads up to the uncured state of the product. In an informational context, this implies some geometrical and topological adjacency of not only the plies but also the fibers contained in them. Since the fibers remain firmly embedded in the hardened plastics after curing, this adjacency information of the plies is inherited by the fibers contained in the plies, both before and especially after curing. In other words, the ply table captures some crucial adjacency relationships among the fibers and these relationships remain invariant under the curing operation.

3. A ply table alone, of course, will not provide all the geometrical and topological information. The accompanying cross-sectional views (such as Fig. 2) and 3D exploded views (such as Fig. 3) supply much needed details. From a geometrical modeling perspective, it is interesting to note that a ply is depicted as a curve in a cross-sectional view in Fig. 2 and as a surface in a 3D exploded view in Fig. 3, even though a ply is a three-dimensional object with some definite thickness. This type of abstraction has a deeper implication than being just a convenient way for graphical presentation.

Further insights from the use of ply tables can be gained from examples shown in Figs. 5 and 6 taken from the ASME standard [3].



-1 BOND ASSY				
SEQUENCE	PLY LEVEL	PLY/ITEM	ORIENTATION	MATERIAL
A	1	-3		-3 BOND
A	2	-7		-7 CORE
A	3	P1	0°	10721
A	4	P2	45°	10721
B	5	-5		-5 BOND

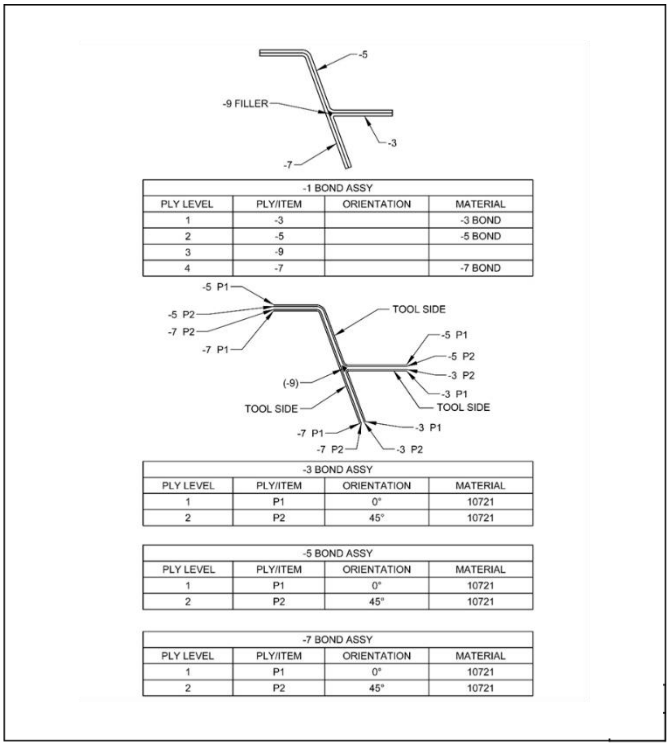
-3 BOND ASSY			
PLY LEVEL	PLY/ITEM	ORIENTATION	MATERIAL
1	P1	0°	10721
2	P2	45°	10721
3	P3	45°	10721
4	P4	0°	10721
5	P5	OPT	PEEL PLY

-5 BOND ASSY			
PLY LEVEL	PLY/ITEM	ORIENTATION	MATERIAL
1	P1	OPT	10745
1	P2	OPT	10745
2	P3	0°	10721
3	P4	45°	10721
4	P5	45°	10721
5	P6	0°	10721
6	P7	OPT	PEEL PLY

Fig. 5 A multi-stage bonded assembly [3].

Figure 5 shows a cross-sectional view and a set of ply tables for a multi-stage bonded assembly, which means that sub-assemblies such as '-3 BOND ASSY' and '-5 BOND ASSY' can be precured and then be bonded into the '-1 BOND ASSY.' It is interesting to observe that the ply table for '-1 BOND ASSY' contains links to other ply tables for '-3 BOND ASSY' and '-5 BOND ASSY,' thus suggesting a hierarchical tree structure for organizing this type of information. A similar example for a complex bonded assembly is shown in Fig. 6, where one ply table contains links to other ply tables.

In addition to the plies, other items such as '-7 CORE' in Fig. 5 and '-9 FILLER' in Fig. 6 can be included in a ply table. These cores and fillers can be modeled as 3D solids; even the precured subassemblies (such as '-3 BOND ASSY' and '-5 BOND ASSY' in Fig. 5; and '-3 BOND ASSY,' '-5 BOND ASSY,' and '-7 BOND ASSY' in Fig. 6) can be modeled as 3D solids. Representations of these solids then become both physical and information containers for other objects, just as the plies discussed earlier.



-1 BOND ASSY			
PLY LEVEL	PLY/ITEM	ORIENTATION	MATERIAL
1	-3		-3 BOND
2	-5		-5 BOND
3	-9		
4	-7		-7 BOND

-3 BOND ASSY			
PLY LEVEL	PLY/ITEM	ORIENTATION	MATERIAL
1	P1	0°	10721
2	P2	45°	10721

-5 BOND ASSY			
PLY LEVEL	PLY/ITEM	ORIENTATION	MATERIAL
1	P1	0°	10721
2	P2	45°	10721

-7 BOND ASSY			
PLY LEVEL	PLY/ITEM	ORIENTATION	MATERIAL
1	P1	0°	10721
2	P2	45°	10721

Fig. 6 A complex bonded assembly [3].

The notion of ply tables and their engineering use have by now been well established and deeply entrenched in composite manufacturing industry. In fact, this practice is so stable that all the earlier and current versions of the ASME and ISO standards have codified them and expanded them to cover other manufacturing processes such as braiding and pultrusion [8, 9]. So, it is reasonable to propose a constructive composite geometry (CCG) representation in the form of a hierarchical tree that is equivalent to a ply table.

Figure 7 shows a CCG tree for the '-101 BOND ASSEMBLY' associated with the ply table in Table 1 and the exploded view in Fig.3. The root node in the tree is the composite product denoted as -101 BOND ASSEMBLY. The leaf nodes are the plies, the core, and the tool. More information, such as 'orientation' and 'material' found in Table 1, can be associated with the leaf nodes as entities and attributes. The interior nodes in the CCG tree in Fig. 7 are denoted by the symbol ⊕ and it stands for an 'adjoin' operation.

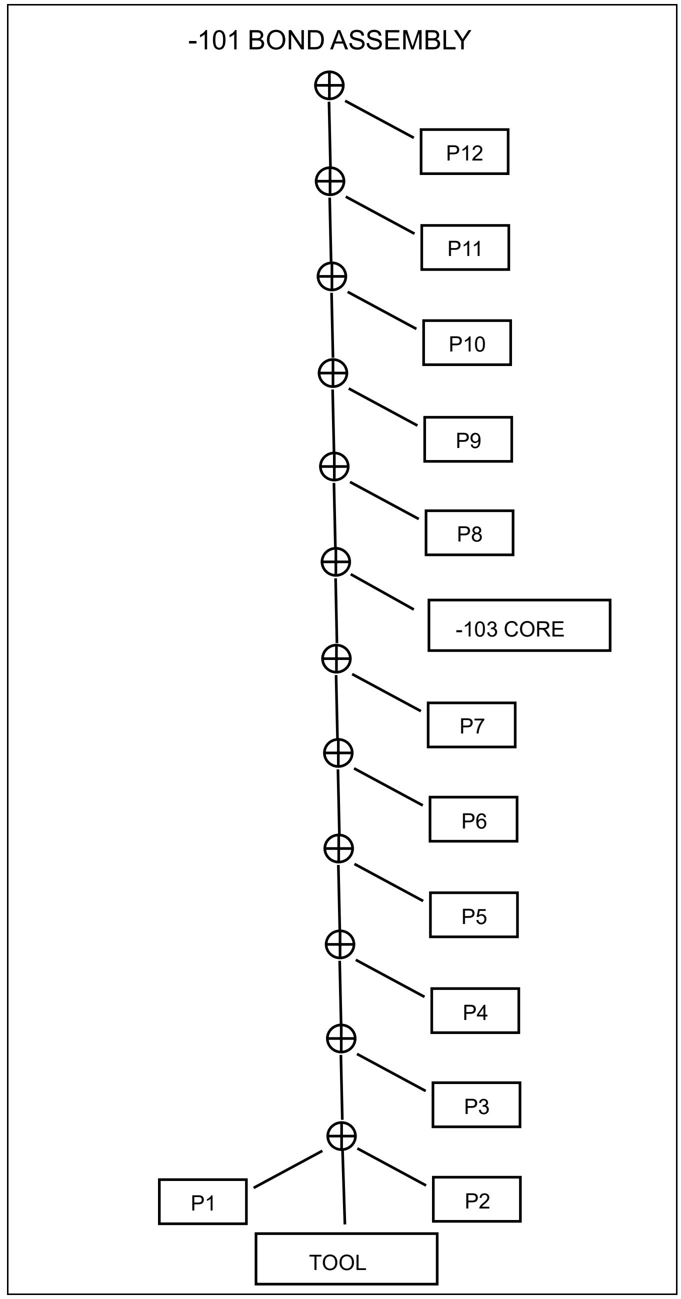


Fig. 7 A CCG tree for the -101 BOND ASSEMBLY in Table 1 and Fig. 2.

The adjoin operation, denoted by the symbol ⊕, is equivalent to a physical bonding (or gluing) of various plies and other items in the *uncured* state in a laminate (see Fig. 4). After curing, the thermosets or thermoplastics undergo chemical bonding, thereby rendering the adjoin operation as an approximation to a 'set union' operation among these plies in the *cured* state. Thus, the interpretation of the adjoin operation is state-dependent.

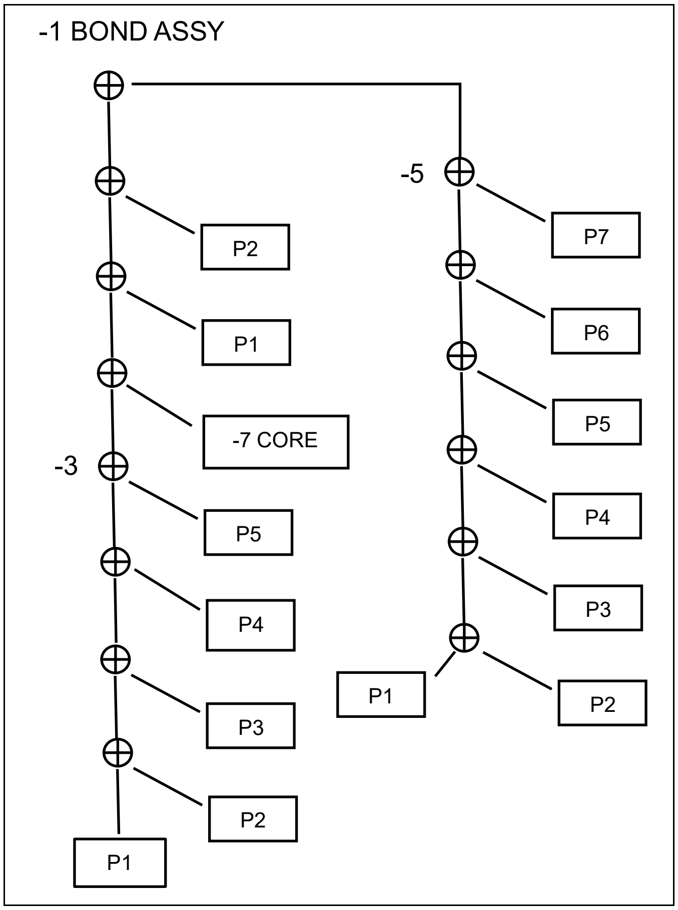


Fig. 8 A CCG tree for the -1 BOND ASSY in Fig. 5.

As a further example, Fig. 8 shows the CCG tree associated with the ply tables and figure in Fig. 5. At the root is the multi-stage bonded assembly denoted as '-1 BOND ASSY.' Similarly, Fig. 9 shows the CCG tree associated with the ply tables and figures in Fig. 6, with the root node denoted as '-1 BOND ASSY.' The hierarchical tree structures seen in Figs. 7, 8, and 9 are a direct consequence of the ply table structures standardized and shown in Table 1 and Figs. 5 and 6.

It is instructive to examine the three CCG trees in Figs. 7, 8, and 9 in some detail, along with their corresponding ply tables and figures.

1. *Tool*: Ply lay-up starts with a tool surface on a tool. The 'tool side' surface (often denoted as in Figs. 2 and 6) plays the role of a 'datum feature' in the composite product definition; a physical tool (with a surface on which the plies are laid) serves as the manufacturing/assembly fixture for the ply lay-up. Using this tool surface as a datum simulator, proper datums and datum systems can be established for the composite product definition [10, 11]. 3D modeling of the tool and the tool surface can be accomplished using currently available solid and geometrical modeling systems.

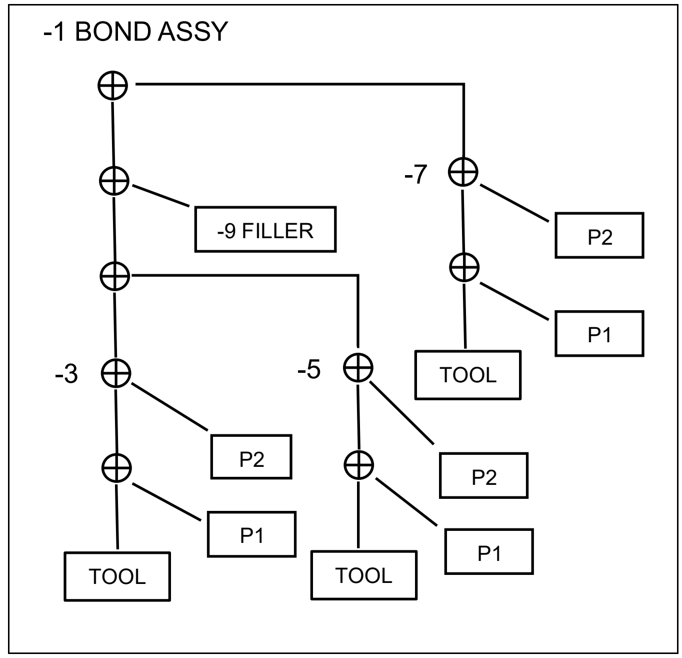


Fig. 9 A CCG tree for the -1 BOND ASSY in Fig. 6.

2. *Core/Filler*: Items such as core and filler can be modeled as 3D solids. Some of the cores, such as the one with honeycomb structure shown in the exploded view of Fig. 3, can have interior cavities that are filled with air or special gases. Figure 1 shows the 'core ribbon direction' for proper orientation of the core with respect to a datum reference frame, which can be established using the 'tool side' information mentioned earlier.

3. *Adjoin*: The adjoin operation, denoted by the symbol ⊕, is a binary operation like 'addition' and 'union.' It can be viewed as a simple gluing operation preserving a physical bonding in an uncured state; the interface between the adjoined objects are preserved in this state. In a cured state, the interfaces between the plies disappear due to chemical bonding and this is similar to the result of a 'set union' operation.

   The algebra of the adjoin operation requires some careful consideration. It is tempting to assign commutativity (that is, $P1 \oplus P2 = P2 \oplus P1$) and associativity (that is, $P1 \oplus (P2 \oplus P3) = (P1 \oplus P2) \oplus P3$) to this operation. But, it is not advisable to do so, due to the strong order dependency in the ply lay-up as described below.

4. *Ply*: As mentioned earlier, ASME defines a ply as 'one discrete piece of manufactured material (e.g., fabric, tape,

adhesive film)' [3]. The primary geometrical representation of a ply is that of a surface with certain thickness. Using the textile metaphor, a ply is like a cloth or a fabric. It can be viewed as a flat surface before a lay-up, and it can be draped as a curved surface on what has been already laid-up or on a tool surface.

The ply is flexible, and it can be tucked and squeezed to conform to the surface to which it is adjoined. The ply thickness may not remain constant during this lay-up operation; the main objective is to make the 'bottom side' of the ply stick to the surfaces on which it is laid without leaving any voids or holes. This poses some interesting challenges in modeling a ply as a 3D solid. This also contributes to the caution about commutativity and associativity of the adjoin operation raised earlier.

5. *Orientation*: Plies also encapsulate reinforcement fibers, both physically and informationally. This will be addressed later in Section 5 in some detail. An important information related to the fibers in a ply is their relative orientation with respect to an external reference frame, which can be established using datums mentioned earlier.

   The angles mentioned in the 'orientation' column of the ply tables seen thus far provide such information. Additionally, a local reference frame called a 'rosette' (as shown in the center of Fig. 1) can be affixed on a ply surface – sometimes in many places on a ply surface – to specify the orientation of the fibers within a ply. Figure 10 shows several examples of such ply orientation symbols associated with rosettes. An example of the use of a guide curve and rosettes is illustrated in Fig. 11. More rosette types and their usage can be found in the recent ASME standard [3]. A rosette is like a two-dimensional compass placed on an undulating terrain, indicating the local directions along which the fibers in a ply should be oriented.

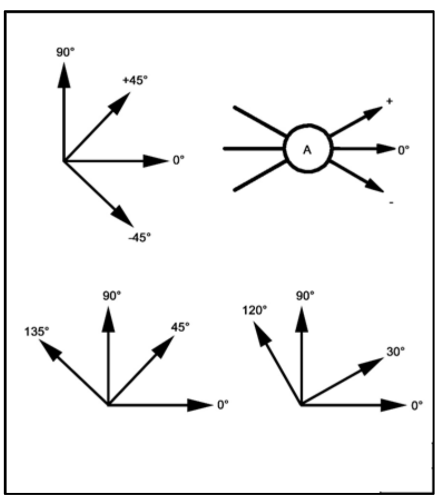


Fig. 10 Examples of ply orientation symbols [3].

6. *Material*: The identifier mentioned in the 'material' column of the ply tables provides the link to much richer information to various other material information about the plies, including critical information about the reinforcement fibers and resins.

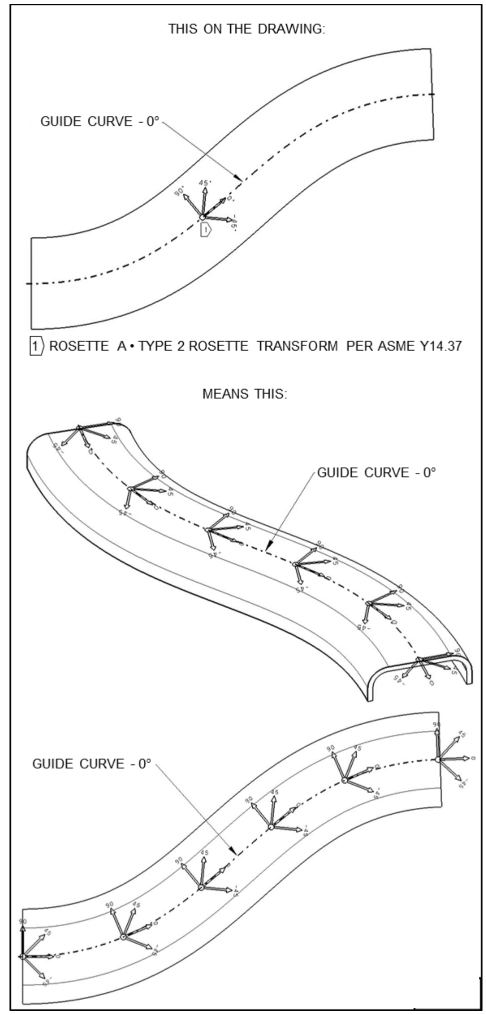


Fig. 11 An example of using a particular type of rosette [3].

The CCG tree is similar to the more familiar constructive solid geometry (CSG) tree [6, 7]. Both are procedural representations from which other representations can be derived. For example, a boundary representation (also known as B-rep) of a solid can be derived from its CSG representation to facilitate several important applications. Following this line of thinking, one may expect that an explicit cell-complex representation of a composite product may be derived from its CCG representation. It turns out that this requires a careful consideration of the topology of the composite products, which will be addressed next.

4 Constructive Composite Topology

The concept of topology captures the notion of adjacency of geometrical objects without paying too much attention to the underlying geometrical details. This does not mean that the geometrical details do not matter; the adjacency of 3D cells in a composite product critically depends on how various plies and other items are positioned in an assembly. Topology, represented as an adjacency graph, turns out to be one of the most robust properties that should be captured in a composite product definition and controlled in its manufacturing.

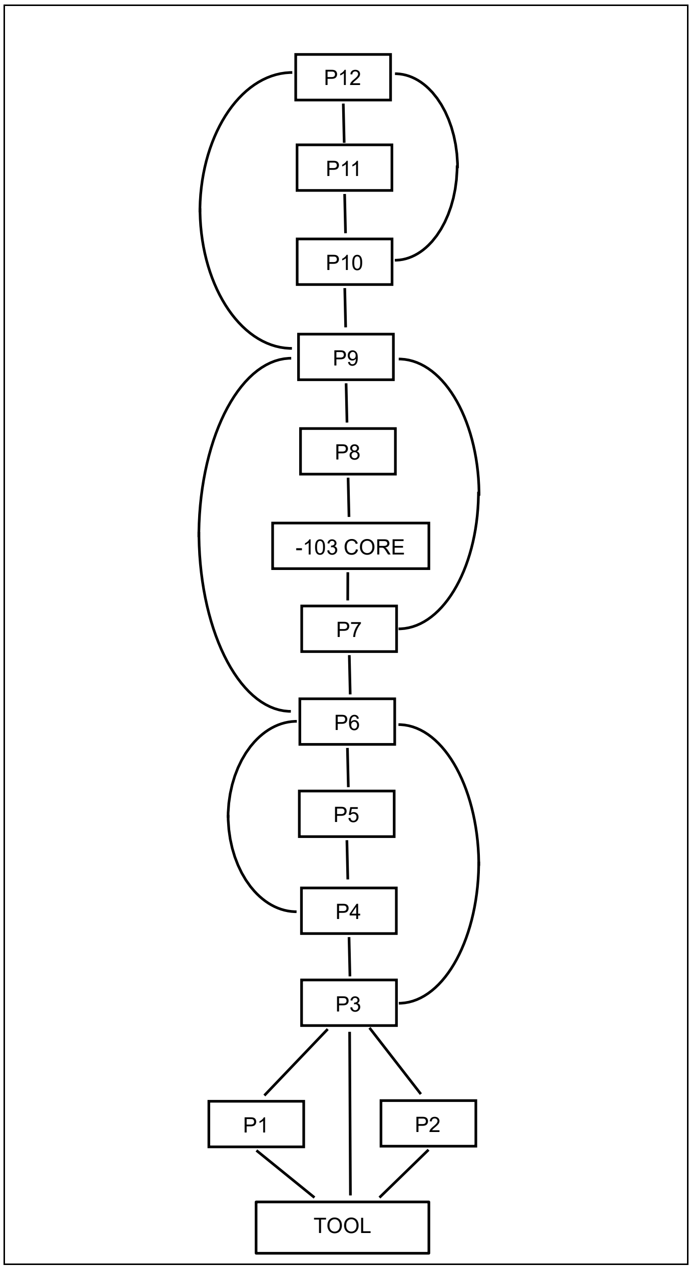


Fig. 12 A CCT graph for the -101 BOND ASSEMBLY in Table 1 and Figs. 2 and 7.

To establish the adjacency of 3D objects in a composite product, consider the plies, cores, and fillers as 3D solids. Figure 12 shows an adjacency graph of the constructive composite topology (CCT) for the -101 BOND ASSEMBLY in Table 1 and Figs. 2 and 7. A similar CCT graph for the -1 BOND ASSY seen earlier in Figs. 5 and 8 is shown in Fig. 13. Also, the CCT graph for the -1 BOND ASSY encountered earlier in Figs. 6 and 9 is shown in Fig. 14. In all these three CCT graphs, the nodes are the plies, cores, fillers, tools, and precured subassemblies; these are displayed as rectangular boxes. The arcs represent the fact that the solids in the nodes joined by each arc have a finite surface area of contact in the uncured state of the bonded assembly.

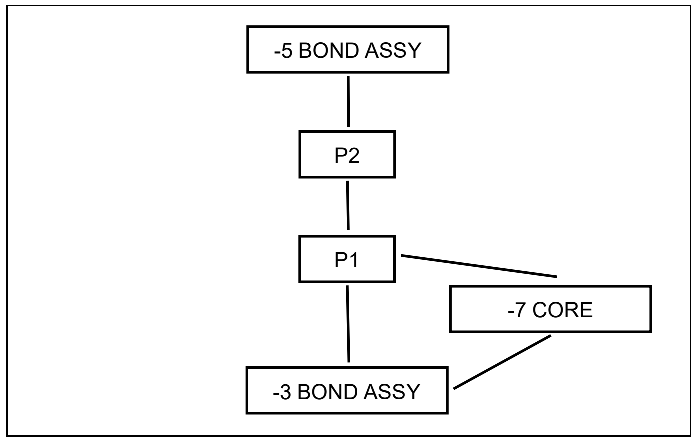


Fig. 13 A CCT graph for the -1 BOND ASSY in Figs. 5 and 8.

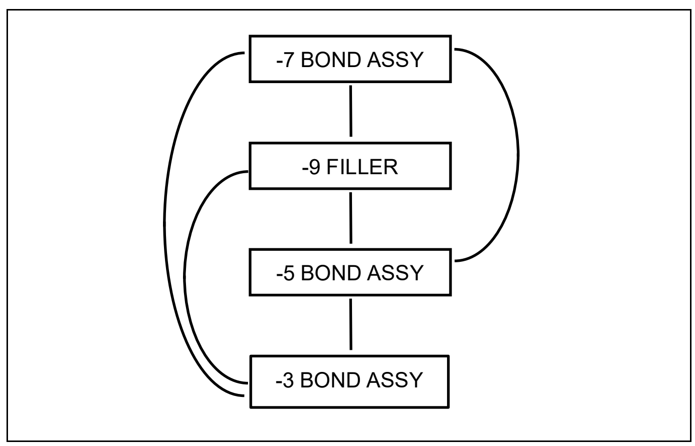


Fig. 14 A CCT graph for the -1 BOND ASSY in Figs. 6 and 9.

Figure 15 illustrates the cell structure and the adjacency relationship using an example previously encountered in Figs. 5, 8, and 13. The cross-sectional view in Fig. 15 captures only a small portion of the bonded assembly, and the dimensions are exaggerated for clarity. The core and the plies in this example are bonded to the objects on which they are laid. It is important that no ‘air bubble’ or ‘air hole’ is introduced in the lay-up process or in the 3D model creation. This poses a challenging problem when approximations to the 3D models, such as tessellated polyhedra, are used for 3D representations of various constituents in a bonded assembly.

Each of the four objects in Fig. 15 is considered as a 3D cell. If any two of these 3D cells have a contact over a surface area, then they are joined by an arc in the CCT graph shown in Fig. 13. This illustrates the concept of adjacency used in all the three CCT graphs shown in Figs. 12, 13, and 14.

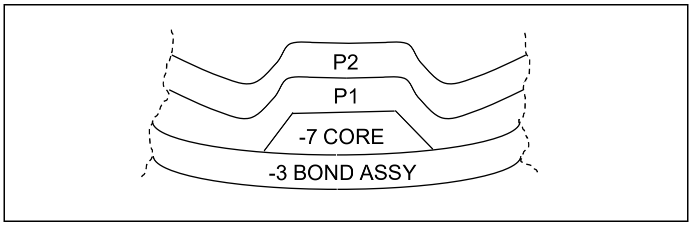


Fig. 15 Cross-section of a portion of an approximate 3D model of the cell-complex of the '-1 BOND ASSY' in Figs. 5 and 8. Drawing is not to scale.

As a further illustration of the CCT graph, consider the example shown in Figs. 1, 2, and 3. Figure 16 shows the cell structure and the adjacency relationship using a cross-sectional view of an approximate 3D model. Such topological information is represented only in a CCT graph; it is not available explicitly either in an exploded view such as Fig. 3 or in a ply table such as Table 1.

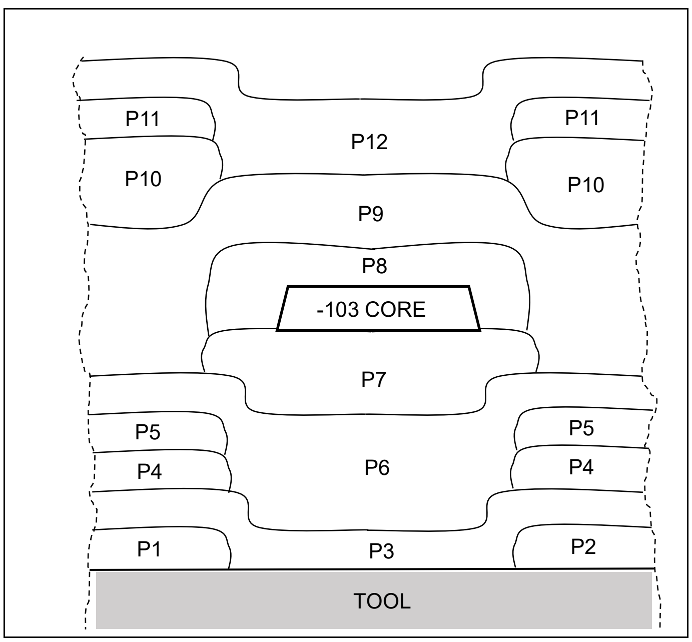


Fig. 16 Cross-section of a portion of an approximate 3D model of the cell-complex of the '-101 BOND ASSEMBLY' in Figs. 1, 2, and 3. Drawing is not to scale.

It is also clear from Figs. 15 and 16 that 'ply thickness' is not a simple number after the ply is stacked up in a laminate in an uncured state. These plies are indeed pliable, and they undergo a homeomorphic transformation as they go from the state of a flat fabric into a stacked-up ply in a laminate in an uncured state. Again, it is important to avoid any air pockets or air holes (neither 'through holes' nor 'blind holes') during the stack-up process.

With these examples as preliminaries, the following observations can be made about the structure and property of the CCT graph.

1. *The CCG tree provides a useful input to build the CCT graph*. The adjoin operation in the CCG tree suggests an adjacency relationship between its operands. CCT graph contains these relationships, but it also captures more adjacency relationships.

2. *The CCT graph is a subgraph of the dual graph of the cell complex*. Algebraic topology of a cell complex, as applied to a composite product, consists of a hierarchy of 3-cells (solids) bounded by 2-cells (surfaces) bounded by 1-cells (edges) bounded by 0-cells (vertices) [14]. A dual graph of this cell complex will include cells of all these dimensions; but the CCT graph captures only the 0 and 1 dimensional subsets of this dual graph.

3. *The plies undergo a homeomorphic transformation in the lay-up process*. The plies start out as flat fabrics. As a ply is draped in the lay-up process by tucking, stretching, and squeezing, it undergoes a homeomorphic transformation, which is a basic notion in topology [14]. Under such a homeomorphic transformation, neighboring points of a ply remain as neighbors. This is an important property that is observed during the lay-up process to ensure the integrity of the reinforcement fibers (e.g., no tearing) contained in the plies. It is also an important property that can be used in 3D modeling of the laid-up plies in an uncured state.

   Such homeomorphic transformations are shown, albeit with some exaggeration, in Figs. 15 and 16. The ply thickness can undergo changes under this transformation. But it can be argued that the volume of the ply remains the same, obeying the conservation of mass (before curing). Thus, the most robust statement that can be made is that the plies undergo a volume-preserving homeomorphic transformation during the lay-up process.

4. *The CCG tree and CCT graph serve as important primary elements of the procedural representation*. The robustness of CCG trees and CCT graphs has been well established in the industrial practice though decades of use in the form of ply tables and accompanying figures. Taking them as important primary representations, explicit 3D representations may be derived as needed for the uncured laminates and cured bonded assemblies. This is similar to the derivation of a B-rep from a CSG representation of a

solid, where the CSG is the primary (procedural) representation and B-rep is the derived (explicit) representation [6, 7]. Specific composite manufacturing properties, such as volume-preserving homeomorphic transformations mentioned above, may be utilized for constructing the 3D representations of such cells.

5 Geometry and Topology of Reinforcement Fibers

Plies contain reinforcement fibers that contribute much of the strength and stiffness to the composite products. These fibers are combined using textile technologies, such as weaving, stitching, seaming, and braiding, to form various patterns in a fabric. Such fabrics can be created as 'preforms' as shown in Fig. 17, where each colored strip contains many fibers. These preforms can also be impregnated in resins to create plies as 'prepregs' that can be handled as cloths, as shown in Fig. 18.

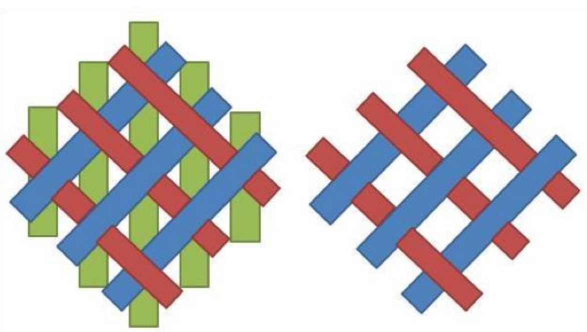


Fig. 17 Examples of preforms in creating plies [15].

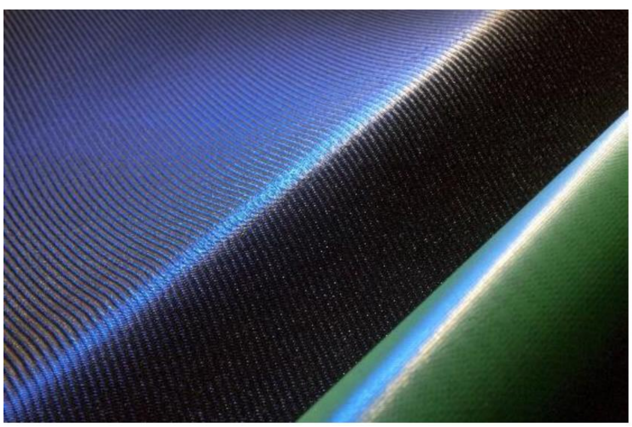


Fig. 18 Example of prepregs in creating plies [16].

The ASME and ISO standards do not address the details of the fiber arrangements in plies. Instead, they refer to other standards such as the ASTM standard on stiches and seams [17]. The 'material' attribute in the ply table provides links to such fiber arrangement details within each ply.

The geometry and topology of the fiber arrangements in a ply can benefit from research literature on textiles [18]. When a ply is draped as a cloth to form a laminate, the local orientations of the fibers receive a great deal of attention in practice, as illustrated in Fig. 11 using rosettes. From such standardized practices, a theory for the geometry and topology of bundles of reinforcement fibers can be derived using the following two important invariance properties:

1. *Conformal mapping to preserve local fiber orientations*. When a flat fabric containing reinforcement fibers is draped onto a curved surface, as shown in Fig. 11 for example, the local orientations of fibers are preserved within each ply. This practice can be directly related to the theory of conformal mapping of oriented surfaces, which preserves the angles and shapes of infinitesimally small figures but not necessarily their sizes or curvatures [19]. In fact, the standardized definitions of rosettes [3] and their usage in practice can be formalized mathematically as conformal mapping of oriented surfaces. Such mappings occur in the lay-up of each ply to form a laminate before the curing operation. These fiber orientations are also expected to be preserved in the cured state of the composite product, within some orientation angle tolerance.

   In a typical conformal mapping of oriented surfaces, the 'domain' surface is parameterized using (u, v) coordinates, and it is mapped to a 'range' surface that is parameterized using (u', v') coordinates. It is then customary to talk about u-curves being mapped to u'-curves, and v-curves being mapped to v'-curves. In the context of manufacturing of composite products, these u-curves, u'-curves, v-curves, and v'-curves take on a physical meaning. These curves are the representations of the fibers in a semi-discrete form. An interesting constraint that arises in such conformal mapping is that the fiber lengths are preserved in the lay-up process.

2. *Preservation of fiber adjacency between layers*. As noted in Fig. 4, the cured composite part does not preserve the interfaces between the plies. However, the adjacency relationships among the plies and other items established in the CCT graph (such as Figs. 12, 13, and 14) are inherited by the bundles of fibers contained in these plies. This provides two important types of information about the bundles of fibers in a cured composite product: (1) Each bundle of fiber is given an identifier that is inherited from the ply within which it is impregnated; (2) A partial spatial ordering of these bundles of fibers in the cured composite product can be derived from the CCT graph. Such information is important for inspection (e.g., nondestructive testing) and structural analysis purposes. For example, a cluster analysis of fibers in a 3D computed tomography image of a cured composite product may benefit greatly from a prior knowledge of the adjacency of bundles of the fibers.

6 Summary and Directions for Future Research

This paper took some initial steps towards a theory of constructive composite geometry and topology. This theory is synthesized from industrial best practices codified in recent ASME and ISO standards [3, 4]. It is also motivated by the need for better scientific tools to explore new frontiers in modeling material structures that are produced by modern manufacturing technologies [2]. The major features of this theory are:

1. A CCG tree, which maintains a one-to-one mapping with the ply/laminate table standardized by ASME and ISO. An ‘adjoin’ operation in the CCG tree defines the bonding (both physical and chemical) between various plies and other items in the assembly.

2. A CCT graph, which captures the adjacency relationship among plies and other items in an uncured state of a laminate, and this adjacency relationship is inherited by the bundles of fibers in the cured composite product.

3. A conformal mapping between a flat ply surface and a draped ply surface in a laminate; this mapping preserves the local orientations of fibers (in terms of angle between fibers) as standardized by ASME and ISO using rosettes and other orientation specifications.

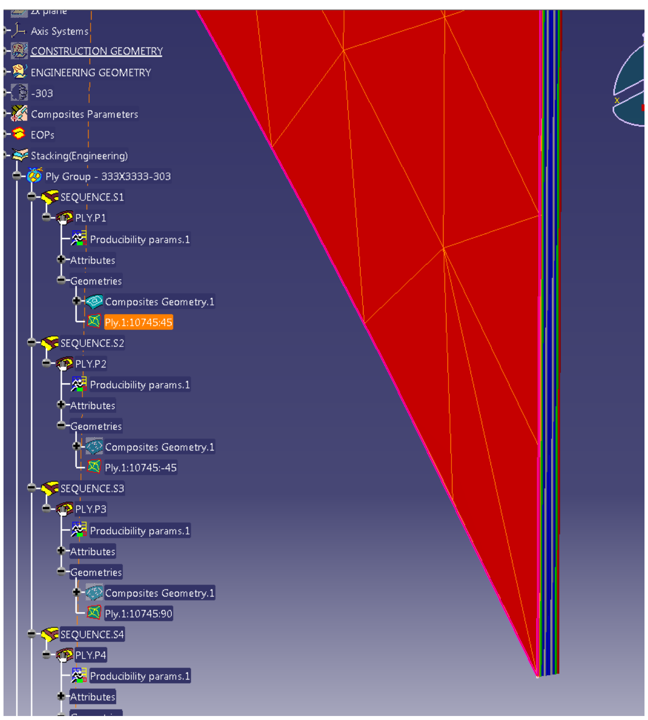


Fig. 19 Example of a composite part used for testing standardized 3D model data exchange [20].

Some of these theoretical abstractions have been implemented already in practice. In major CAD systems that support composite products, a ‘model tree’ is a prominent visual component of the user interface; this can be easily related to the CCG tree. Figure 19 shows a screen shot of a composite part used for testing standardized 3D model exchange [20]. It clearly shows a tree-like representation for the ply stack-up under the ‘Stacking (Engineering)’ node in the model tree displayed on the left.

While these are encouraging signs, more research and standardization need to be undertaken along the following lines:

- The CCG tree and CCT graph can be extended to cover more composite manufacturing technologies such as infusion, pultrusion, and braiding. Recent ASME and ISO standards have already taken initial steps in this direction [3, 4].

- 3D modeling of uncured and cured states of composite products can be improved considerably, with proper attention to the conformal mapping of ply surfaces and the homeomorphic transformation of ply (and other) solid cells, as described in the body of the paper.

- Better harmonization of ASME and ISO standards is needed so that the composite product definition practices and 3D models for information exchange are properly aligned.

Acknowledgments and a Disclaimer

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References

[1] Grandine, T., 2013, “Geometric Interoperability for Composite Manufacturing,” Panel on Geometric Interoperability, ASME Man. Sci. Eng. Conf., Madison, WI.
[2] Regli, W., Rossignac, J., Shapiro, V., and Srinivasan, V., 2016, “The New Frontiers in Computational Modeling of Material Structures,” Com. Aided Des., **77**, pp. 73-85.
[3] ASME Y14.37-2019, 2019, *Product Definition for Composite Parts*, The American Society of Mechanical Engineers, New York.
[4] ISO 10303-242:2019, 2019, *Industrial automation systems and integration – Product data representation and exchange –Part 242: Application protocol: Managed model-based 3Dengineering*, International Organization for Standardization, Geneva, Switzerland.
[5] Hedberg Jr, T.D., Feeney, A.B., and Srinivasan, V., 2019, “An Analysis of Recent Standards on Composite Product Models to Enable Digital Transformation of Composite Product Manufacturing,” MSEC2019-2783, ASME Man. Sci. Eng. Conf., Erie, PA.
[6] Hoffmann, C.M., 1989, *Geometric and Solid Modeling: An Introduction*, Morgan Kaufmann, Palo Alto, CA.
[7] Srinivasan, V., 2004, *Theory of Dimensioning: An Introduction to Parameterizing Geometric Models*, Marcel Dekker, NY.
[8] Campbell, F.C., 2004, *Manufacturing Processes for Advanced Composites*, Elsevier, Oxford, U.K.

[9] Strong, A.B., 2008, *Fundamentals of Composites Manufacturing*, 2nd Edition, Society of Manufacturing Engineers, Dearborn, MI.
[10] ASME Y14.5-2009, 2009, *Dimensioning and Tolerancing*, The American Society of Mechanical Engineers, New York.
[11] ISO 1101:2017, 2017, *Geometrical Product Specifications (GPS) – Geometrical Tolerancing – Tolerances of Form, Orientation, Location and Run-out*, International Organization for Standardization, Geneva, Switzerland.
[12] ISO 10303-209:2001, 2001, *Industrial Automation Systems and Integration – Product Data Representation and Exchange – Part 209: Application Protocol: Composite and Metallic Structural Analysis and Related Design*, International Organization for Standardization, Geneva, Switzerland.
[13] ASME Y14.37-2012, 2012, *Composite Part Drawings*, The American Society of Mechanical Engineers, New York.
[14] Hatcher, A., 2002, *Algebraic Topology*, Cambridge University Press, U.K.
[15] Wphallig. "Braids." Digital image. *Wikimedia Commons*. November 26, 2013. Accessed October 21, 2018. Licensed under CC BY-SA 3.0. https://commons.wikimedia.org/w/index.php?curid=29864409.
[16] Cross, N., 2009. "Gurit - a World Leader in Epoxy Prepreg Technology." Digital image. *Flickr*, January 30, 2009. Accessed October 25, 2018. Licensed under CC BY-ND 2.0. www.flickr.com/photos/80188450@N03/8138514559.
[17] ASTM D6193-16, 2016, *Standard Practice for Stiches and Seams*, ASTM International, West Conshohocken, PA.
[18] Grunbaum, B., and Shephard, G.C., 1980, "Satins and Twills: An Introduction to the Geometry of Fabrics," Mathematics Magazine, **53**(3), pp. 139-161.
[19] Kreyszig, E., 1991, *Differential Geometry*, Dover, New York.
[20] https://www.cax-if.org/production_model.html. Accessed Jan. 23, 2019.