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Using radiation pressure to develop a radio-frequency power measurement technique traceable to the redefined SI

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We discuss a power-measurement technique traceable to the International System of Units (SI) based on radiation pressure (or radiation force) inherent in an electromagnetic wave. A measurement of radiation pressure offers the possibility for a power measurement traceable to the kilogram and to Planck's constant through the redefined SI. Towards this goal, we performed measurements of the radiation pressure in a radio-frequency (RF) electromagnetic field at three frequencies (26.5 GHz, 32.5 GHz, and 40.0 GHz) and power levels ranging from 2 W to 25 W using a commercially available mass scale. We show comparisons between the RF power obtained with this technique and those obtained with a conventional power meter. The results in this paper represent the first step towards the realization of a more direct link to RF power within the newly redefined SI.
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One of the keys to developing science and technologies is to have sound metrology tools and techniques. Fundamental to all electromagnetic (EM) measurements is having accurately calibrated probes, antennas, and power meters in order to measure either electric fields or power. A stated goal of international metrology organizations, including the National Institute of Standards and Technology (NIST), is to make all measurements directly traceable to the International System of Units (SI) to ensure a common international basis for accurate measurements. The method for performing absolute power measurements [for both optical and radio-frequency (RF) spectra] has not changed in over 100 years.[1–3] The current method of power traceability is typically based on an indirect traceability path through a thermal measurement using a calorimeter, in which a temperature rise created by absorbed microwave energy is compared to the DC electrical power used to create an identical temperature change.

The world of measurement science is changing rapidly due to the International System of Units (SI) redefinition planned for late 2018.[4,5] As a result of the shift towards fundamental physical constants, the role of primary standards must change. This includes radio-frequency (RF) power. In this work, a direct SI-traceable measurement of RF power is accomplished by the use of the radiation pressure carried in an EM wave, which results in a direct traceable path to the kilogram and to Planck's constant through the redefined SI.

Measurements of optical power using radiation pressure have been demonstrated periodically over the course of a century.[6,7] Recently, laser power has been measured under a variety of conditions using radiation pressure in a portable format that allows a measurement of a horizontally or vertically directed force, thus permitting direct traceability to the kilogram.[8–10] In the 1950s, there was an attempt to use radiation pressure at radio frequencies,[11–13] in which measurements were performed at a power level of tens of watts. However, these measurements were carried out with torsion or pendulum style balances which limit operation in practical measurement conditions and preclude traceability to the kilogram through direct weighing of a calibrated mass. Here, we will use a commercially available mass scale to perform RF measurements in the range of 2 W to 25 W at three different frequencies.

The concept of measuring radiation pressure is based on the fact that EM fields carry a momentum as they propagate through space, and this momentum results in an EM pressure expressed as (in units of N/m^2)[14]

$$\text{Pressure} = \frac{\langle \mathbf{E} \times \mathbf{H} \rangle}{v}, \tag{1}$$

where the symbol "$\langle\ \rangle$" represents the time averaged, $\mathbf{E}$ (in units of V/m) and $\mathbf{H}$ (in units of A/m) are the electric and magnetic fields, and v is the speed of light of the EM wave (in units of m/s). If in free space, v is c (the speed of light *in vacuo*), but it is modified if the EM wave propagates in other environments (see below).

This pressure can be determined using a force measurement, from which the power carried in the RF field can be obtained. It can be shown that when the EM field is normally incident on a device to measure force with a reflecting surface (i.e., a scale), the radiation force is given by (in units of N)[8]

$$F = \frac{2\,P}{v}\left(R + (1-R)\,\frac{\alpha}{2}\right), \tag{2}$$

where P (in units of W) is the RF power incident on the scale, R is the reflectance of the surface (i.e., the power reflection coefficient), and α is the fraction of non-reflected RF that is absorbed. In this experiment, we assume that all of the non-reflected RF energy is absorbed or $\alpha = 1$. The factor of 2 in front of P results from the conservation of linear momentum (i.e., when a particle is totally reflected from a surface, the surface absorbs twice the momentum). By measuring the force (or mass), the RF power is obtained from

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$$P = \frac{mgv}{2\left(R + (1-R)\frac{\alpha}{2}\right)}, \tag{3}$$

where m is the measured mass (in units of g) and g is the acceleration due to gravity (in units of m/s^2).

It is instructive to consider the magnitude of the force (or mass) one must be able to detect for a given power level. For the power levels ranging from 1 W to 25 W (the range used in our experiments below), we calculate a force range of 6.667 nN to 166.667 nN (or 0.6803 μg to 17.01 μg). These force and mass values were obtained from Eq. (2), assuming $R=1$, $v=c$, and the acceleration due to gravity of 9.8 m/s^2. For comparison, a human eyelash weighs approximately 686 nN (or 70 μg) and a fruit fly weighs 1960 nN (or 200 μg). Thus, 1 W is equivalent to about 1/100 of a human eyelash, and 25 W is equivalent to about 1/10 of a fruit fly.

In order to demonstrate the ability to measure the radiation pressure and, in turn, the power carried by an RF wave, we use a commercially available mass scale in the experimental setup shown in Figs. 1 and 2. The experimental setup includes a RF signal generator (SG), a RF power amplifier, a circulator, a high-power load, sections of WR28 rectangular waveguides, an open-ended waveguide probe, and a commercially available Mettler mass scale (the Mettler UMX2 Ultra Microbalance; mention of this product does not imply an endorsement but serves to clarify the equipment used in this experiment). The output of the power amplifier is connected to the waveguide/coax adapter via a cable. The cable is used to minimize mechanical vibrations (i.e., isolating the mass scale from any vibrations caused by the power amplifier). The waveguide/coax adapter is connected to a RF circulator. The circulator is used to keep RF power reflected off the scale's balance pan from entering the high-power amplifier. The reflected power is dumped into a high-power load. The output end of the circulator is connected to three sections of the WR28 waveguide. These waveguide sections are connected to an open-ended waveguide probe. In order to insure maximum reflection of the RF fields from the scale, a 25 mm by 19 mm copper plate (mirror) is placed on the balance pan of the scale (see Fig. 3). The open-ended waveguide is placed close ($\approx$0.1 mm) to the mirror.

The WR28 waveguide has dimensions of $a = 7.112$ mm (the larger cross-sectional dimension) and $b = 3.556$ mm (the smaller cross-sectional dimension), which allows for only one propagating mode [fundamental transverse electric (TE_{10}) mode] between 26.5 GHz and 40.0 GHz. Placing the open-ended waveguide probe 0.1 mm from the copper plate (this is over an order of magnitude smaller than the shortest wavelength used in these experiments) helps us to ensure that approximately 100% of the RF power is incident onto the copper plate and allowed the scale to interact with 100% of the RF power. The RF power is reflected from the copper plate and travels back down the waveguide where it travels through the circulator and is absorbed by the high-power load. In this configuration, v in Eqs. (2) and (3) is the speed of propagation inside the waveguide, and for a TE_{10} mode, it is given by[15]

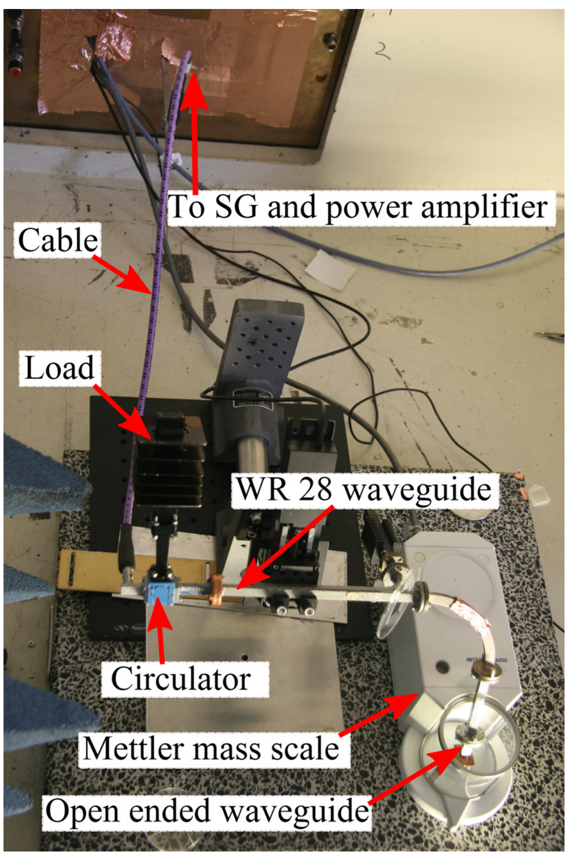

FIG. 1. Photographs of the experimental setup with a mass scale.

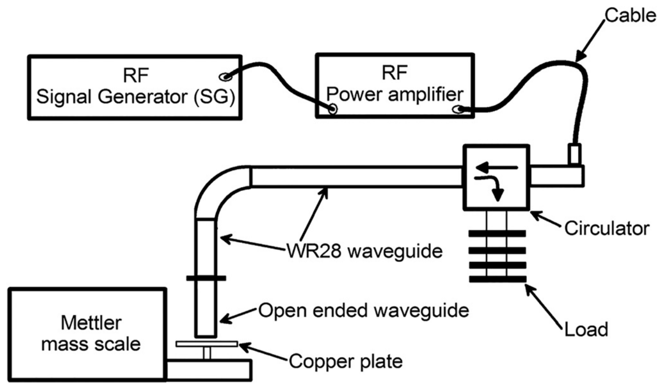

FIG. 2. Block diagram of the experimental setup.

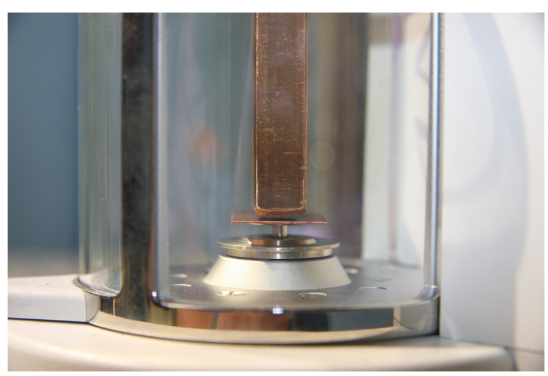

FIG. 3. Close-up of the open-ended waveguide pointed at a mass scale with a copper plate.

$$v = c\sqrt{1-\left(\frac{c}{2af}\right)^2}, \qquad (4)$$

where f is the frequency, and for a WR28 waveguide, $a = 7.112$ mm.

We first performed experiments at 32.5 GHz. During the experiments, the output of the SG was varied such that the power (measured with a conventional power meter) at the output of the SG ranged from 3.55 μW (−24.5 dBm) to 10.0 μW (−20 dBm). At 32.5 GHz, these SG output levels correspond to approximately 13 W and 43 W at the output of the power amplifier. Note that at 32.5 GHz, there is about 4.3 dB of loss in the system from the amplifier to the open-ended waveguide probe. For these relatively high power measurements, we need to ensure that the RF energy does not heat up the mass scale due to absorption of RF power (i.e., microwave heating of the force sensor). This is accomplished by pulsing the RF power, turning the SG on for 2 s and then off for about 20 s. This was repeated five times. Figure 4 shows the measured mass for five pulsed measurements at one SG power level. The five peaks correspond to when the RF power is on and represent the maximum scale deflection. From these pulse measurements, we see that there appears to be little to no effect of heating because the momentum time constant is much faster than the thermal time constant. The change in the mass reading for a given RF power is obtained by taking the difference between the mass when the RF is turned on (the start of a pulse) and the mass at the maximum of the peak (indicated by the arrows in the figure to the left of each pulse). Similarly, data were collected for a range of SG powers. These mass measurements were used in Eq. (3) to obtain a measurement of the RF power at the output of the open-ended waveguide probe. In this expression, we used $R = 0.999$. This is the value of R calculated analytically for a copper plate at the frequencies used in these experiments, and this was confirmed by measuring the reflected power when the copper plate was placed at the open-ended waveguide probe. The measured RF power for a range of SG powers is shown in Fig. 5. The error bars correspond to the standard deviation of seven datasets. Also shown here are measurements obtained with a conventional power meter (measured at the input to the open-ended waveguide probe), where some deviations between the two measurements are seen (more on this below). The measurement using the conventional power meter was performed with the RF energy on continuously; the error bars correspond to a 0.75 dB measurement uncertainty. The power meter we used could not handle the large output power from the amplifier. Thus, for these conventional power meter experiments, the open-ended probe was removed and the section of the waveguide was connected to a RF attenuator system (directional couplers and attenuators). The power meter was then connected to the output of the attenuator system, and the output power was measured for different SG powers. The loss in this RF attenuator system was calibrated and added to the power meter reading. Hence, the final input power (measured with the conventional power meter) at the open-ended waveguide was determined.

We performed the same set of experiments at both 26.5 GHz and 40.0 GHz. The measured RF power (the radiation pressure) obtained with the Mettler mass scale is also shown in Fig. 5. There is a frequency dependence in the output power from the RF amplifier, which is why the measured output power at 32.5 GHz is the highest and 26.5 GHz is the lowest. We also show the results for these two frequencies obtained with a conventional power meter. At all three frequencies, we see that the radiation pressure measurements and the power meter measurements are similar in that they track each other. However, we see that when comparing the measurements from the radiation pressure to those obtained with the power meter, the radiation pressure measurements are

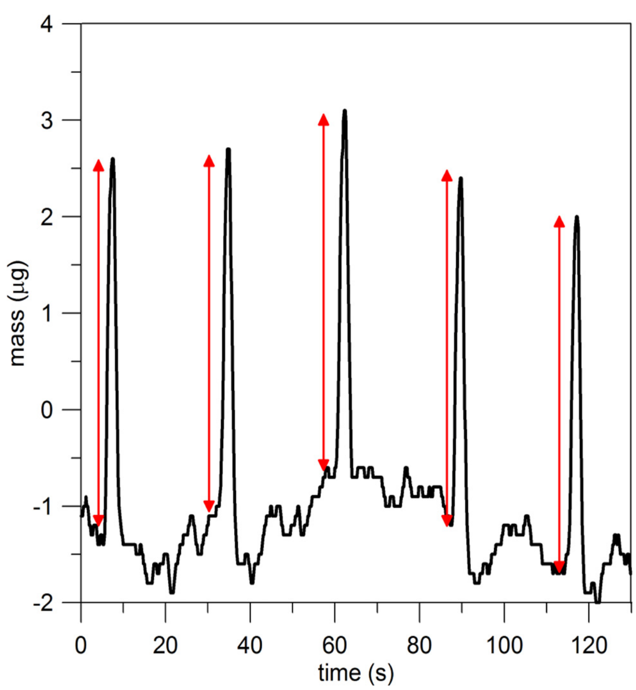

FIG. 4. Measured mass for pulse RF power. The arrows indicate the measured mass due to the RF radiation pressure.

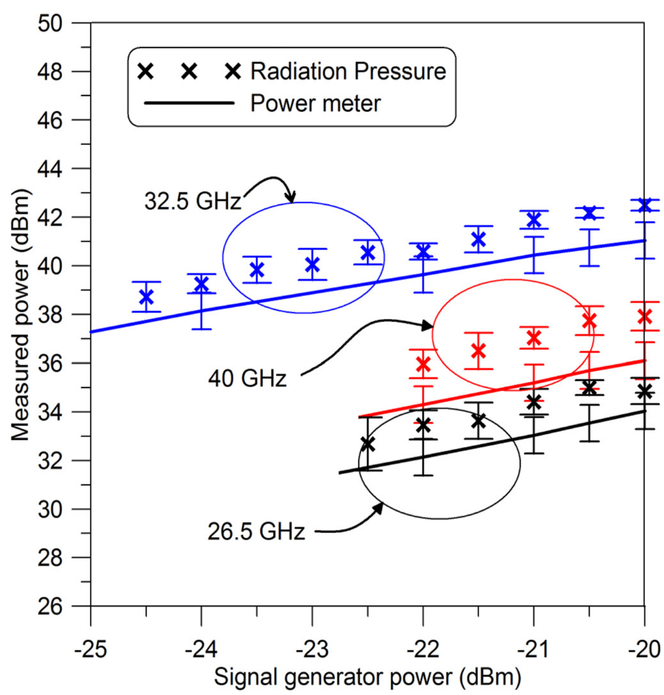

FIG. 5. Measured RF power with comparison to measurements obtained with a conventional power meter.

always higher (by about 1 dB to 1.5 dB) than those obtained with the power meter. A further study will be needed to verify this difference, but we expect that the higher radiation pressure values could be due to the RF energy reflecting multiple times between the open-ended waveguide and the mirror. It is unique to radiation pressure-based power measurements that RF power can be "double-counted" like this, and this effect is used in optical radiation pressure measurements to increase the measurement sensitivity. Such an effect has been seen in optical radiation pressure experiments in the past.[16] This problem may be mitigated in future experiments by changing the location of the scale and waveguide probe. We should also add that, while the RF system was calibrated when performing the power measurements with the convectional power meter, calibration uncertainties of 0.5 dB to 1 dB are possible (see the 0.75 dB error bars shown in Fig. 5). We should also add that the change measured in the mass scale is close to the balance resolution (0.1 μg). It is currently difficult to assess the linearity of the balance for changes in the mass reading that are this small. All these possible sources of error will be investigated in future work. With that said, while there are differences in the two measurements, the results in this paper are the first step towards the development of a direct link to the newly redefined SI.

In the experiments shown in this paper, we did not measure the temperature increase in the copper mirror (i.e., reflector), but a conservative estimate based on calorimetry puts the maximum rise at 3 °C. Our estimate of mirror reflectance came from the theoretical prediction of reflectivity of a copper mirror as better than 0.999 for the frequencies of interest. The overall level of uncertainty appropriate for this paper is indicated by the level of discrepancy between the conventional power measurement and the radiation pressure approach, which is on the order of 1 dB. For the sake of the comparison, the reflectivity only needs to be known to a level of 1 dB, which is on the order of 10%. The expected temperature change will not affect the reflectivity of the copper mirror at this level. Furthermore, although thermal effects are clearly present, they appear at longer timescales. The thermal time constant of the balance and reflector is much longer than the duration of the exposure to RF power and becomes apparent if pulses longer than 5 s are used.

Furthermore, to distinguish between a true radiation pressure based force and forces due to thermal effects alone, we consider two possible ways that heat from the RF power could generate a force on the scale. First, the so-called "radiometric force" occurs when colder air molecules at the back of the mirror flow around the edge to the hotter (lower density) molecules at the front of the mirror, generating a force on the mirror. For this effect to be significant, the distance from the back to the front of the mirror must be on the order of a mean free path (MFP) of the air molecules. For our ∼1 mm thick mirror at atmospheric pressure (MFP ∼10 s of nm), the effect is negligible. This was investigated by Williams *et al.*, in a paper examining radiation pressure for the measurement of laser power.[8] The second thermal force effect could be thermal distortion of the copper mirror. Our conservative estimate has the copper temperature rising by no more than 3 °C due to the incident RF power. Furthermore, in this preliminary testing, the copper mirror was not attached to the scale shaft but merely sitting on top. So, distortion of the copper would be unlikely to couple as a force into the shaft. Williams *et al.* performed a direct thermal injection onto a scale in our previous (laser power) publication.[8] That test was not performed for this particular scale, but similar operating conditions allow us to use the previous result[8] as an indication that thermal effects are negligible in this case.

It has been shown that synchronous detection can be used to accurately measure the radiation pressure force from a modulated laser source.[17,18] It is possible that an analogous technique can be used to improve the signal to noise ratio in the analogous RF power measurement as well.

The major uncertainties in this approach stem from the uncertainty of the small force measurement. With that said, future work will include a detailed uncertainty analysis of this approach. When compared to conventional power metrology approaches, this approach (1) has the possibility of having much lower uncertainty, (2) exhibits a much better frequency range (basic independent of frequency), (3) has a much better dynamic range (i.e., power-level ranges), and (4) is a more direct SI traceable approach.

In this paper, we have demonstrated the ability to use a commercially available mass scale to measure radiation pressure (and force) carried by RF energy. While the perfect agreement is not shown for all the power levels tested and more work is needed to understand all the sources of error for this approach, the results here demonstrated the ability to measure RF power using radiation pressure and can lead to a direct SI-traceable approach for power metrology. This technique could potentially allow electromagnetic power measurements and calibrations from 1 mW to 1 MW (and higher) regardless of frequency (from UV to RF) with one traceability chain.

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