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On Mitigation Inefficiency of Selfish Investment in Network Recovery from High Loss SIS Infection

V. Marbukh
National Institute of Standards and Technology
100 Bureau Drive, Stop 8910
Gaithersburg, MD 20899-8910
E-mail: marbukh@nist.gov

Abstract—**This paper reports on quantification and mitigation of the inefficiency of selfish investment in network recovery from Susceptible Infected Susceptible (SIS) infection in a practically important case of high losses due to infection. In this case, both socially optimal and selfish investments in the infection loss mitigation keep the system close to the boundary of the infection-free region. However, our analysis reveals that while socially optimal investments result in asymptotically zero infection losses, this is not the case for selfish investments. The inefficiency of selfish investments, which is measured by the corresponding Price of Anarchy (PoA), is due to positive externalities. In heterogeneous networks, positive externalities result in finite infection losses despite aggregate overinvestment due to imbalances of selfish investments. While the infection losses can be eliminated with "small" increase in the selfish investments, dealing with imbalances of selfish investments is more challenging. This assessment challenges conventional view that inefficiency of selfish investment in network security is due to aggregate underinvestment, at least in a practically important case of large infection losses. We discuss possible approaches to reduction of the second inefficiency component through regulations, incentives, or their combination, and outline directions of future research.**

Keywords-Susceptible-Infected-Susceptible (SIS) infection; selfish investment in recovery capability; inefficiency evaluation and mitigation.

I. INTRODUCTION

Economic and convenience benefits of interconnectivity drive current explosive emergence and growth of networked systems. However, these benefits of interconnectivity are inherently associated with various risks, including risk of undesirable contagion [1]. Due to its reliance on networked infrastructures, understanding and managing the fundamental risk/benefit tradeoffs of interconnectivity is one of the most important challenges faced by modern society. In this paper, we consider a specific case of infection described by a Susceptible Infected Susceptible (SIS) model, where strategic nodes have the ability to mitigate their losses due to infection by investing in the node recovery capabilities.

Assuming a certain cost/benefit structure, game theoretic analysis of this model [2] numerically evaluated the corresponding Nash equilibria and their inefficiency measured by the corresponding Price of Anarchy. The inefficiency is due to the positive externalities since an investment in infection risk mitigation by a network component reduces likelihood of the infection and thus benefits other system components [3]. It is known [3]-[4] that while in homogeneous networks, this inefficiency is due to aggregate underinvestment by selfish components, in heterogeneous networks investments by some selfish components may exceed the corresponding socially optimal levels [4]. However, the relative contribution of these two sources of inefficiency of selfish investments in the recovery capability in a general network remains an open issue. An even more pressing issue is elimination or at least mitigation of this inefficiency with a viable combination of regulations and incentives. Unfortunately, the computational intractability of the corresponding realistic game-theoretic models makes achieving this goal a serious challenge.

This paper demonstrates that inefficiency of selfish investments can be effectively evaluated for realistic networks under the practically important scenario of large infection losses, when the system operates at the edge of the systemic infection. Major outcomes of our analysis in a case of large infection losses in a general heterogeneous network are as follows. First, contrary to the conventional view, in a heterogeneous network, selfish investments result in aggregate overinvestment. Second, inefficiency of selfish investments is due to (a) this overinvestment, and (b) positive infection losses due to an imbalance of selfish investments. Third, while inefficiency due to infection losses can be "easily" eliminated by a small increase in the investments compared to the selfish equilibrium levels, mitigation of the investment imbalances is a more challenging problem. We suggest approaches to developing practical inefficiency mitigation solutions.

Since an infection-free region in the system parameter space for a SIS infection model is determined by the condition that the corresponding Perron-Frobenius eigenvalue does not exceed unity [2], regulations and incentives/pricing preventing systemic infection can be based on the Gershgorin circle theorem [5] and the matrix perturbation theorem [6]. For a random uncorrelated network, where node centrality is characterized by node degree, we propose adaptive regulation and incentive/pricing strategies.

The paper is organized as follows. Section II briefly describes mean-field models of SIS infection for a general topology and random uncorrelated networks, and solves these

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models at the edge of systemic infection. Section III introduces an economic model, where nodes can control their expected recovery time through investment. Section IV evaluates inefficiency of selfish vs. socially optimal investments in the practically important case of large infection losses. Section IV also proposes and discusses various techniques intended to mitigate inefficiency of the selfish investments. Finally, Section V outlines directions of future research.

II. SIS Infection

Subsection A briefly introduces the Markov model and mean-field approximation of SIS infection in a general network, where nodes can control their expected recovery time through investment. Subsection B discusses this model and mean-field approximation in a particular case of uncorrelated random network. Subsection C shows that the mean-field approximation can be effectively solved at the edge of systemic infection, when the portion of infected nodes is small.

A. General Network

Consider a SIS model on an undirected connected graph with N nodes and irreducible symmetric incidence matrix $A=(A_{ij})_{i,j=1}^{N}$, where $A_{ij}=A_{ji}=1$ if nodes i and j are connected by a link and $A_{ij}=A_{ji}=0$ otherwise. Each node i is either "healthy" or "infected". Introduce indicator $\delta_i=0$ if node i is healthy at moment t, and $\delta_i=1$ otherwise. Once node i becomes infected, it spreads infection to each of its uninfected neighboring nodes j, $A_{ij}=A_{ji}=1$, $\delta_j=0$, at fixed rate $\lambda>0$. Node i recovery time is distributed exponentially with average T_i.

Under these assumptions, vector $\delta(t)=(\delta_i(t), i=1,..,N)$ is a controlled Markov process with 2^N states $\delta\in\{0,1\}^N$ and continuous time $t\geq 0$. While only the steady-state of process $\delta(t)$ is infection-free: $\delta=0$, for large number of nodes N, process $\delta(t)$ may have metastable states describing persistent systemic infection on a practically important time scale [2]. Due to very high dimension 2^N of the corresponding Kolmogorov system, we consider mean-field approximation for probabilities of nodes being infected at moment t, $P_i=P\{\delta_i(t)=1\}$ [2]:

$$P_i'(t)=-(1/\mathrm{T}_i)P_i(t)+\lambda[1-P_i(t)]\sum_{j\neq i}A_{ij}P_j(t), \quad (1)$$

and identify metastable probabilities P_i with non-trivial equilibria of (1):

$$P_i=\frac{\lambda\mathrm{T}_i\sum_{j\neq i}A_{ij}P_j}{1+\lambda\mathrm{T}_i\sum_{j\neq i}A_{ij}P_j}. \quad (2)$$

It is known [2] that for $\lambda<1/\gamma$ where γ is the Perron-Frobenius eigenvalue of matrix $B=(B_{ij})_{i,j=1}^{N}$ with components $B_{ij}=\mathrm{T}_iA_{ij}$, system (2) has only trivial solution $P_i=0$, $i=1,..,N$. It follows from concave Perron-Frobenius theory [7] due to concavity, irreducibility, and some other more technical properties of mapping (2) that for $\lambda>1/\gamma$, in addition to the trivial solution, system (2) also has unique non-trivial solution $P_i>0$, $i=1,..,N$ with non-zero portion of infected nodes:

$$\Omega=(1/N)\sum_i P_i. \quad (3)$$

Figure 1 shows portion (3) as a function of the infection propagation rate λ.

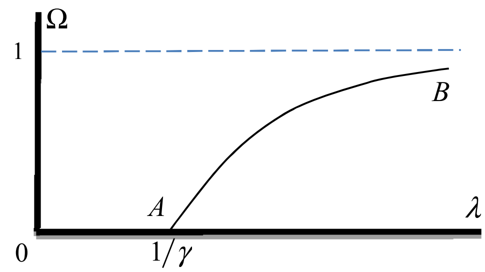

Fig. 1. Portion of infected nodes vs. infection propagation rate.

B. Random Uncorrelated Network

Consider random uncorrelated network with node degree distribution q_d, $d=1,2,..$, $\sum_{d\geq 1}q_d=1$, where node i investment C_i only depends on the node i degree d_i: $C_i=c_{d_i}$. We also assume that node i average recovery time T_i only depends on the node i degree d_i and investment $C_i=c_{d_i}$: $\mathrm{T}_i(c)=\tau_{d_i}(c_{d_i})$.

Under these assumptions, system (1) has solution $P_i(t)=p_{d_i}(t)$, where d_i is the node i degree. This corresponding equilibrium satisfies the following system [8]:

$$p_i=\frac{\lambda i\tau_i\Theta}{1+\lambda i\tau_i\Theta}, \quad (4)$$

where the probability that any given link points to an infected node is

$$\Theta=(1/d_{ave})\sum_{i\geq 1}iq_ip_i \quad (5)$$

and average node degree is $d_{ave}=\sum_{i\geq 1}iq_i$.

Substituting (4) into the right-hand side of (5) we obtain the following equation for Θ:

$$\Theta=\varphi(\Theta), \quad (6)$$

where function

$$\varphi(\Theta)=\frac{\lambda}{d_{ave}}\Theta\sum_{i\geq 1}\frac{i^2\tau_iq_i}{1+\lambda\Theta i\tau_i}. \quad (7)$$

Equation (6) always has trivial solution $\Theta = 0$ describing infection-free equilibrium $p_i = 0$, $i = 1,2,..$. Function $\varphi(\Theta)$ is continuous, increasing and concave with respect to $\Theta > 0$, and thus equation (6) may have at most one non-trivial solution $\Theta^* > 0$, which satisfies the equation

$$\frac{\lambda}{d_{ave}} \sum_{i \geq 1} \frac{i^2 \tau_i q_i}{1 + \lambda \Theta i \tau_i} = 1 . \tag{8}$$

It is easy to verify that this unique non-trivial solution $\Theta^* > 0$ exists for $\lambda > \lambda_*$, where

$$\lambda_* = d_{ave} \big/ \left\langle i^2 \tau_i \right\rangle . \tag{9}$$

We use notation $\langle . \rangle$ for the averaging over node degree distribution q_i. After solving equation (8) with respect to Θ, expressions (4) explicitly identify metastable probabilities p_i and the average portion of persistently infected nodes

$$\Omega = \lambda \Theta \sum_i \frac{i \tau_i q_i}{1 + \lambda \Theta i \tau_i} . \tag{10}$$

Equation (8) can be solved explicitly only in some particular cases. If $\tau_i = \tau_1 / i$, threshold (9) is $\lambda_* = 1/\tau_1$, and node infection probabilities (4) are independent on the node degree: $p_i = \Omega = 1 - 1/(\lambda \tau_1)$, where $\lambda > 1/\tau_1$. For a regular network, where all nodes have the same degree d, threshold (9) is $\lambda_* = 1/(d \tau_d)$, and node infection probabilities (4) are: $p_i = \Omega = 1 - 1/(\lambda d \tau_d)$, where $\lambda > 1/(d \tau_d)$.

C. Edge of Systemic Infection

Consider the solution to mean-field system (2) at the edge of systemic infection, when $\lambda \downarrow 1/\gamma$. Let $\alpha = (\alpha_1, .., \alpha_N)$ and $\beta = (\beta_1, .., \beta_N)$ be the left and right eigenvectors of matrix $B(c) = (\mathrm{T}_i A_{ij})_{i,j=1}^N$, associated with Perron-Frobenius eigenvalue γ, and normalized as follows: $\alpha \beta^T = 1$. The following asymptotic expression for the node infection probabilities at the edge of systemic infection is a direct result of matrix perturbation theory [6]:

$$P_i = \frac{\alpha_i}{\sum_j \alpha_j^2 \beta_j} \left(\lambda - \frac{1}{\gamma} \right) + o\left(\lambda - \frac{1}{\gamma} \right) \text{ as } \lambda \downarrow 1/\gamma . \tag{11}$$

The average portion of infected nodes (3) is

$$\Omega = \frac{\sum_i \alpha_i}{\sum_j \alpha_j^2 \beta_j} \left(\lambda - \frac{1}{\gamma} \right) + o\left(\lambda - \frac{1}{\gamma} \right) \text{ as } \lambda \downarrow 1/\gamma . \tag{12}$$

Note that expressions (11)-(12) generalize asymptotic expressions in [9].

For a random uncorrelated network expanding right-hand side of equation (7) about $\Theta = 0$, we obtain solution to (8) at the edge of systemic infection:

$$\Theta^* = \left(\frac{\lambda}{\lambda_*} - 1 \right) \frac{d_{ave}}{2\lambda^2 \left\langle i^3 \tau_i^2 \right\rangle} + o\left(\frac{\lambda}{\lambda_*} - 1 \right) \text{ as } \lambda / \lambda_* \downarrow 1 , \tag{13}$$

where λ_* is given by (9). Substituting (13) into (4) we obtain the following expression for the infection probabilities:

$$p_i^* = (\lambda/b)(\lambda/\lambda_* - 1) i \tau_i \tag{14}$$

up to the terms of the second order of $\lambda/\lambda_* - 1$ as $\lambda \downarrow \lambda_*$. Note that in a particular case $\tau_i = 1$, expressions (13)-(14) reproduce known result, e.g., systemic infection threshold [8]: $\lambda_* = d_{ave} \big/ \left\langle i^2 \right\rangle$.

III. Economic Model

Subsection A introduces performance models for socially-optimal and selfish investments in the node recovery capabilities for a general network under mean-field approximation. Subsection B demonstrates that for an uncorrelated random network, these models can be solved efficiently, which allows for quantifying the inefficiency of selfish investments. Subsection C discusses various techniques for mitigation of these inefficiencies.

A. General Network

We assume that node i average recovery time T_i is a decreasing and strongly convex function of the node i investment $C_i \geq 0$: $\mathrm{T}_i = \mathrm{T}_i(C_i)$. Also, "large" investment makes recovery "very fast" and "small" investment makes recovery time "very slow," i.e., $\mathrm{T}_i(C_i) \uparrow \infty$ as $C_i \downarrow 0$, and $\mathrm{T}_i(C_i) \downarrow 0$ as $C_i \uparrow \infty$, $i = 1,.., N$.

If an infected node i suffers loss $H_i \geq 0$, the expected node i loss

$$Loss_i(C) = H_i P_i(C) + C_i \tag{15}$$

depends on the entire vector of investments $C = (C_1, .., C_N)$ through infection probability $P_i(C)$. Socially optimal investments $C^{opt} = (C_1^{opt}, .., C_N^{opt})$ minimize the aggregate loss:

$$C^{opt} = \arg\min_{C_i \geq 0} \sum_i [H_i P_i(C) + C_i] \tag{16}$$

subject to constraints (2).

Following [2], we model selfish node investment in the recovery capability as a non-cooperative game G, where each node $i = 1,.., N$ attempts to minimize its expected individual expected loss (14) over this node investment C_i, given investments by other nodes $C_{-i} := (C_j, j \neq i)$. Since increase in the investment C_i by any node $i=1,..,N$ benefits other nodes by reducing the ability of infection to propagate, game G has positive externalities, i.e., $dP_i(C)/dC_j<0$, $i,j=1,..,N$. It can be shown that under our and some additional technical assumptions, game G is strictly concave [11], and thus has unique pure Nash equilibrium $C^*=(C_1^*,..,C_N^*)$, which solves the following optimization problem:

$$C_i^* = \arg\min_{C_i\geq 0}[H_i P_i(C_i, C_{-i}^*) + C_i]. \tag{17}$$

Conventional metric of inefficiency of selfish equilibrium (16) relative to socially optimal equilibrium (16) is "Price of Anarchy" (PoA) [2]-[4]:

$$PoA(C^*|C^{opt}) := \frac{\sum_i [H_i P_i(C^*) + C_i^*]}{\sum_i [H_i P_i(C^{opt}) + C_i^{opt}]}, \tag{18}$$

where $PoA(C^*|C^{opt})\geq 1$, and $PoA(C^*|C^{opt})=1$ means no inefficiency.

B. Random Uncorrelated Network

Consider a random uncorrelated network, where node i loss due to infection H_i and investment C_i only depends on the node i degree d_i: $H_i=h_{d_i}$ and $C_i=c_{d_i}$ respectively. In this network, the probability that a node of degree i is infected is given by (4), and thus the expected loss for this node is

$$loss_i(c) = \lambda h_i \frac{i\tau_i(c_i)\Theta(c)}{1+\lambda i\tau_i(c_i)\Theta(c)} + c_i, \tag{19}$$

where $\Theta(c)$ is given by equation (6).

Thus, for random uncorrelated network, social optimization (16) becomes:

$$c^{opt} = \arg\min_{c_i\geq 0} \sum_i \left(\lambda h_i \frac{i\tau_i(c_i)\Theta}{1+\lambda i\tau_i(c_i)\Theta} + c_i \right) q_i \tag{20}$$

subject to (6). Equation (17) for selfish investments takes the following form:

$$c_i^* = \arg\min_{c_i\geq 0} \left(\lambda h_i \frac{i\tau_i(c_i)\Theta}{1+\lambda i\tau_i(c_i)\Theta} + c_i \right), \tag{21}$$

where $\Theta=\Theta(c^*)$ is the unique solution to equation (6). Price of Anarchy (17) becomes:

$$PoA(c^*|c^{opt}) := \frac{\sum_i [h_i p_i(c^*) + c_i^*] q_i}{\sum_i [h_i p_i(c^{opt}) + c_i^{opt}] q_i}. \tag{22}$$

IV. High Infection Losses

This section considers a practically important case of large infection losses of the order of ε^{-1}, where $\varepsilon\to 0$. It is natural to develop expansions of the socially optimal and selfish node investments as well as the corresponding infection probabilities and expected losses in power series with respect to ε. Due to space limitations, we report some of our results on the leading terms in these expansions. Subsection A considers a general network, and section B considers a random uncorrelated network.

A. General Network

We assume that an infected node i suffers a "large" loss:

$$H_i = H_{i0}/\varepsilon,\ H_{i0}=O(1),\ \varepsilon\to 0, \tag{23}$$

which results in the node i expected loss

$$L_i(\varepsilon, C) = (H_{i0}/\varepsilon) P_i(\varepsilon, C) + C_i. \tag{24}$$

Our goal is expansion of node infection probabilities

$$P_i(\varepsilon, C) = P_{i0}(C) + P_{i1}(C)\varepsilon + P_{i1}(C)\varepsilon^2 + ... \tag{25}$$

for socially optimal and selfish investments.

It can be shown that as $\varepsilon\to 0$, the socially optimal investment by node i, $C_i^{opt}(\varepsilon)$ ensures node infection probability of the order of ε^2:

$$P_i^{opt}(\varepsilon) = P_{i2}^{opt}\varepsilon^2 + O(\varepsilon^3), \tag{26}$$

which results in *asymptotically zero* infection losses:

$$\lim_{\varepsilon\to 0} L_i[\varepsilon, C^{opt}(\varepsilon)] = C_{i0}^{opt}, \tag{27}$$

where $C_{i0}^{opt} = \lim_{\varepsilon\to 0} C_i^{opt}(\varepsilon)$.

On the other hand, as $\varepsilon\to 0$, selfish investment by node i, $C_i^*(\varepsilon)$ ensures node infection probability of the order of ε:

$$P_i^*(\varepsilon) = P_{i1}^*\varepsilon + O(\varepsilon^2), \tag{28}$$

which results in *finite* expected infection loss:

$$\lim_{\varepsilon\to 0} L_i[\varepsilon, C^*(\varepsilon)] = H_{i0}P_{i1}^* + C_{i0}^*, \tag{29}$$

where $C_{i0}^* = \lim_{\varepsilon\to 0} C_i^*(\varepsilon)$.

It follows from (26)-(29) that as $\varepsilon\to 0$, PoA (18) is separated in two components:

$$PoA_0 := \lim_{\varepsilon\to 0} PoA_\varepsilon = PoA_0^{(1)} + PoA_0^{(2)}, \tag{30}$$

where component $PoA_0^{(1)} = (1/C_{\Sigma 0}^{opt})\sum_i H_{i0}P_{i1}^*$ quantifies inefficiency due to infection losses, and component $PoA_0^{(2)} = C_{\Sigma 0}^*/C_{\Sigma 0}^{opt}$ quantifies inefficiency due to aggregate overinvestment by selfish nodes. Here $C_{\Sigma 0}^{opt} := \sum_i C_{i0}^{opt}$ and $C_{\Sigma 0}^* := \sum_i C_{i0}^*$ are aggregate optimal and selfish investments respectively.

It is easy to see that "small," of the order of ε increase in the selfish investments to $C_{i0}^{**}(\varepsilon) = C_{i0}^*(0) + O(\varepsilon)$, $\varepsilon\to 0$ through incentives or regulations or some of their combinations can move the system to the infection-free region eliminating the infection losses and thus resulting in

$$PoA_0 = C_{\Sigma 0}^*/C_{\Sigma 0}^{opt} + O(\varepsilon),\ \varepsilon\to 0. \tag{31}$$

Since Price of Anarchy cannot be less than one, this implies that selfish nodes on average overinvest as compared to the socially optimal level: $C^*_{\Sigma 0} \ge C^{opt}_{\Sigma 0}$.

Due to (26)-(27), the optimal node investment minimizes the aggregate investment by all nodes

$$C^{opt} = \arg\min_{C_i \ge 0} \sum_i C_i \quad (32)$$

subject to condition on the Perron-Frobenius eigenvector $\gamma = \gamma(C)$ ensuring infection free regime:

$$\gamma(C) \le 1/\lambda. \quad (33)$$

Optimization problem (31)-(32) is convex, and thus unique solution, which lies on the boundary of infection free region (33), can be found using Lagrange multipliers [10]. Results of this subsection are illustrated in Figure 2.

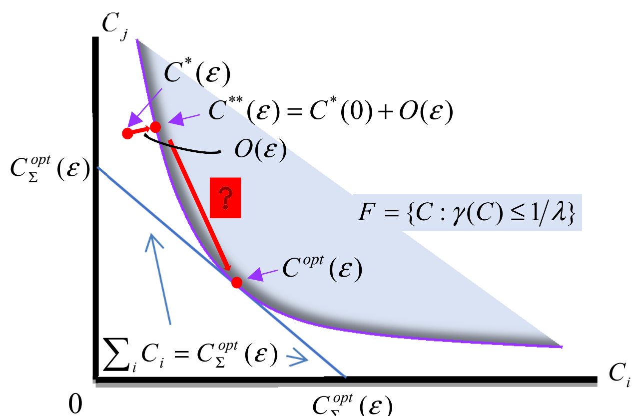

Fig. 2. Social optimization: large infection losses.

Evaluation of the equilibrium selfish investments C^*_{i0} and thus PoA as $\varepsilon \to 0$ for a general network is involved and will be considered elsewhere.

In the rest of this subsection we consider random uncorrelated network in the limit of large infection losses

$$h_i = h_{i0}/\varepsilon,\ i = 1,2,..,\ \text{where } \varepsilon \to 0,\ h_{i0} = O(1). \quad (34)$$

In this case, optimization problem (32)-(33) for the socially optimal investments becomes:

$$c^{opt} = \arg\min_{c_i \ge 0} \sum_i q_i c_i \quad (35)$$

subject to the following condition

$$\sum_{i\ge1} i^2 \tau_i(c_i) q_i \le d_{ave}/\lambda, \quad (36)$$

which follows from $\lambda \le \lambda_*$, where λ_* is given by (9), and guarantees that the network is infection free.

It can be shown that optimization problem (35)-(36) is convex, and thus can be solved using Lagrange multipliers. The first order optimality condition yield the following equation for the socially optimal investments $c_i = c_i^{opt}$:

$$\tau_i'(c_i) = -1/(\mu\lambda i^2), \quad (37)$$

where Lagrange multiplier μ is determined by condition that solution lies on the boundary of the infection-free region (34):

$$\sum_{i\ge1} i^2 \tau_i(c_i) q_i = d_{ave}/\lambda. \quad (38)$$

Equations (21) yield the following system for equilibrium selfish investments $c^* = (c_i^*)$ under (34):

$$c_i^* = \arg\min_{c_i \ge 0} [\lambda\Theta\varepsilon^{-1} h_{i0} i \tau_i(c_i) + c_i]. \quad (39)$$

Since (39) is a convex optimization problem, unique solution to (39) can be found from the following first order optimality condition:

$$\tau_i'(c_i) = -1/(\nu\lambda h_{i0} i), \quad (40)$$

where parameter $\nu = \Theta\varepsilon^{-1}$ is determined by (38).

Comparing equations (37) and (40) we conclude that if node loss due to infection is proportional to the node degree: $h_i \sim i$, $i = 1,2,..$, equilibrium selfish investments are socially optimal: $c_i^* = c_i^{opt}$. Otherwise, inefficiency of selfish investments, measured by PoA (18), is separated in two components (30), where component $PoA_0^{(1)} = (1/c^{opt}_{\Sigma 0})\sum_i h_{i0} p^*_{i1} q_i$ quantifies inefficiency due to infection losses, and component $PoA_0^{(2)} = c^*_{\Sigma 0}/c^{opt}_{\Sigma 0}$ quantifies inefficiency due to imbalances of the selfish investments. Here $c^{opt}_{\Sigma 0} := \sum_i c^{opt}_{i0} q_i$ and $c^*_{\Sigma 0} := \sum_i c^*_{i0} q_i$ are average optimal and selfish investments respectively. As in a general topology network, "small," of the order of ε increase in the selfish investments can move system to the infection-free region eliminating the infection losses.

B. Towards Mitigation Imbalances of Selfish Investments

Inefficiency of selfish investments can be reduced through regulations, market mechanisms, or their combination. Probably, the simplest regulation, which eliminates systemic infection, imposes the following low bound on the node i investment C_i:

$$\mathrm{T}_i(C_i) \le \hat{\mathrm{T}}_i := \left(\lambda \sum_{j\ne i} A_{ij}\right)^{-1}, \quad (41)$$

$i = 1,..,N$. Indeed, the Gershgorin circle theorem [5] ensures that the corresponding Perron-Frobenius eigenvalue $\gamma \le 1$. However, regulations (43) generally do not keep the network close to the optimal operating points even in a case of large infection losses, when the optimal operating point lies at the edge of systemic infection. Market mechanisms with properly designed incentives/penalties have more flexibility and thus may be more economically efficient than regulations in reducing inefficiencies of selfish behavior. One of the major contributions of the Coase theorem [12] is identifying obstacles to ability of market mechanisms to reduce inefficiencies due to externalities. Note in passing that cyber insurance can be viewed as a market mechanism intended to eliminate or at least mitigate some of these obstacles created by incomplete information availability to the selfish participants.

Probably the simplest incentives/penalties scheme for a general network penalizes violation of the constraints (41) once systemic infection emerges by redistribution of the losses due to systemic infection. Broader perspective on incentives/penalties can be gained from the observation that regulations (41) force/incentivize more systemically important nodes to invest more in the recovery capability, where node $i = 1,..,N$ systemic importance is quantified by this node degree $d_i = \sum_{j \neq i} A_{ij}$. This broader perspective assumes that a "central planner," being capable of measuring the aggregate loss due to contagion $H_{ave}^{agg} := NH_{ave}P_{ave}$, where portion of infected nodes is $P_{ave} = N^{-1}\sum_i P_i$, and overall average node loss due to infection is $H_{ave} := (NP_{ave})^{-1}\sum_i H_i P_i(c)$, allocates cost/penalty to each specific node $i = 1,..,N$ according to this node centrality measure π_i, where $\sum_i \pi_i = 1$. This means that the central planner (a) imposes additional cost/penalty on node i equal to $\pi_i H_{ave} - P_i H_i$ if $\pi_i H_{ave} > P_i H_i$, and (b) the central planner provides credit/payment $P_i H_i - \pi_i H_{ave}$ to node i if $\pi_i H_{ave} < P_i H_i$. An example of a centrality measure is $\pi_i = d_i / (Nd_{ave})$, where d_i is node i degree, and d_{ave} is the average node degree. In this short paper we only note that proper notion of node centrality in a general network can be derived from matrix perturbation theory [6]. In the rest of this subsection we demonstrate how a node degree based centrality measure can eliminate inefficiency of selfish investment in a random uncorrelated network.

For a random uncorrelated network, we assume that the "central planner" is aware of parameters h_i, $i = 1,2,..$, and can monitor the average loss per node

$$h_{ave} = \sum_{i \geq 1} h_i p_i q_i . \qquad (42)$$

The central planner compensates all infected nodes for their losses, and then imposes tax/penalty on each infected node of degree $i = 1,2,..$ proportional to i: $t_i = Zi$, where constant Z is chosen to balance the tax inflow and compensation outflow:

$$Z = \sum_{j \geq 1} h_j p_j q_j \Big/ \sum_{j \geq 1} j p_j q_j . \qquad (43)$$

Thus, node $i = 1,2,..$ individual optimization problem becomes

$$\min_{c_i \geq 0} [Zip_i(c_i) + c_i], \qquad (44)$$

and according to (40), for $h_i >> 1$, can be rewritten as follows:

$$\min_{c_i \geq 0} [Z\lambda i^2 \tau_i(c_i) + c_i]. \qquad (45)$$

The solution to optimization problem (45) is given by

$$\tau_i'(c_i) = -1 / (Z\lambda i^2), \qquad (46)$$

where Z is selected according to (43), ensures that then the system stays on the boundary of the infection free region due to our assumption $h_i >> 1$, $i = 1,2,..$.

We end this subsection with the following observations for this scheme. In practical situations, a central planner can determine the aggregate parameter Z adaptively by increasing (decreasing) Z in the presence (absence) of a systemic infection. We leave specific adjustment algorithms for future work. Note that the proposed algorithm compensates for node underinvestment/overinvestment over the socially optimal levels. When $h_i \sim i$, all nodes underinvest, and the proposed algorithm forces all nodes to proportionally increase their investments. In other cases, the proposed algorithm not only raises the aggregate investment, but also rebalances investments by nodes of different degrees.

V. Future Research

Our current research concentrates on performance evaluation, optimization, and practical implementation of the proposed incentive/penalty scheme for random uncorrelated network and generalization of this scheme to arbitrary networks. Results in this paper owe their computational tractability to the assumption of very large infection losses, which results in the solutions on the boundary of the infection-free region. A possibility of relaxing this assumption is an open question.

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