Uncertainty Quantification of Failure Probability and a Dynamic Risk Analysis of Decision-Making for Maintenance of Aging Infrastructure

Jeffrey T. Fong, James J. Filliben, N. Alan Heckert, Dennis D. Leber, Paul A. Berkman and Robert E. Chapman

Abstract Risk, as the product of failure probability and failure consequence, has been estimated and applied by engineers and managers to help make critical decisions on (a) maintenance of aging plants, and (b) planning of new infrastructure. For aging plants, failure probabilities are more difficult to estimate than consequences, primarily because of a shortage of time-varying data on the condition of the complex systems of hardware and software at varying scales after years of service. A different argument holds for yet-to-be-built infrastructure, since it is also hard to estimate the time-varying nature of future loadings and resource availability. A dynamic, or, time-dependent risk analysis using a time-varying failure probability and a consequence with uncertainty estimation is an appropriate way to manage aging infrastructure and plan new ones. In this paper, we first introduce the notion of a time-varying failure

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probability via a numerical example of a multi-scale fatigue model of a steel pipe, and then the concept of a dynamic risk for decision-making via an application of the analysis to the inspection strategy for a cooling piping system of a 40-year-old nuclear power plant. Significance and limitations of the multi-scale fatigue life model and the risk analysis methodology are presented and discussed.

Keywords Aging structures · Coverage · Dynamic risk analysis · Engineering decision-making · Failure probability · Fatigue · Inspection strategy · Maintenance engineering · Predictive limits · Reliability · Risk analysis · Statistical analysis · Tolerance limits · Uncertainty quantification

1 Introduction

The failure of a complex engineering structure, such as a long-span suspension bridge, or a simple component such as an aircraft window, has a common feature, namely the initiation and propagation of one or more microscopic discontinuities such as voids, micro-cracks, etc.

To illustrate this common root cause of failure known as "fatigue," we show in Fig. 1 the distribution of micro-cracks as a function of crack length at three stages (48%, 60%, 100%) of life of a corrosion fatigue test specimen of steel as recorded by Kitagawa and Suzuki [1]. In Fig. 2, we show a statistical representation of the distribution of fiber lengths in a sample of currency paper before and after 80,000 flexes in a folding fatigue test, as reported by Fong et al. [2, 3]. In Fig. 3, we show a series of images, observed at Level 1 (Micro) by Nisitani et al. [4], of crack initiation and growth in a steel specimen undergoing a cyclic stress test at Level 2 (Specimen). In addition, we characterize in Fig. 3 a 3-part fundamental model of structural fatigue testing, namely,

(Part 1): The microscopic data set collected at Level 1 provides a *scientific* basis, with statistical representation and analysis, for the fatigue mechanisms discovered in test specimens at Level 2,
(Part 2): The fatigue failure data set collected at Level 2 from a sample of n specimens provides a *statistical* basis for predicting the fatigue lives of an infinite number of specimens at Level 2 with uncertainty estimated by "predicted limits," and
(Part 3): The same set of fatigue failure data set collected at Level 2 from a sample of n specimens provides a *statistical* basis for predicting, with a new statistical concept known as "*coverage*," the fatigue lives of an infinite number of full-size structure or component of the same material at Level 3 (Component) with uncertainty estimated by "tolerance limits."

The second and third parts of the model provide a methodology for engineers to use the fatigue test data of a finite number of specimens at Level 2 to predict the fatigue life of a full-size structure or component at Level 3 with uncertainty estimated by "tolerance limits" for any specific coverage less than 100%.

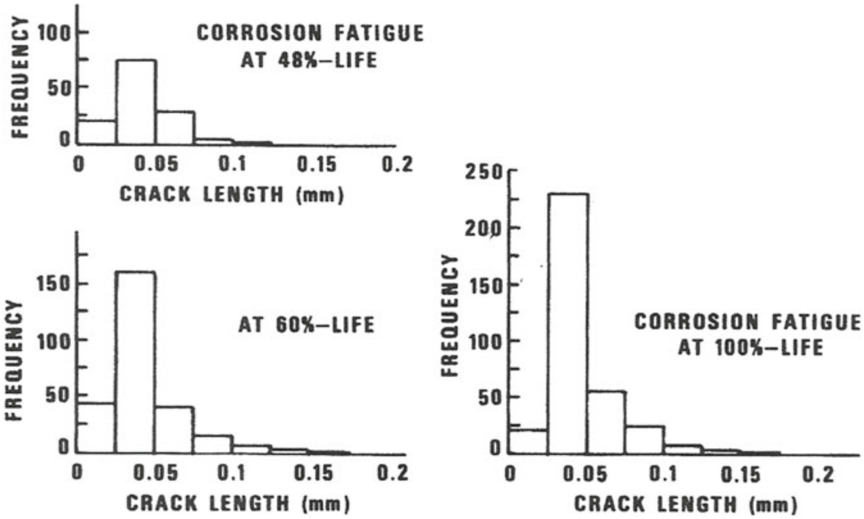

Fig. 1 Histograms of microcrack lengths of HT 50 high-tensile strength steel in tap water at selected stages of corrosion fatigue for a cyclic stress of 12 ± 12 kg/mm^2 (after Kitagawa and Suzuki [1])

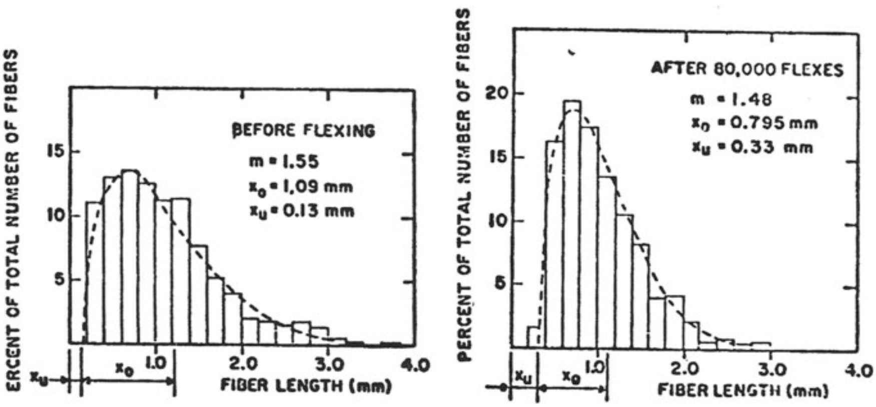

Fig. 2 Frequency distribution of fiber length for high-grade rag paper before and after Flexes (after Fong et al. [2])

By postulating a fatigue failure with a lack of coverage at Level 3 as a criterion for full-size structural fatigue failure, we can derive a time-dependent Level 3 fatigue failure probability model to yield a new approach to risk-informed decision-making for maintenance of aging and planning of new infrastructures.

In Sect. 2, we present the concepts and the methods of computing the “predictive limits” and the “tolerance limits” of the fatigue lives at Levels 2 and 3, respectively. In Sect. 3, we present the development of a multi-scale fatigue life model in five steps. In Sect. 4, we present a numerical example of a multi-scale fatigue life model

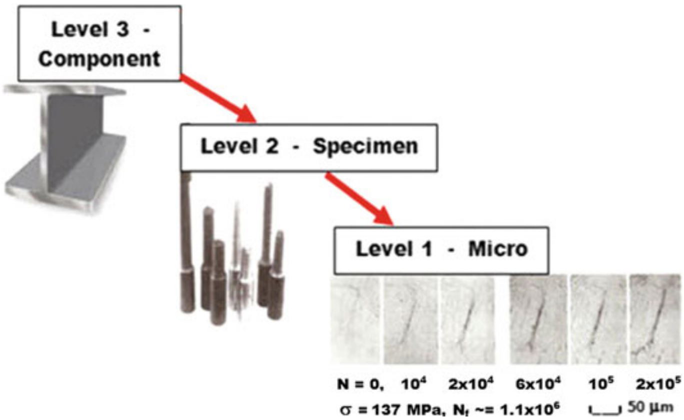

Fig. 3 A multi-scale representation of a typical fatigue test information flow with images of crack initiation and growth at Level 1 of the test as recorded by Nisitani et al. [4]

by applying the first three steps leading to an estimate of the fatigue life at Level 2 (Specimen) with uncertainty estimated by "predictive limits." In Sect. 5, we present the same numerical example using Step 4 leading to an estimate of the fatigue life at Level 3 (Component) with uncertainty estimated by "tolerance limits" for a range of coverages between 0.75 and 0.999. In Sect. 6, we present Step 5 by introducing a physical assumption that the fatigue lives at very low failure probabilities cannot be negative and must approach zero as the failure probability approaches zero. The consequence of that assumption allows us to fit the nonnegative life results of Sect. 5 with a nonlinear least squares 3-parameter logistic model, and therefore to obtain a failure probability versus time-to-failure curve for a full-size component at Level 3 based on fatigue test data at Level 2. In Sect. 7, we apply the new time-dependent failure probability result to a new approach of risk-informed decision-making for maintenance of a critical structure or component. Significance and limitations of the multi-scale fatigue life model and the risk analysis methodology are presented in Sect. 8. Some concluding remarks and a list of references are given in Sects. 9 and 10, respectively.

2 A Statistical Analysis Methodology for a Multi-scale Fatigue Model

We begin with an introduction to the notion of a "predictive interval" in statistics that is used in Part 2 of our fatigue model to estimate the Level 2 uncertainty by "predictive limits".

Let us consider a cycles-to-failure prediction at Level 2 to be at a 95% confidence level, with the symbol α defined by $95\% = (1-\alpha)100\%$, or, $\alpha = 0.05$. As shown by

Nelson et al. [5, pp. 179–180], when the true mean, μ, and standard deviation, σ, of a normal distribution are not known, the so-called $(1 - \alpha)100\%$ predictive interval is given by the following expression:

$$\bar{y} \pm t(\alpha/2; n - 1)s\sqrt{1 + \frac{1}{n}}, \tag{1}$$

where $\bar{y}$ is the estimated mean, s, the estimated standard deviation, n, the sample size, t, the well-known Student's distribution function, and α, the quantity associated with the confidence level given by $(1-\alpha)$ 100%. For engineers dealing with experimental data at the specimen Level 2, the estimated predictive interval given in Eq. (1) for a normally distributed sample data is valid only at the Level 2 scale, and not at a higher level such as Level 3, the level of a full-size component. In short, *a predictive interval is only valid for a single-scale model*.

To extrapolate a Level 2 estimate to that of a higher level, we need to introduce a new concept, i.e., the concept of "*coverage*", p, which is defined as the proportion of the population that is covered by a new statistical interval known as the "tolerance interval," (see, e.g., Nelson et al. [5, pp. 179–180]). The upper and lower limits of the tolerance interval are known as the upper tolerance limit (*UTL*) and lower tolerance limit (*LTL*), respectively. It is the one-sided *LTL* for a given coverage, p, and the $(1- \alpha)$ 100% confidence level that engineers are most interested in, whether it is for finding a code-allowable minimum strength of a material for structural design, or the minimum cycles-to-failure, minNf, of a material for a rotary equipment.

The reason for choosing the one-sided *LTL* to work with is that the statistical quantity called the confidence level, γ, or, $(1 - \alpha)$, is commonly associated with engineering reliability, which is a safety concept based on the assumed existence of a minimum strength of a structure, or, in the case of fatigue life design, a minimum cycles-to-failure, minNf.

The theory of one-sided or two-sided tolerance intervals for a normal population is well-established in the statistics literature (see, e.g., Prochan [6], Natrella [7], and Nelson et al. [5]). For example, as shown by Nelson et al. [5], the tolerance interval of fatigue life, Nf_3, for an infinitely large normal population of full-size components at Level 3, can be expressed in terms of the estimated sample mean cycles to failure, $\bar{y}$, or, Nf_2, and the sample standard deviation, s, or, sdNf_2, of the experimental data derived from n specimens at Level 2, as shown below:

$$Nf_3 = \bar{y} \pm r\ u\ s, \tag{2}$$

where $\bar{y} = Nf_2$, $s = sdNf_2$, the factor, $r\ (n, p)$, depends on the sample size, n, and the coverage, p, and the factor, $u\ (df, \gamma)$, depends on the degrees of freedom, df, defined by $n - 1$, and the confidence level, γ, defined by $1 - \alpha$.

Both factors of r and u in Eq. (2) for a normal population are available for a broad range of n, p, and γ, in tabular forms in many statistics books such as Natrella [7] and Nelson et al. [5]. Unfortunately, Nelson et al. [5] gives only tables of the two-sided *LTL*, whereas Natrella [7] gives both two-sided and one-sided *LTL*. As mentioned earlier, for engineering applications, it is the one-sided *LTL* that is of interest, so in this paper, we will only use tables from Natrella [7] to develop a multi-scale fatigue life model where the uncertainty in the fatigue life, Nf_3, at the full-size component level (Level 3) is quantified by applying the one-sided *LTL* formula of Eq. (2) using the mean fatigue life, Nf_2, and its standard deviation, sdNf_2, as computed from data at the specimen level (Level 2).

3 Development of a Multi-scale Fatigue Model in Five Steps

Using the statistical tools of "predictive intervals" and "tolerance intervals", we develop a multi-scale fatigue model in five steps:

Step 1: **Level 2 Life versus Stress Model**. Identify and adopt a fatigue model based on a fatigue life formula at the specimen level (Level 2).
Step 2: **Collect experimental data at Level 2**. Run fatigue experiments to obtain cycles-to-failure, Nf_2, as a function of the applied stress amplitude, σ_a, or, in the absence of available experimental data, compute Nf_2 using the formula identified in Step 1 with the parameters in the formula estimated from either available experimental data or handbook values at specimen Level 2.
Step 3: **Level 2 Life with Uncertainty Quantification at Operating Stress**, $(\sigma_a)_{\mathrm{op}}$. Use the linear least squares fit algorithm to obtain a log-log plot of Nf_2 versus σ_a, and obtain, for some operating stress amplitude, $(\sigma_a)_{\mathrm{op}}$, an estimate of the predicted fatigue life, $(Nf_2)_{\mathrm{op}}$, and its standard deviation, $(\mathrm{sd}Nf_2)_{\mathrm{op}}$.
Step 4: **Level 3 Life with Uncertainty Quantification at Operating Stress**, $(\sigma_a)_{\mathrm{op}}$. Apply the tools of tolerance intervals and use the tables of the one-sided Lower Tolerance Limits, *LTL*, of Natrella [7], to compute the minimum fatigue life of a full-size component, $(\mathrm{min}Nf_3)_{\mathrm{op}}$, at the operating stress amplitude, $(\sigma_a)_{\mathrm{op}}$, as a function of the sample size, n, the confidence level, γ, and the lack or "Failure" of coverage, Fp $(= 1 - p)$.
Step 5: **Minimum Level 3 Life at Operating Stress**, $(\sigma_a)_{\mathrm{op}}$, **and Extremely Low Failure of Coverage**. Using a nonlinear least squares fit algorithm and the physical assumption that the one-sided Lower Tolerance Limit (*LTL*), at 95% confidence level, of the fatigue life, i.e., the minimum cycles-to-failure, minNf_3, of a full-size component, *cannot be negative* as the lack or "Failure" of coverage (Fp), defined as $1 - p$, approaches zero, we estimate the minimum cycles-to-failure, minNf_3, at extremely low "Failure" of coverage, Fp, say, between 10^{-3} and 10^{-7}.

4 Steps 1–3 of a Fatigue Model for a Steel Pipe—A Numerical Example

For Step 1, we choose to work a simple model described in a book by Dowling [8, p. 364]. As shown below in Eq. (3), the number of cycles of a constant-amplitude fatigue fracture failure, N_f, and the applied stress amplitude, σ_a, are in a power-law relationship:

$$\sigma_a = A(Nf)^B, \text{or equivalently,} \quad Nf = (\sigma_a/A)^{1/B}, \tag{3}$$

where A and B are two empirical material property parameters that can either be estimated with uncertainties from a linear least squares fit of a set of log (Nf) versus log (σ_a) data, or obtained from material properties handbooks and databases for specific materials.

After a Level 2 (specimen) life formula is identified (Step 1), we begin our Step 2 by either running fatigue experiments to obtain cycles-to-failure, Nf_2, as a function of the applied stress amplitude, σ_a, or compute Nf_2 using the formula identified in Step 1 with the material property parameters in the formula estimated from either available experiments or handbooks.

In this paper, we choose to work with finding the minimum cycles-to-failure of a critical nuclear power plant component made of an alloy steel named AISI 4030. Its fatigue life formula is a power-law relationship as shown in Eq. (3). The fatigue experimental data for that material (after Dowling [8, 9]) are listed in Table 1.

In Step 3, we apply a standard linear least squares fit algorithm (see, e.g., Draper and Smith [10]) to obtain first a log-log plot of Nf_2 versus σ_a, as shown in Fig. 4, and then an estimate of the predicted fatigue life, $(Nf_2)_{\text{op}}$, and its standard deviation, $(\text{sd}Nf_2)_{\text{op}}$, for some operating stress amplitude, $(\sigma_a)_{\text{op}}$, as shown in Fig. 5. We assume in our numerical example that the operating stress amplitude, $(\sigma_a)_{\text{op}}$, is 398 MPa, with the corresponding value of the quantity, $\log_{10} \{(\sigma_a)_{\text{op}}\}$, equal to 2.60. A complete listing of a computer code that solves the linear least squares fit with uncertainty problem and is written in an open-source language named DATAPLOT [11, 12], is available upon request.

Table 1 Fatigue Data for AISI 4340 Steel (Dowling [8, 9])

Stress amplitude σ_a, MPa	Cycles-to-failure Nf_2, Cycles
948	222
834	992
703	6004
631	14,130
579	45,860
524	132,150

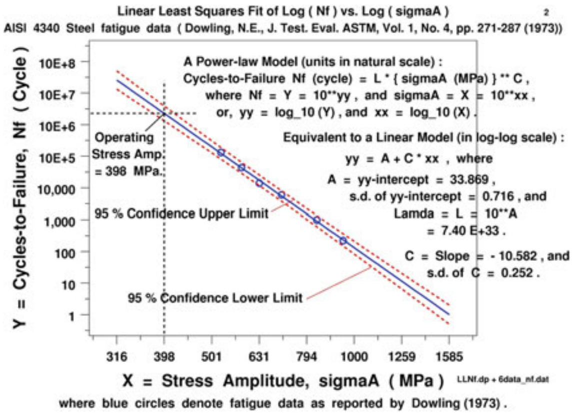

Fig. 4 A linear least squares fit of six fatigue specimen data (after Dowling [8, 9])

```
---------------------------
            Y (log(Nf))       X (log(sigmaA))
---------------------------
     2.3464           2.9768
     2.9965           2.9212
     3.7784           2.8470
     4.1501           2.8000
     4.6421           2.7627
     5.1211           2.7193

'* NOTE--
   SUBSET VARIABLE = X2
   SUBSET MINIMUM  =   0.2599997400E+01
   SUBSET MAXIMUM  =   0.2600002600E+01
   INPUT  NUMBER OF OBSERVATIONS  =        71
   NUMBER OF OBSERVATIONS IGNORED =        70
   OUTPUT NUMBER OF OBSERVATIONS  =         1

----------------------------------------
        PRED2             SD2            X2
     ----------------------------------
```

$\log_{10}\{(Nf_2)_{op}\}$ = 6.3560   0.0844   2.6000 = $\log_{10}\{(\sigma_a)_{op}\}$

Fig. 5 A screen output of a linear least squares fit code written in Dataplot and used to produce the graphical results shown in Fig. 4

5 Step 4 (Life at Level 3) of a Fatigue Model for a Steel Pipe—A Numerical Example

In Step 4, we apply the statistical theory of tolerance intervals (see, e.g., Nelson et al. [5]) and use the tables of the one-sided Lower Tolerance Limits, *LTL*, of Natrella [7], to compute the minimum fatigue life of a full-size component, $(\min Nf_3)_{op}$, at the operating stress amplitude, $(\sigma_a)_{op}$, as a function of the sample size, n, the confidence level, γ, and the lack or "Failure" of coverage, Fp $(= 1 - p)$. The result of our calculations is given in Table 2, using Fig. 5.

Table 2 One-sided *LTL* versus $(1-p)$ between $p = 0.75$ and 0.999 for $n = 6$ and $\gamma = 0.95$

	Confidence level, $\gamma = 0.95$				
Coverage, p	0.75	0.90	0.95	0.99	0.999
Lack or "Failure" of coverage $Fp = 1-p$	0.25	0.10	0.05	0.01	**0.001**
For $n = 6$					
From Natrella [6] K	1.895	3.006	3.707	5.062	6.612
From Step 3 $(Nf_2)_{op}$	2.26986 E + 6	2.26986 E + 6	2.26986 E + 6	2.26986 E + 6	2.26986 E + 6
From a special computational procedure given in Eq. (8)** $(sdNf_2)_{op}$	0.44112 E + 6	0.44112 E + 6	0.44112 E + 6	0.44112 E + 6	0.44112 E + 6
K* $(sdNf_2)_{op}$	0.83592 E + 6	1.32601 E + 6	1.63523 E + 6	2.23295 E + 6	2.91669 E + 6
$(\min Nf_3)_{op}$ = one-sided *LTL* = $(Nf_2)_{op}$—K* $(sdNf_2)_{op}$	**1.43393 E + 6**	**0.94385 E + 6**	**0.63463 E + 6**	**0.03691 E + 6**	**− 0.64683 E + 6**

**The estimation of the standard deviation of $(Nf_2)_{op}$ from a log-log plot of Nf_2 versus σ_a requires a special computational procedure as described below

From Fig. 5, we obtain $\log_{10}[(Nf_2)_{op}] = 6.3560$, and sd $\{\log_{10}[(Nf_2)_{op}]\} = 0.0844$

From the statistical theory of error propagation (see, e.g., Ku [12]), we find a closed-form relationship between the standard deviation of $\log_e(Nf)$, or, sd$\{\log_e(Nf)\}$, and sd (Nf) as follows:

$sd\{\log_e(Nf)\} = \{\mathrm{sd}(Nf)\}/Nf$. Since $\log_e(Nf) = \log_e 10 * \log_{10}(Nf)$, we now have

$\log_e 10 * \mathrm{sd}\{\log_{10}(Nf)\} = \{\mathrm{sd}(Nf)\}/Nf$, and

$$(sdNf_2)_{op} = \log_e 10 * \mathrm{sd}\{\log_{10}(Nf_2)\} * Nf_2 = 2.30259 * 0.0844 * 2.26986\,\mathrm{E} + 6$$

$$= \mathbf{0.44112\,E + 6}$$

6 Step 5 (Life at Small Failures of Coverage) of a Fatigue Model for a Steel Pipe

It is interesting to observe that in the last Step 4, an estimate of the quantity, $(\min Nf_3)_{op}$, at small "Failure" of coverage, Fp, say, 0.001, turns out to be negative. This is physically meaningless, because the fatigue life of an engineered product cannot be negative. In this final tep 5, we first ignore the estimates of $(\min Nf_3)_{op}$ at low Fp such as 0.01 and 0.001, and recalculate $(\min Nf_3)_{op}$ at a reasonable range of Fp, namely, between 0.25 and 0.05, to obtain a revised result of Table 2 as shown in Table 3.

We then use a nonlinear least squares algorithm and a 3-parameter logistic function (see, e.g., Fong et al. [14, 15]) to fit the five data points of $(\min Nf_3)_{op}$ versus Fp in Table 3 with the assumption that the one-sided Lower Tolerance Limit (*LTL*), at 95%

Table 3 One-sided *LTL* versus $(1 - p)$ between $p = 0.75$ and 0.95 for $n = 6$ and $\gamma = 0.95$

Coverage, p	Confidence Level, $\gamma = 0.95$				
	0.75	0.80	0.85	0.90	0.95
Lack or "Failure" of Coverage $Fp = 1 - p$	0.25	0.20	0.15	0.10	0.05
For $n = 6$					
From Natrella [6] K	1.895	2.265#	2.635#	3.006	3.707
From Step 3 $(Nf_2)_{op}$	2.26986 E + 6	2.26986 E + 6	2.26986 E + 6	2.26986 E + 6	2.26986 E + 6
See Step 4, Eq. 8 $(sdNf_2)_{op}$	0.44112 E + 6	0.44112 E + 6	0.44112 E + 6	0.44112 E + 6	0.44112 E + 6
K* $(sdNf_2)_{op}$	0.83593 E + 6	0.99914 E + 6	1.16235 E + 6	1.32601 E + 6	1.63523 E + 6
$(\min Nf_3)_{op} = (Nf_2)_{op}$—K* $(sdNf_2)_{op}$	**1.43393 E + 6**	**1.27072 E + 6**	**1.10751 E + 6**	**0.94352 E + 6**	**0.63463 E + 6**

#Values of K for $p = 0.80$ and 0.85 are obtained by interpolating tabulated values in Natrella [7]

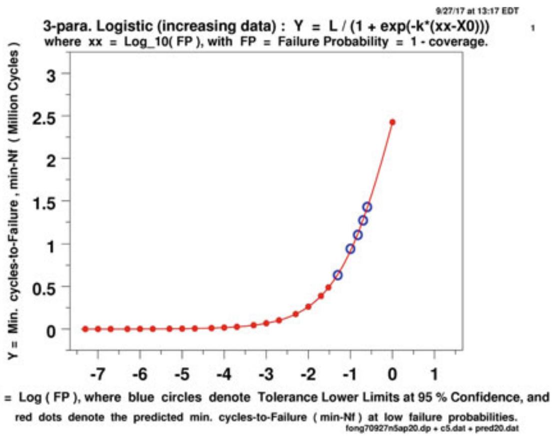

Fig. 6 A nonlinear least squares fit of five Lower Tolerance Limit data (denoted by blue circles) with a series of predicted minimum cycles-to-failure, min*Nf*, by red dots

confidence level, of the fatigue life, i.e., the minimum cycles-to-failure, $(\min Nf_3)_{op}$, of a full-size component approaches zero as the lack or "Failure" of coverage *(Fp)*, defined as $1 - p$, approaches zero. This nonlinear fit allows us to estimate $(\min Nf_3)_{op}$ at extremely low "Failure" of coverage, *Fp*, say, between 10^{-3} and 10^{-7}. The result is shown in Fig. 6, and this completes our 5-step multi-scale fatigue life modeling of a full-size component or structure.

In Figs. 7 and 8, we show the results of applying the multi-scale model to a real-life situation, where a nuclear power plant rotary equipment made of AISI 4340

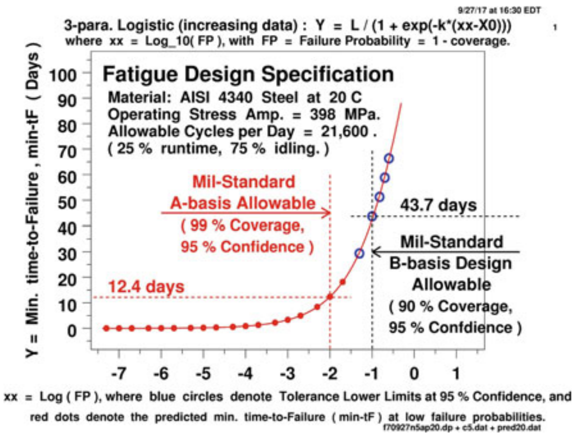

Fig. 7 Predicted minimum time-to-failure, $mint_F$, versus Log_{10} (failure probability)

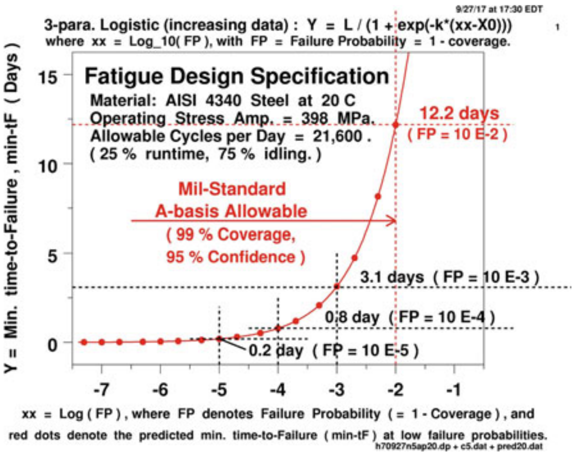

Fig. 8 Predicted minimum time-to-failure, $mint_F$, versus Log_{10} (failure probability) at very low failure probabilities

steel is designed to run at 60 RPM for 25% of continuous runtime at the operating stress amplitude of 398 MPa. We also assume that the lack or "Failure" of coverage, *Fp*, can be equated to a fatigue failure probability, *FP*, with the implication that our multi-scale model can become a probabilistic failure model as the basis for a risk analysis.

In Fig. 9, we complete a failure probability versus minimum time-to-failure plot based on a numerical example of a multi-scale fatigue life model for a full-size component made of AISI 4340 steel. A comparison of the curve in Fig. 9 with Fig. 10 the bathtub curve (see, e.g., Wilkins [16]) at the End-of-Life Wear-Out (fatigue) regime shows a very good agreement.

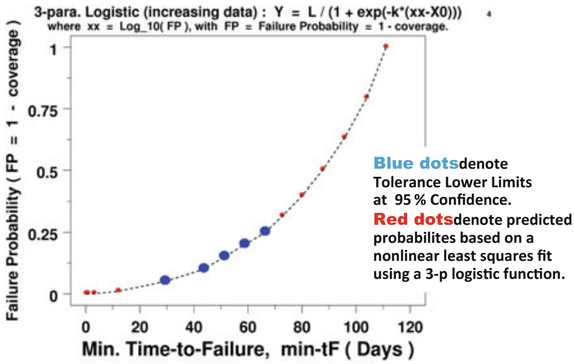

Fig. 9 A time-dependent failure probability plot based on a multi-scale fatigue life model with the blue dots denoting the tolerance lower limits at 95% confidence and the red dots the predicted failure probabilities due to fatigue and lack of coverage

7 From a Multi-scale Fatigue Model to a Dynamic Risk Analysis of a Maintenance Strategy

To illustrate this new approach of linking a fatigue model with a risk analysis, we continue our numerical example on the prediction of a time-to-failure (days) of a critical nuclear power plant equipment versus failure probability, *FP*, as shown in Fig. 9. Assuming that the consequence of an accident due to the failure of that equipment varies from a low of \$10 million to a high of \$100 million with a median of \$50 million, and accepting the validity of the simple equation that risk is the product of failure probability and consequence, we arrive at a graphical plot, as shown in Fig. 11, of an estimate of risk with uncertainty versus a predicted most likely date of a high consequence failure event at a nuclear power plant. This plot, and similar ones for other critical components, can become a valuable tool for a risk-informed inspection strategy associated with the maintenance of any aging plant (Fig. 10).

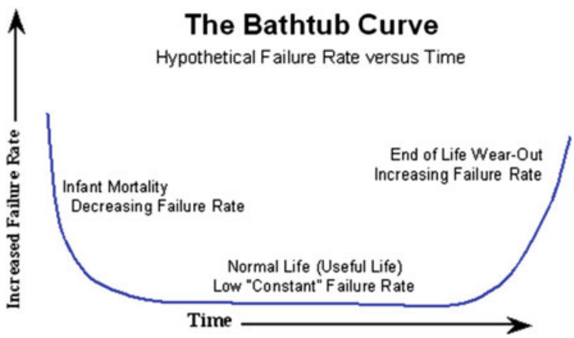

Fig. 10 A graphical representation of a hypothetical product failure behavior of a population of products in the form of a bathtub curve with 3 regimes: Infant Mortality, Normal Life, and End-of-Life Wear-Out (fatigue) (after Wilkins [16])

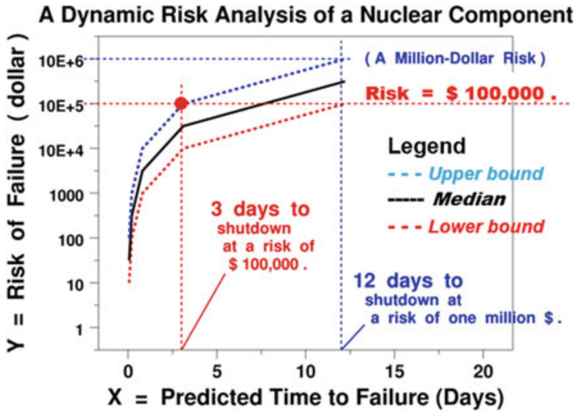

Fig. 11 A dynamic risk analysis of a nuclear component made of an AISI 4340 alloy steel

8 Significance and Limitations of the Multi-scale Fatigue Life Model and Risk Analysis

Statistical methods and concepts have been known to and applied by workers in fatigue for at least 70–80 years. A 1977 review of the literature by Harter [17] on the specialized topic of the size effect on material strength alone, for example, listed about a thousand papers. The subject of a multi-scale fatigue life modeling based on measurement data and imaging at microscopic, specimen, and component levels was addressed by the first author [3] in 1979 with a concluding remark that said,

> … There is a qualitative difference between the use of statistical tools in mechanism research and that in fatigue specimen and component life testing.

The model presented in this paper clearly belongs to the second category. Nevertheless, the idea of using quantitative information at one level, say, Level 1-Micro, to predict fatigue life with uncertainty quantification at a higher level, say, Level 2-Specimen, at extremely high coverage or, equally plausible, high reliability, is generic. The modeling methodology presented in this paper is, therefore, *significant* not only to the advancement of knowledge in the second category, but also in the first, namely, fatigue mechanism research, where a huge amount of information is available at Level 1-Micro, and life prediction at Level 2-Specimen may similarly be modeled with uncertainty quantification.

The multi-scale life model presented in this paper is also new and significant, because for the first time, a physical assumption on the impossibility of a negative life at extremely high coverage has been made to extract from the model new life predictions that are useful to planning inspection of critical components. For high consequence systems with very low failure probability events, a credible risk analysis is generally very difficult because of the lack of data at low failure probabilities. The results of our 5-step multi-scale model should help engineers in making better risk-informed design and maintenance decisions.

However, the proposed model does have limitations that need to be discussed. First of all, the use of the one-sided lower tolerance limit tables of Natrella [7] is strongly linked to the assumption of a normal distribution for the fatigue life. Recent work by Fong et al. [18] on relaxing the normality assumption to include 2-parameter Weibull, 3-parameter Weibull, 2-parameter Lognormal, and 3-parameter Lognormal, should be capable of addressing that shortcoming. Second, the use of a linear least squares fit for the specimen fatigue life data implies a linear model without the existence of an endurance limit. A recent paper by Fong et al. [19] using a nonlinear least squares logistic fit for plain concrete fatigue data showed that endurance limit could exist for that material.

9 Concluding Remarks

An uncertainty-based multi-scale fatigue life model has been presented with a numerical example using the 1973 published fatigue data of six specimens of an AISI 4340 alloy steel.

The modeling methodology is presented in five steps, with the first three describing the statistics and uncertainty quantification of Level 2, the specimen level, and the last two, that of Level 3, the component level. The effort of the first three steps is innovative, because it allows the modeler to estimate the uncertainty of the predicted Level 2 life at any operating stress or stress amplitude. The effort of the last two steps is also new, because it transforms the uncertainty of the predicted Level 2 life into that of the predicted Level 3 life with an added uncertainty due to a new statistical concept known as "coverage."

The combined effort of the five modeling steps is to yield a predicted minimum life vs. failure of coverage or failure probability curve such that for the first time it is feasible for an engineer to predict minimum life at extremely low "failure" of coverage or failure probability between, say, 10^{-3} and 10^{-7}. This curve has been found to be useful to engineers when they are required to make risk-informed decisions on operation and maintenance.

Disclaimer Certain commercial equipment, instruments, materials, or computer software are identified in this paper in order to specify the experimental or computational procedure adequately. Such identification is not intended to imply recommendation or endorsement by the U.S. National Institute of Standards and Technology, nor it is intended to imply that the materials, equipment, or software identified are necessarily the best available for the purpose.

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