Probability of Coexistence of LTE-LAA and WLAN Systems Based on Delay Constraints*

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Abstract— **To support efficient spectrum sharing and related standardization efforts in unlicensed spectrum, it is important to develop analytical tools to accurately quantify coexistence performance between long-term evolution license assisted access (LTE-LAA) and incumbent systems, such as wireless local area network (WLAN). Though joint throughput of spectrum sharing LTE-LAA and WLAN systems has been extensively studied, there lacks a systematic study on a high level metric – the probability of coexistence (PoC), which indicates whether coexistence is successful or not probabilistically. Another problem is that the majority of available results either ignored delay constraints, or studied only the mean (or variance) of delay, but have not considered the delay distribution and its impact on throughput. To address these problems, we define and analyze the original PoC metrics between LTE-LAA and WLAN systems based on two practical delay constraints. The first PoC is derived from the joint distribution probability of delays for successful transmissions; and the second PoC is defined upon the joint probability of delay-constrained throughput (DCT) of LAA and WLAN systems. To address the technical difficulties involved, we design a novel analytical framework to evaluate the moment generating function and cumulative distribution function (CDF) of the delay, and a new method to evaluate the DCT and its CDF. Consequently, the PoCs can be evaluated accurately with low complexity. The analytical results are verified by our Monte Carlo simulations, which demonstrate impacts of delay and throughput requirements on the PoCs, and illustrate design tradeoffs and insightful findings. These results provide theoretical and practical value for designing improved LTE-LAA and WLAN systems, and may be extended to other emerging spectrum sharing communication systems.**

Keywords: WLAN; LTE-LAA; Probability of Coexistence; Delay-Constrained Throughput; Delay Outage Probability

I. Introduction

Accurately evaluating spectrum sharing performance between long-term evolution license assisted access (LTE-LAA) and IEEE 802.11 wireless local area network (WLAN) systems [1]–[5] is an important ongoing research topic. The listen before talk (LBT) scheme has been considered as a candidate in LAA to enable constructive coexistence [1], [2]. Category 3 and 4 LBT schemes are system-load based sensing schemes. Similar to WLAN, load-based LAA-LBT schemes may use carrier sense multiple access with collision avoidance (CSMA/CA) in the medium access control (MAC) layer, and their coexistence involves a complicated transmission backoff process.

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Standardization efforts on spectrum sharing and coexistence are under way [1], [3], [6], [7]. In [6], coexistence is defined as "The ability of two or more spectrum-dependent devices or networks to operate without harmful interference." In [7], a likelihood of coexistence (LoC) is conceptually proposed. Yet, a detailed mathematical definition of this likelihood is not addressed in [7] but left open to future research based on system applications and environments. In [8], a logistic regression (LR) approach is developed to estimate the LoC between a WLAN system (as interference) and a Zigbee system (as device under test). This scheme uses a training dataset to track and learn the coexistence behaviors using measurements of several input and output parameters. Then, in the testing phase the LR algorithm estimates an LoC metric and the success or failure of each new transmission of the Zigbee system under WLAN interferences. This method relies on actual measurements and availability of training samples to track and predict coexistence performance.

In this paper, we seek to extend the prior art to the probabilistic coexistence research by introducing probability of coexistence (PoC) metrics, and develop methods to map system and MAC protocol parameters to PoCs, by considering two novel and practical successful transmission delay constraints. Here, we consider the LBT for the LAA system and the distributed coordination function (DCF) for the WLAN system. To distinguish from the LoC defined in [8] which is based on prediction of likelihood using logistic regression of measured data, in this paper we choose to use the term PoC for the probability analysis which is established by strict mathematical modeling of system and protocol parameters.

Both LTE-LAA and WLAN specifications have considered real-time audio and video services which are delay sensitive. We believe that two delay constraints are important for delay sensitive applications. The first constraint is related to the delay outage probability (DOP), defined as the probability that packets have not been successfully transmitted before a given delay threshold. The second constraint is based on the delay-constrained throughput (DCT), which is the packet throughput achieved before a given delay threshold. The DCT is more important than the average throughput in quantifying the coexistence performance, because it shows the impact of delay threshold on the throughput. Furthermore, a limiting case of the DCT (when the delay threshold becomes large) maps to static throughput studied in recent works [9]–[11]. Note that CSMA/CA channel access delay is a major consideration for

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optimizing LAA based spectrum sharing schemes. However, to our knowledge, the DCT has not been clearly defined or analyzed for LAA-LBT and WLAN coexistence systems. The DCT defined here is different from those defined by Shannon information theory, which may involve fading channel distributions and signal to noise ratios (SNRs) of the channels.

To our knowledge, a majority of available methods use average throughput as a key performance metric without considering transmission delay constraints, such as [9]–[16], [18]. To study the impact of delay constraints on coexistence performance, current methods are insufficient. Some CSMA/CA MAC-layer performance analysis methods for WLAN systems were developed in [19], [20], and have been extended in [9]–[11] for coexistence analysis. The methods in [9]–[11] have not modeled transmission backoff wait time, a critical element for the delay analysis. [12] models the backoff counter hold time for the coexisting LAA and WLAN systems, but does not analyze the transmission delays. In [17], a joint optimization of LTE-LAA and WLAN systems is investigated, and the average packet transmission delay is evaluated. In [21], the authors model the backoff hold time, and provide an analysis on the mean and variance of transmission delay for a WLAN system. However, this method does not consider other important metrics, such as the cumulative distribution function (CDF) of delay and its impact on the throughput, a more difficult evaluation.

In this paper, we define new PoC metrics to quantify coexistence performance of LAA and WLAN systems, and design new analytical methods to assess transmission delay statistics to obtain PoCs. The contributions and novelties are highlighted as follows:

- We develop a novel transmission delay analysis for coexisting LAA and WLAN systems, and provide new formulas of DOP and DCT.
- We define and evaluate PoC metrics based on these delay criteria, which involve a joint CDF of delays and a joint CDF of DCTs.
- We implement computer programming and extensive Monte Carlo simulations, which demonstrate a good match between our analytical and simulation results. Numerical results illustrate some insightful findings.

These results fill a major gap in the coexistence analysis of LTE-LAA and WLAN systems with practical delay constraints. They provide new insight into understanding performance uncertainties caused by protocol and system parameters and their impact on the PoC metrics. The results allow us to achieve flexible performance tradeoffs between threshold values of delays and throughput, and between LAA and WLAN systems based on their different requirements.

The technical insight and methods provided by this work may be used for analysis and optimization of other spectrum sharing systems and technologies, such as coexisting WLAN and Bluetooth (or Zigbee) devices. Actually, when the moment generating function (MGF) of a link transmission delay in a different system is obtained, the link delay statistics (such as DOP and DCT) can be readily evaluated by using our technique. Here, the PoC is assessed in terms of delay and payload throughput in the MAC layer. With absorption of more information from application and physical layers, and depending on the particular devices or applications, additional PoC metrics may be defined and evaluated. For ease of reference, some symbols, expressions and their definitions are listed in Table I.

II. System Model

Suppose that in several small cells the LTE-LAA and WLAN systems share the unlicensed spectrum in an industrial, scientific, and medical (ISM) radio band. There are n_L LAA links coexisting with n_W WLAN links. All LAA and WLAN links can hear each other, and at any time successful transmission happens when only one link transmits. Suppose that the LAA-LBT and WLAN systems use similar CSMA/CA MAC schemes, but with different parameters.

In this paper, the delay for a payload is defined as the duration from the time the packet becomes the head of the line (HoL) in the transmit queue until the instant that the payload transmission finishes successfully. The delay threshold is defined as the maximum delay duration allowed for a packet to finish its transmission. Otherwise, it is counted as a packet delay outage. In this model, the delay threshold is used to quantify performance of the MAC schemes, but it is not enforced to drop outdated packets. The LAA and WLAN systems considered still use their original MAC schemes (such as LBT and DCF) to schedule and transmit packets. If the delay thresholds are enforced to drop packets, the resulting MAC schemes will be different, and a new design may lead to better delay statistics. However, this extension and related optimization work are out of the scope of this paper, but would be considered in future work.

The DOP for a payload is defined as:

$$\mathrm{DOP}_D(D_{\mathrm{Th}}) = P\{D > D_{\mathrm{Th}}\}, \tag{1}$$

where D and D_{Th} are the experienced delay and the delay threshold value (aka. permitted maximum delay), respectively.

We define a link's DCT as the total payload that is successfully transmitted by the delay threshold D_{Th}, divided by the average delay duration $\overline{D}_{\mathrm{Th}}$. The DCT is given by

$$\mathrm{DCT}(D_{\mathrm{Th}}) = \frac{E[\text{payload successfully transmitted by } D_{\mathrm{Th}}]}{\overline{D}_{\mathrm{Th}}}, \tag{2}$$

where $\overline{D}_{\mathrm{Th}}$ is the average duration including the effect of delay outage, and we can call it a modified delay threshold. It is given by

$$\overline{D}_{\mathrm{Th}} = D_{\mathrm{Th}} + T_{\mathrm{DOP}}(D_{\mathrm{Th}}). \tag{3}$$

In (3),

$$T_{\mathrm{DOP}}(D_{\mathrm{Th}}) = \int_{D_{\mathrm{Th}}}^{\infty} x f_D(x) dx, \tag{4}$$

where $f_D(x)$ is the probability density function (PDF) of D. The PDF expression can be obtained by using numerical

TABLE I: Definition of some symbols and expressions frequently used in this paper.

Symbol or Expression	Definition
DCT	Delay constrained throughput
DOP	Delay outage probability
MGF	Moment generating function
D_L (or D_W)	Transmission delay in an LAA (or WLAN) link.
$D_{L,\text{Th}}$ (or $D_{W,\text{Th}}$)	Delay threshold of an LAA (or WLAN) link.
$\text{DOP}_L(D_{L,\text{Th}})$ (or $\text{DOP}_W(D_{W,\text{Th}})$)	DOP with threshold $D_{L,\text{Th}}$ (or $D_{W,\text{Th}}$) in an LAA (or WLAN) link.
$G_{D_L}(s)$ (or $G_{D_W}(s)$)	MGF of delay D_L (or D_W).
$P_{D_L}(D_{L,\text{Th}})$ (or $P_{D_W}(D_{W,Th})$	CDF of delay D_L (or D_W) with threshold value $D_{L,\text{Th}}$ (or $D_{W,Th}$).
$\text{PoC}_{\text{DOP}}(D_{L,\text{Th}}, D_{W,\text{Th}})$	PoC based on LAA and WLAN joint DOPs with delay threshold pair $(D_{L,\text{Th}}, D_{W,\text{Th}})$.
$P_{L,\text{EX}}(n, D_{L,\text{Th}})$ (or $P_{W,\text{EX}}(n, D_{W,\text{Th}})$)	Probability that exactly n payloads are transmitted by $D_{L,\text{Th}}$ (or $D_{W,\text{Th}}$) in an LAA (or WLAN) link.
$\text{PoC}_{\text{DCT}}(D_{L,\text{Th}}, D_{W,\text{Th}}, R_{L,\text{Th}}, R_{W,\text{Th}})$	PoC based on joint DCTs with delay threshold pair $(D_{L,\text{Th}}, D_{W,\text{Th}})$ and throughput threshold pair $(R_{L,\text{Th}}, R_{W,\text{Th}})$.
$P_{t,L}$ (or $P_{t,W}$)	Conditional successful transmission probability of an LAA (or WLAN) link.
$R_L(D_{L,\text{Th}})$ (or $R_W(D_{W,\text{Th}})$)	DCT of an LAA (or WLAN) link based on delay threshold $D_{L,\text{Th}}$ or $(D_{W,\text{Th}})$.
τ_L (or τ_W)	Channel access probability of an LAA (or WLAN) link.

differentiation of the CDF of D, and the CDF is evaluated with the inverse Laplace transform (ILT) of the MGF of D, which is derived in detail in Section III. Eq. (3) has two parts: When the delay outage does not happen (with probability $1 - \text{DOP}_D(D_{\text{Th}})$), the delay is given by D_{Th}; and when delay outage happens (aka, no packet was successfully transmitted) with probability $\text{DOP}_D(D_{\text{Th}})$, the involved additional delay is then given by $T_{\text{DOP}}(D_{\text{Th}})$. Note that $\overline{D}_{\text{Th}} \geq D_{\text{Th}}$ holds. Our DCT definition in (2) and (3) is original and non-trivial. It properly accounts for the effect of delay outage on the achieved throughput. Methods to analyze and evaluate $\text{DOP}_D(D_{\text{Th}})$ and $\text{DCT}(D_{\text{Th}})$ for LAA and WLAN links will be provided in Section III.

We use subscripts L, W, i, S, C, p to denote LAA, WLAN, idle, successful transmission, collision, and payload, respectively. Suppose the LAA-LBT process has cutoff stage M_L, and stage m has contention window (CW) size Z_m, for $m = 0, 1, \ldots, M_L$; and the WLAN has cutoff stage M_W, and stage m has CW size W_m. We define δ_W as a backoff idle slot duration, which is identical for both WLAN and LAA systems. $T_{S,L}$ and $T_{C,L}$ are defined as durations for an LAA link's successful transmission and failed transmission, respectively; and $T_{S,W}$ and $T_{C,W}$ as durations of a WLAN link's successful and failed transmissions. Note that $T_{S,L}$ ($T_{S,W}$) and $T_{C,L}$ ($T_{C,W}$) include handshaking duration, and silence period after transmission, such as deferred extended clear channel assessment (DeCCA) duration [1], [11]. Here, the DeCCA duration is assumed to be equal to the WLAN DCF interframe space (DIFS) period. Both basic access and request-to-send and clear-to-send (RTS-CTS) access schemes can be modeled for both LAA and WLAN systems, by selecting proper values of $T_{S,L}$ ($T_{S,W}$) and $T_{C,L}$ ($T_{C,W}$).

III. Delay Performance Analysis

The technical task flow of this work may be summarized as follows:

1) Define system and protocol parameters for LAA and WLAN systems.
2) Compute coexistence performance metrics, such as channel access probabilities, successful transmission probabilities, and average throughput.
3) Analyze delay statistics, such as the MGF and DOP.
4) Evaluate the first PoC based on the joint DOP metric.
5) Analyze the DCT and its distribution. Evaluate the second PoC based on the joint DCT metrics.

In this task list, each step is built on its previous step. Steps 1 and 2 have been implemented in several works [9]–[12], but Steps 3-5 are original in this paper. Analyzing the DCT in Step 5 is especially challenging, and a similar analytical technique could not be found in the available literature. Moreover, even a proper definition of the DCT is non-trivial, as shown by (2) and (3).

Define the conditional successful transmission probabilities (STPs) for an LAA link and a WLAN link as $P_{t,L}$ and $P_{t,W}$, respectively, when their backoff counters reduce to zero. Complementary, $P_{f,L} = 1 - P_{t,L}$ and $P_{f,W} = 1 - P_{t,W}$ are probabilities of collision (or failed transmission). Here, we assume that failed transmissions are only caused by packet collisions. It follows that

$$P_{t,L} = (1 - \tau_L)^{n_L - 1}(1 - \tau_W)^{n_W}, \quad (5)$$
$$P_{t,W} = (1 - \tau_L)^{n_L}(1 - \tau_W)^{n_W - 1}, \quad (6)$$

where τ_L and τ_W are the transmission (or channel access) probabilities of LAA and WLAN systems, respectively. Based on a result in [11], the transmission probability of an LAA link is given by

$$\tau_L \quad = \quad \frac{2(1-P_{f,L}^{M_L+1})}{(1-P_{f,L})\sum_{m=0}^{M_L} P_{f,L}^m (1+Z_m)}. \tag{7}$$

Assuming that WLAN nodes have a similar CSMA/CA backoff algorithm as the LAA nodes (but with different parameters), the transmission probability of a WLAN node is given by

$$\tau_W \quad = \quad \frac{2(1-P_{f,W}^{M_W+1})}{(1-P_{f,W})\sum_{m=0}^{M_W} P_{f,W}^m (1+W_m)}. \tag{8}$$

A. MGF of the Delay

Define $D_{L,m}$ as the transmission delay of an LAA node at backoff stage m ($m = 0, 1, \ldots, M_L$). It can be expressed as

$$D_{L,m} = \begin{cases} T_{L,m} + T_{S,L} & \text{with prob. } P_{t,L}, \\ T_{L,m} + T_{C,L} + D_{L,m+1} & \text{with prob. } P_{f,L}. \end{cases} \tag{9}$$

In (9), $T_{L,m}$ is the backoff hold time which includes the LBT sensing time and counter frozen duration. In the second line of (9), the term $D_{L,m+1}$ means that the backoff stage moves to $m+1$ due to a failed transmission. In the last stage, the delay is given by

$$D_{L,M_L} = \begin{cases} T_{L,M_L} + T_{S,L} & \text{with prob. } P_{t,L}, \\ T_{L,M_L} + T_{C,L} + D_{L,0} & \text{with prob. } P_{f,L}. \end{cases} \tag{10}$$

In the second line of (10), the term $D_{L,0}$ implies that the state moves to initial stage 0 due to a failed transmission and the packet is dropped. However, for fairness of evaluating performance metrics, the delay involved for the dropped packets is counted towards the total delay of the considered LAA link.

Based on (9) and (10), the successful transmission delay per payload in an LAA link is given by $D_{L,0}$. So we can use D_L and $D_{L,0}$ interchangeably. For a WLAN link, the transmission delay is equal to D_W. We want to derive the statistics of D_L, including the MGF and CDF, to evaluate the DOP and DCT metrics. To evaluate (9) and (10), we need to analyze the $T_{L,m}$ and $P_{t,L}$.

To accurately model the interaction between LAA and WLAN systems, we use probabilities $\hat{P}$ and P to denote events observed by a node when observing its own system (e.g., state of LAA system observed by an LAA node), and the other system (e.g., state of LAA system observed by a WLAN node), respectively. Let $\hat{P}_{i,L}$ (or $P_{i,L}$), $\hat{P}_{S,L}$ (or $P_{S,L}$), and $\hat{P}_{C,L}$ (or $P_{C,L}$) be probabilities of non-transmission (idle), successful transmission, and collision in the LAA system observed by an LAA node (or a WLAN node), respectively. It follows that $\hat{P}_{i,L} = (1-\tau_L)^{n_L-1}$ and $P_{i,L} = (1-\tau_L)^{n_L}$ because an LAA node is affected by activity of the other $n_L - 1$ LAA nodes, but a WLAN node is affected by n_L LAA nodes. Also, we obtain $\hat{P}_{S,L} = (n_L-1)\tau_L(1-\tau_L)^{n_L-2}$, $P_{S,L} = n_L\tau_L(1-\tau_L)^{n_L-1}$, $\hat{P}_{C,L} = 1-(1-\tau_L)^{n_L-1} - (n_L-1)\tau_L(1-\tau_L)^{n_L-2}$, and $P_{C,L} = 1-(1-\tau_L)^{n_L} - n_L\tau_L(1-\tau_L)^{n_L-1}$.

Similarly, to model the activity of the WLAN system, we define $\hat{P}_{i,W}$ (or $P_{i,W}$), $\hat{P}_{S,W}$ (or $P_{S,W}$), and $\hat{P}_{C,W}$ (or

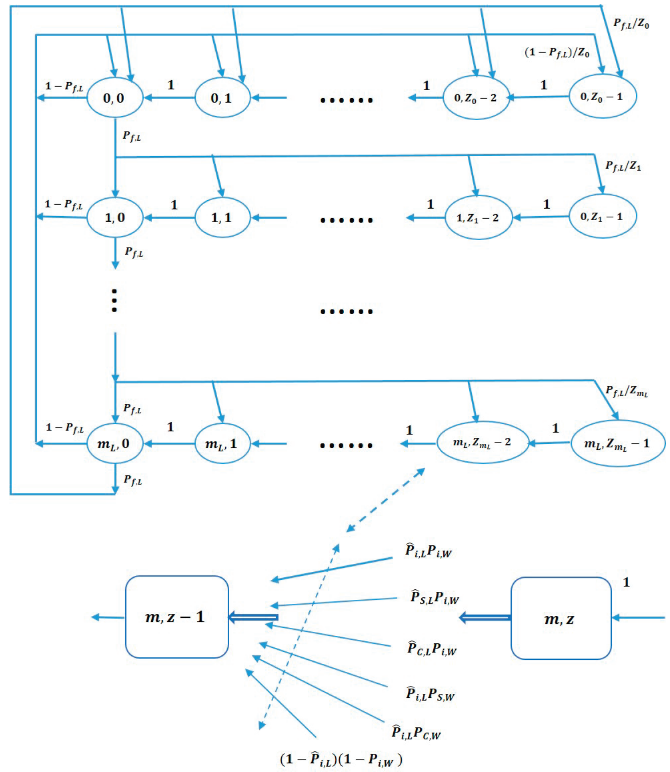

Fig. 1: Our proposed Markov model for the LTE-LAA LBT category 4 procedure (revised from [11]), where six probability paths in the backoff counter reduction process are highlighted.

$P_{C,W}$) be probabilities of non-transmission (idle), successful transmission, and collision in the WLAN system observed by a WLAN node (or an LAA node), respectively. It follows that $\hat{P}_{i,W} = (1-\tau_W)^{n_W-1}$, $P_{i,W} = (1-\tau_W)^{n_W}$, $\hat{P}_{S,W} = (n_W-1)\tau_W(1-\tau_W)^{n_W-2}$, $P_{S,W} = n_W\tau_W(1-\tau_W)^{n_W-1}$, $\hat{P}_{C,W} = 1-(1-\tau_W)^{n_W-1} - (n_W-1)\tau_W(1-\tau_W)^{n_W-2}$, and $P_{C,W} = 1-(1-\tau_W)^{n_W} - n_W\tau_W(1-\tau_W)^{n_W-1}$. Refer to Fig. 1, where the counter reduction (from value z to $z-1$) at backoff stage m in an LAA transmit node is depicted. We model the feedforward path for one backoff counter reduction as six mutually exclusive sub-events: all LAA and WLAN nodes are idle (with probability $\hat{P}_{i,L}P_{i,W}$ and duration δ_W), successful transmission of an LAA node (with probability $\hat{P}_{S,L}P_{i,W}$ and duration $T_{S,L}$), collision of LAA nodes while WLAN system is idle (with probability $\hat{P}_{C,L}P_{i,W}$ and duration $T_{C,L}$), successful transmission of a WLAN node (with probability $P_{S,W}\hat{P}_{i,L}$ and duration $T_{S,W}$), collision of WLAN nodes while LAA system is idle (with probability $P_{C,W}\hat{P}_{i,L}$ and duration $T_{C,W}$), and LAA-WLAN inter-system transmission collision (with probability $(1-P_{i,W})(1-\hat{P}_{i,L})$ and duration $T_{C,M}$), where $T_{C,M} = \max(T_{C,L}, T_{C,W})$.

Define T_{LCR} and T_{WCR} as the average hold time per counter reduction for an LAA node and a WLAN node, respectively. For an LAA node, its counter reduction must experience one of six mutually exclusive events, and T_{LCR} is thus given by

$$\begin{aligned} T_{\text{LCR}} &= \hat{P}_{i,L}P_{i,W}\delta_L + \hat{P}_{S,L}P_{i,W}T_{S,L} + \hat{P}_{C,L}P_{i,W}T_{C,L} \\ &+ \hat{P}_{i,L}P_{S,W}T_{S,W} + \hat{P}_{i,L}P_{C,W}T_{C,W} \\ &+ (1-\hat{P}_{i,L})(1-P_{i,W})T_{C,M}. \end{aligned} \quad (11)$$

We can verify that the probability mass function (PMF) involved in (11) sums up to unity and is valid, as shown by

$$\begin{aligned} &\hat{P}_{i,L}P_{i,W} + \hat{P}_{S,L}P_{i,W} + \hat{P}_{C,L}P_{i,W} + P_{S,W}\hat{P}_{i,L} \\ &+P_{C,W}\hat{P}_{i,L} + (1-\hat{P}_{i,L})(1-P_{i,W}) = 1. \end{aligned} \quad (12)$$

The MGF of T_{LCR} is defined as its Laplace transformation as $G_{T_{\text{LCR}}}(s) = E[\exp(sT_{\text{LCR}})]$. By modeling T_{LCR} as a random variable with the six PMF-and-duration pairs shown in (11), we obtain

$$\begin{aligned} G_{T_{\text{LCR}}}(s) &= \hat{P}_{i,L}P_{i,W}\exp(s\delta_L) + \hat{P}_{S,L}P_{i,W}\exp(sT_{S,L}) \\ &+\hat{P}_{C,L}P_{i,W}\exp(sT_{C,L}) + \hat{P}_{i,L}P_{S,W}\exp(sT_{S,W}) \\ &+\hat{P}_{i,L}P_{C,W}\exp(sT_{C,W}) \\ &+(1-\hat{P}_{i,L})(1-P_{i,W})\exp(sT_{C,M}). \end{aligned} \quad (13)$$

Similarly, the MGF of T_{WCR} is derived as

$$\begin{aligned} G_{T_{\text{WCR}}}(s) &= E[\exp(sT_{\text{WCR}})] \\ &= \hat{P}_{i,W}P_{i,L}\exp(s\delta_W) + \hat{P}_{S,W}P_{i,L}\exp(sT_{S,W}) \\ &+\hat{P}_{C,W}P_{i,L}\exp(sT_{C,W}) + \hat{P}_{i,W}P_{S,L}\exp(sT_{S,L}) \\ &+\hat{P}_{i,W}P_{C,L}\exp(sT_{C,L}) \\ &+(1-\hat{P}_{i,W})(1-P_{i,L})\exp(sT_{C,M}). \end{aligned} \quad (14)$$

At backoff stage m, the total wait time $T_{L,m}$ for an LAA link is a function of CW size Z_m and per-counter reduction time T_{LCR}, and can be expressed as

$$T_{L,m} = \frac{1}{Z_m}\sum_{m=0}^{Z_m-1}(x_m + \delta_W), \quad (15)$$

where x_m is the wait time if the backoff starts at the mth counter, and is the sum of m independent and identically distributed (i.i.d.) waiting slots (each with duration T_{LCR}) and one additional δ_W duration. Thus, the MGF of x_m is given by $G_{x_m}(s) = G^m_{T_{\text{LCR}}}(s)$. The MGF of $T_{L,m}$ is obtained as

$$\begin{aligned} G_{T_{L,m}}(s) &= \frac{1}{Z_m}\sum_{m=0}^{Z_m-1} G^m_{T_{\text{LCR}}}(s)e^{s\delta_W} \\ &= \frac{e^{s\delta_W}}{Z_m}\frac{1-G^{Z_m}_{T_{\text{LCR}}}(s)}{1-G_{T_{\text{LCR}}}(s)}. \end{aligned} \quad (16)$$

At stage m, total backoff wait time $T_{W,m}$ for a WLAN link is given by

$$G_{T_{W,m}}(s) = \frac{e^{s\delta_W}}{W_m}\frac{1-G^{W_m}_{T_{\text{WCR}}}(s)}{1-G_{T_{\text{WCR}}}(s)}. \quad (17)$$

Finally, by using the recursive relation shown in (9) and (10), the MGF of the delay D_L is derived as

$$G_{D_L}(s) = \frac{\sum_{m=0}^{M_L}\left(\prod_{k=0}^{m}G_{T_{L,m}}(s)\right)e^{s(mT_{C,L}+T_{S,L})}(1-P_{t,L})^m P_{t,L}}{1-\left(\prod_{k=0}^{M_L}G_{T_{L,m}}(s)\right)e^{s((M_L+1)T_{C,L})}(1-P_{t,L})^{M_L+1}}. \quad (18)$$

When $M_L = 0$, (18) simplifies to

$$G_{D_L}(s) = \frac{\exp(sT_{S,L})P_{t,L}}{1-\exp(sT_{C,L})(1-P_{t,L})}. \quad (19)$$

The delay of successful transmission for a WLAN node is denoted by D_W. Similarly, its MGF is derived as

$$G_{D_W}(s) = \frac{\sum_{m=0}^{M_W}\left(\prod_{k=0}^{m}G_{T_{W,m}}(s)\right)e^{s(mT_{C,W}+T_{S,W})}(1-P_{t,W})^m P_{t,W}}{1-\left(\prod_{k=0}^{M_W}G_{T_{W,m}}(s)\right)e^{s(M_W+1)T_{C,W}}(1-P_{t,W})^{M_W+1}}. \quad (20)$$

B. CDF of Delay and the First PoC

The DOP of D_L is equal to its CDF for a given threshold $D_{L,\text{Th}}$, and can be obtained by using the ILT of the MGF $G_{D_L}(s)$, as shown by

$$P_{D_L}(D_{L,\text{Th}}) = \text{ILT}[\exp(sD_{L,\text{Th}})G_{D_L}(s)/s]. \quad (21)$$

An efficient ILT numerical formula for evaluating CDF of a variable from its MGF was developed in [22], and used for wireless communication outage probability computation in several works, such as [23], [24]. When a variable D has MGF $G_D(s)$, its CDF $P_D(D_{\text{Th}})$ is given by

$$\begin{aligned} P_D(D_{\text{Th}}) &= P\{D < D_{\text{Th}}\} \\ &= 2^{-Q}e^{\frac{A}{2}}\sum_{q=0}^{Q}\binom{Q}{q}\sum_{n=0}^{N+q}(-1)^n\beta_n \\ &\times \Re\left(\frac{G_D\left(\frac{A+jn2\pi}{2D_{\text{Th}}}\right)}{A+jn2\pi}\right) + E_{A,N,Q}, \end{aligned} \quad (22)$$

where $\beta_n = \begin{cases} 1, & n=0 \\ 2, & n=1,\ldots,N+Q \end{cases}$, $A, N,$ and Q are parameters used to control the convergence, and $E_{A,N,Q}$ is an error term which diminishes as N and Q increase.

By replacing D and D_{Th} in (22) with D_L and $D_{L,\text{Th}}$, we readily obtain the CDF $P_{D_L}(D_{L,\text{Th}})$. Define D_W and $D_{W,\text{Th}}$ as the successful transmission delay and delay threshold for a WLAN link. The CDF of D_W is given by

$$P_{D_W}(D_{W,\text{Th}}) = \text{ILT}[\exp(sD_{W,\text{Th}})G_{D_W}(s)/s], \quad (23)$$

which can be efficiently evaluated by using (22). The DOPs of LAA and WLAN systems are defined as the complementary CDF (CCDF) of D_W and D_L, given by

$$\begin{aligned} \text{DOP}_L(D_{L,\text{Th}}) &= 1 - P_D(D_{L,\text{Th}}), \\ \text{DOP}_W(D_{W,\text{Th}}) &= 1 - P_{D_W}(D_{W,\text{Th}}). \end{aligned}$$

Based on joint CDFs of both LAA and WLAN systems, we define the first PoC as

$$\mathrm{PoC}_{\mathrm{DOP}}(D_{L,\mathrm{Th}}, D_{W,\mathrm{Th}}) = P\{D_L \le D_{L,\mathrm{Th}}, D_W \le D_{W,\mathrm{Th}}\}.$$

Since the interactions between LAA and WLAN links are modeled in the backoff delay statistics, as shown by (18) and (20), it is reasonable to assume that DOPs of the LAA and WLAN links are conditionally independent. We obtain that

$$\begin{aligned}&\mathrm{PoC}_{\mathrm{DOP}}(D_{L,\mathrm{Th}}, D_{W,\mathrm{Th}})\\ &\qquad\simeq P\{D_L \le D_{L,\mathrm{Th}}\} \cdot P\{D_W \le D_{W,\mathrm{Th}}\}\\ &\qquad= \mathrm{ILT}[\exp(sD_{L,\mathrm{Th}})G_{D_L}(s)/s]\\ &\qquad\quad\times \mathrm{ILT}[\exp(sD_{W,\mathrm{Th}})G_{D_W}(s)/s]. \end{aligned} \tag{24}$$

Eq. (24) is a new result which demonstrates how the joint DOPs of LAA and WLAN systems can be mapped to the PoC.

C. Delay Constrained Throughput and the Second PoC

For a given delay threshold $D_{L,\mathrm{Th}}$ of an LAA link, we denote its DCT as $R_L(D_{L,\mathrm{Th}})$, which is defined as the successfully transmitted total payload with transmission delay no more than $D_{L,\mathrm{Th}}$. Similarly, the DCT of a WLAN node is defined as $R_W(D_{W,\mathrm{Th}})$. Define the sum DCTs of n_L LAA links and n_W WLAN links, respectively, as

$$\begin{aligned} R_{L,\mathrm{All}}(D_{L,\mathrm{Th}}) &= n_L R_L(D_{L,\mathrm{Th}}),\\ R_{W,\mathrm{All}}(D_{W,\mathrm{Th}}) &= n_W R_W(D_{W,\mathrm{Th}}). \end{aligned} \tag{25}$$

In general, we define the second PoC based on sum DCTs of all LAA and WLAN links as

$$\begin{aligned}&\mathrm{PoC}_{\mathrm{DCT}}(D_{L,\mathrm{Th}}, D_{W,\mathrm{Th}}, R_{L,\mathrm{Th}}, R_{W,\mathrm{Th}}),\\ &= P\{R_{L,\mathrm{All}}(D_{L,\mathrm{Th}}) > R_{L,\mathrm{Th}}, R_{W,\mathrm{All}}(D_{W,\mathrm{Th}}) > R_{W,\mathrm{Th}}\},\end{aligned} \tag{26}$$

which indicates the joint probability that both LAA and WLAN systems can fulfill their required sum DCTs.

We propose to analytically evaluate $R_L(D_{L,\mathrm{Th}})$ by

$$R_L(D_{L,\mathrm{Th}}) = \sum_{n=1}^{N_{m,L}} nP_{L,\mathrm{EX}}(n, D_{L,\mathrm{Th}})T_{P,L}/\overline{D}_{L,\mathrm{Th}}, \tag{27}$$

where $T_{P,L}$ is the payload duration of an LAA link, $P_{L,\mathrm{EX}}(n, D_{L,\mathrm{Th}})$ is a short-hand form for the probability that the LAA node has successfully transmitted exactly n payloads with a total transmission delay of no more than $D_{L,\mathrm{Th}}$, and $\overline{D}_{L,\mathrm{Th}}$ is a modified delay threshold. Based on (3), we obtain

$$\overline{D}_{L,\mathrm{Th}} = D_{L,\mathrm{Th}} + \int_{D_{L,\mathrm{Th}}}^{\infty} x f_{D_L}(x)dx, \tag{28}$$

where $f_{D_L}(x)$ is the PDF of D_L.

Here, the total transmission delay of n payloads is defined as the duration from the instant that the first payload becomes the HoL in the transmit queue until the instant the transmission of the nth payload is successfully completed (without collision). The $N_{m,L}$ is the maximum number of payloads that can be transmitted before $D_{L,\mathrm{Th}}$. The exact value of $N_{m,L}$ is not needed in computing (27), because if we use the value $\hat{N}_m$ in (27) with $\hat{N}_{m,L} > N_{m,L}$, then $P_{L,\mathrm{EX}}(n, D_{L,\mathrm{Th}}) = 0$, $(n = N_m + 1, \ldots, \hat{N}_m)$. A simple method to determine a realistic $\hat{N}_{m,L}$ is given by

$$\hat{N}_{m,L} = \lfloor D_{L,\mathrm{Th}}/T_{S,L} \rfloor. \tag{29}$$

Mathematically, $P_{L,\mathrm{EX}}(n, D_{L,\mathrm{Th}})$ can be expressed as

$$\begin{aligned} P_{L,\mathrm{EX}}(n, D_{L,\mathrm{Th}}) &= \int_0^{D_{L,\mathrm{Th}}} \cdots \int_0^{D_{L,\mathrm{Th}}} F_L(\Delta t_1) \cdots F_L(\Delta t_n)\\ &\times [1 - F_L(D_{L,\mathrm{Th}} - \sum_{k=1}^{n} \Delta t_k)] d\Delta t_1 \cdots d\Delta t_n / D_{L,\mathrm{Th}}^n\\ &\text{subject to } \sum_{k=1}^{n} \Delta t_k \le D_{L,\mathrm{Th}}, \end{aligned} \tag{30}$$

where $F_L(\Delta t_1), \cdots, F_L(\Delta t_n)$ refer to probabilities of the first n successful transmissions, respectively, and $1 - F_L(D_{L,\mathrm{Th}} - \sum_{k=1}^{n} \Delta t_k)$ refers to probability that the $(n+1)$th transmission is not finished before the threshold $D_{L,\mathrm{Th}}$. However, (30) involves an n-dimensional integral and is very difficult to evaluate numerically.

To bypass this technical difficulty, we develop a novel approach to evaluate (30) accurately. We define $P_{\mathrm{AL}}(n, D_{L,\mathrm{Th}})$ as the probability that the LAA node has successfully transmitted *at least* n payloads with a total transmission delay of no more than $D_{L,\mathrm{Th}}$. Thus, we have $N_{m,L}$ equalities (for $n = 1, \ldots, N_{m,L}$), shown by

$$P_{\mathrm{AL}}(n, D_{L,\mathrm{Th}}) = \sum_{m=n}^{N_{m,L}} P_{L,\mathrm{EX}}(m, D_{L,\mathrm{Th}}). \tag{31}$$

From (31) we obtain

$$P_{L,\mathrm{EX}}(N_{m,L}, D_{L,\mathrm{Th}}) = P_{\mathrm{AL}}(N_{m,L}, D_{L,\mathrm{Th}}),$$

and

$$P_{L,\mathrm{EX}}(n, D_{L,\mathrm{Th}}) = P_{\mathrm{AL}}(n, D_{L,\mathrm{Th}}) - P_{\mathrm{AL}}(n+1, D_{L,\mathrm{Th}}), \tag{32}$$

for $n = 1, \ldots, N_{m,L} - 1$. When considering only one payload, we have

$$P_{\mathrm{AL}}(1, D_{L,\mathrm{Th}}) = \mathrm{ILT}[e^{sD_{L,\mathrm{Th}}} G_{D_L}(s)/s].$$

To analyze $P_{\mathrm{AL}}(n, D_{L,\mathrm{Th}})$ for $n > 1$, we realize that in this case $D_{L,\mathrm{Th}}$ is the duration which allows at least n successful transmissions. Then $D_{L,\mathrm{Th}}$ is the sum of n i.i.d. delay variables, and each of such a variable has the MGF $G_{D_L}(s)$. Thus, $G_{D_{L,\mathrm{Th}}}(s) = G_{D_L}^n(s)$ holds. Consequently, we have (for $n = 1, \ldots, N_{m,L}$)

$$P_{\mathrm{AL}}(n, D_{L,\mathrm{Th}}) = \mathrm{ILT}[e^{sD_{L,\mathrm{Th}}} G_{D_L}^n(s)/s], \tag{33}$$

which can be evaluated efficiently by use of (22).

By substituting (32) and (33) into (27), we obtain the DCT for an LAA link as

$$R_L(D_{L,\text{Th}}) = \sum_{n=1}^{N_{m,L}} P_{\text{AL}}(n, D_{L,\text{Th}}) T_{P,L}/\overline{D}_{L,\text{Th}}$$
$$= \sum_{n=1}^{N_{m,L}} \text{ILT}[e^{sD_{L,\text{Th}}} G_{D_L}^n(s)/s] \frac{T_{P,L}}{\overline{D}_{L,\text{Th}}}. \quad (34)$$

Similarly, for a given delay threshold $D_{W,\text{Th}}$, the DCT for a WLAN link is obtained as

$$R_W(D_{W,\text{Th}}) = \sum_{n=1}^{N_{m,W}} \text{ILT}[e^{sD_{W,\text{Th}}} G_{D_W}^n(s)/s] \frac{T_{P,W}}{\overline{D}_{W,\text{Th}}}, \quad (35)$$

where $N_{m,W} = \lfloor D_{W,\text{Th}}/T_{S,W} \rfloor$, $T_{P,W}$ is payload duration of a WLAN transmission, and

$$\overline{D}_{W,\text{Th}} = D_{W,\text{Th}} + \int_{D_{W,\text{Th}}}^{\infty} x f_{D_W}(x) dx, \quad (36)$$

with $f_{D_W}(x)$ being the PDF of D_W.

To our knowledge, (34) and (35) are new results on the DCT in spectrum sharing LAA and WLAN systems. After the DCTs of LAA and WLAN systems are obtained, we are also interested in their distributions. Here, we consider $R_L(D_{L,\text{Th}})$ and $R_W(D_{W,\text{Th}})$ as random variables, and study their CDFs. Consider an LAA link first. From (27), it is observed $\sum_{n=0}^{N_{m,L}} P_{L,\text{EX}}(n, D_{L,\text{Th}}) = 1$, where $P_{L,\text{EX}}(0, D_{L,\text{Th}})$ corresponds to the probability of zero DCT.

By using $\{P_{L,\text{EX}}(n, D_{L,\text{Th}})\}_{n=0,\ldots,N_{m,L}}$ as the PMF of the DCT $R_L(D_{L,\text{Th}})$, we derive its MGF as

$$G_{R_L(D_{L,\text{Th}})}(s) = \sum_{n=0}^{N_{m,L}} P_{L,\text{EX}}(n, D_{L,\text{Th}}) \exp(snT_{P,L}/\overline{D}_{L,\text{Th}}).$$

Similarly, the MGF of $R_W(D_{W,\text{Th}})$ is obtained as

$$G_{R_W(D_{W,\text{Th}})}(s) = \sum_{n=0}^{N_{m,W}} P_{W,\text{EX}}(n, D_{W,\text{Th}}) \exp\left(\frac{snT_{P,W}}{\overline{D}_{W,\text{Th}}}\right).$$

For the LAA sum DCT with threshold delay $D_{L,\text{Th}}$ and threshold throughput $R_{L,\text{Th}}$, we define the CDF of the sum DCT as $P\{n_L R_L(D_{L,\text{Th}}) < R_{L,\text{Th}}\}$, which can be evaluated as follows:

When $n_1 n_L T_{P,L}/\overline{D}_{L,\text{Th}} \leq R_{L,\text{Th}} \leq (n_1 + 1) n_L T_{P,L}/\overline{D}_{L,\text{Th}}$, aka., $n_1 = \lfloor R_{L,\text{Th}} \overline{D}_{L,\text{Th}}/(n_L T_{P,L}) \rfloor$, based on the PMF, we have

$$P\{R_{L,\text{All}}(D_{L,\text{Th}}) < R_{L,\text{Th}}\} = \sum_{n=0}^{n_1} P_{L,\text{EX}}(n, D_{L,\text{Th}}). \quad (37)$$

Similarly, the CDF of the sum DCT of n_W WLAN links is derived as

$$P\{R_{W,\text{All}}(D_{W,\text{Th}}) < R_{W,\text{Th}}\} = \sum_{n=0}^{n_2} P_{W,\text{EX}}(n, D_{W,\text{Th}}), \quad (38)$$

where $n_2 = \lfloor R_{W,\text{Th}} \overline{D}_{W,\text{Th}}/(n_W T_{P,W}) \rfloor$, and $D_{W,\text{Th}}$ and $R_{W,\text{Th}}$ are the WLAN threshold values of delay and sum throughput, respectively.

To obtain the CDF of DCT of a single LAA link and a WLAN link, respectively, we just need to set $n_1 = \lfloor R_{L,\text{Th}} \overline{D}_{L,\text{Th}}/T_{P,L} \rfloor$, and $n_2 = \lfloor R_{W,\text{Th}} \overline{D}_{W,\text{Th}}/T_{P,W} \rfloor$ in (37) and (38), respectively.

Note that the CDFs in (37) and (38) are equivalent to outage probabilities (or CDFs) of the sum DCTs, because they give the probabilities that the achieved joint DCTs are less than the required DCT threshold values. Based on the CCDF of the sum DCTs, the second PoC defined in (26) can be evaluated as

$$\begin{aligned} &\text{PoC}_{\text{DCT}}(D_{L,\text{Th}}, D_{W,\text{Th}}, R_{L,\text{Th}}, R_{W,\text{Th}}) \\ &\simeq P\{R_{L,\text{All}}(D_{L,\text{Th}}) > R_{L,\text{Th}}\} \cdot P\{R_{W,\text{All}}(D_{W,\text{Th}}) > R_{W,\text{Th}}\} \\ &= [1 - \sum_{n=0}^{n_1} P_{L,\text{EX}}(n, D_{L,\text{Th}})] \cdot [1 - \sum_{n=0}^{n_2} P_{W,\text{EX}}(n, D_{W,\text{Th}})]. \end{aligned} \quad (39)$$

This is a function of delay threshold pairs $(D_{L,\text{Th}}, D_{W,\text{Th}})$, and throughput threshold pairs $(R_{L,\text{Th}}, R_{W,\text{Th}})$.

For comparison purposes, it is important to show the static sum throughput of LAA and WLAN systems (without delay constraints), and check if our derived DCTs converge to the static throughput when the delay thresholds become large. The average successful transmission probabilities for the LAA system and the WLAN system are given by

$$P_{S,L} = n_L \tau_L P_{t,L}, \quad (40)$$
$$P_{S,W} = n_W \tau_W P_{t,W}, \quad (41)$$

where $P_{t,L}$ and $P_{t,W}$ are given by (5) and (6). Using a procedure similar to that given in [11], the static sum throughput of LAA and WLAN systems can be computed as

$$R_{S,L} = P_{S,L} T_{P,L}/T_{\text{ave}}, \quad (42)$$
$$R_{S,W} = P_{S,W} T_{P,W}/T_{\text{ave}}. \quad (43)$$

Here, T_{ave} is the average duration to enable a successful transmission in either LAA or WLAN system, given by

$$\begin{aligned} T_{\text{ave}} &= P_{i,L} P_{i,W} \delta_L + P_{S,L} P_{i,W} T_{S,L} + P_{C,L} P_{i,W} T_{C,L} \\ &+ P_{i,L} P_{S,W} T_{S,W} + P_{i,L} P_{C,W} T_{C,W} \\ &+ (1 - P_{i,L})(1 - P_{i,W}) T_{C,M}. \end{aligned} \quad (44)$$

Numerical comparisons of DCTs and static throughput of LAA and WLAN systems are provided in Section IV.

IV. Numerical Results

In this section, we provide both analytical and simulation results on the delay-related PoC performance metrics of the spectrum sharing LAA-LBT and WLAN links. The parameters used for analysis and simulation are listed in Table II. We assume that the RTS/CTS type access is used for both LAA and WLAN systems. The DCT of a link shown here is the achieved time-efficiency, which is the time proportion of successful payload transmission of that link divided by the total simulation time. We assume $n_L = n_W = 3$. Here we set $n_L > 1$ due to assumption of multiple overlapped small cells. In our Monte Carlo computer simulation, we

TABLE II: LTE-LAA and WLAN Parameters Used for Simulation

LTE-LAA parameters

Parameter	Value
Payload per transmission	1 ms
LBT defer period: T_{Defer} (=T_{DIFS})	34 μs
LBT eCCA period: T_{eCCA} (=δ_W)	9 μs
CW size Z_0	8
Cutoff stage m_L	1

WLAN parameters

Parameter	Value
Payload per transmission	1 ms
Idle slot duration δ_W	9 μs
CW size W_0	16
Cutoff stage m_W	3

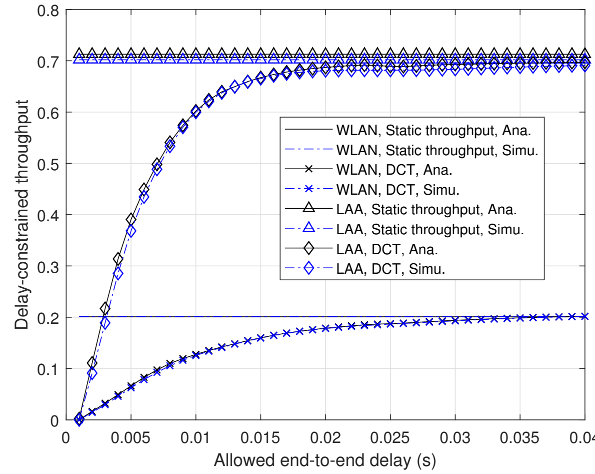

Fig. 3: Normalized sum DCTs (and static throughput) of LTE-LAA and WLAN systems vs. delay threshold.

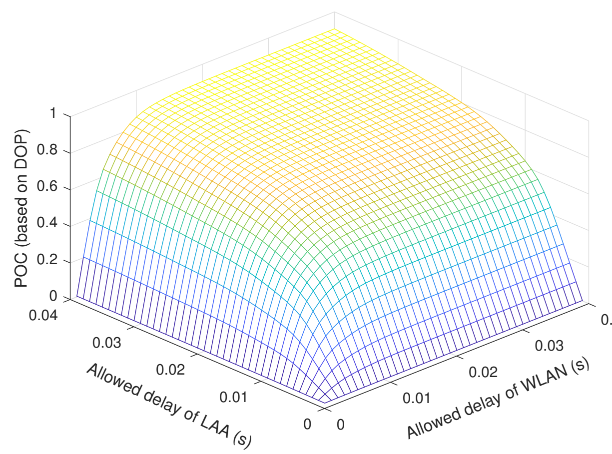

Fig. 4: DOP-based probability of coexistence of LTE-LAA and WLAN systems vs. delay threshold.

track the numbers of local events for each WLAN and LTE node: channel idle, counter freezing (due to channel being busy), successful transmission, and collision. We also track the experienced delays of all the successfully transmitted payloads in LAA and WLAN systems, and compare these with specified delay thresholds to obtain simulated DOP and DCT statistics of each link. The simulation results were obtained by running for 10^6 time slots on each parameter setting.

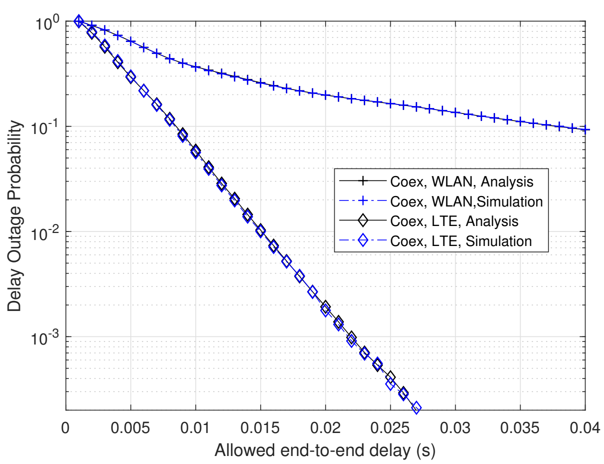

Fig. 2: Delay outage probabilities of LTE-LAA and WLAN systems vs. allowed transmission delays, when $n_L = n_W = 3$.

We provide the DOP and DCT results of LTE-LAA and WLAN systems in Figs. 2 and 3, respectively. The delay threshold vector for the x-axis is generated from $(1 \sim 40)$ ms and has step size of 1 ms. To compute the infinite range integrals in (4), (28), and (36), the integration upper-bound is chosen as 200 ms. This is sufficiently large as demonstrated by the achieved very low DOP. Figs. 2 and Fig. 3 show that as the delay threshold increases, both LAA and WLAN systems have decreased DOPs and increased DCTs, as expected. Furthermore, it is observed that all the analytical and simulation results match well with each other. Since we assume that the LAA system has smaller CW and cutoff stage than the WLAN system, as shown in Table II, Figs. 2 and Fig. 3 demonstrate that the LAA system has much lower DOP and higher DCT than those of the WLAN. In Fig. 3, the static throughput is defined as the throughput of LAA and WLAN systems without delay constraints, which can be computed by using (42) and (43). As the delay threshold increases, the DCTs of both LAA and WLAN systems smoothly converge to their static throughput, respectively. This result illustrates a design tradeoff of achieved DCT vs. delay threshold in comparison with throughput without delay constraint.

In Figs. 4 and 5, we present the PoCs of LAA and WLAN systems based on DOP and DCT criteria, respectively. Fig.

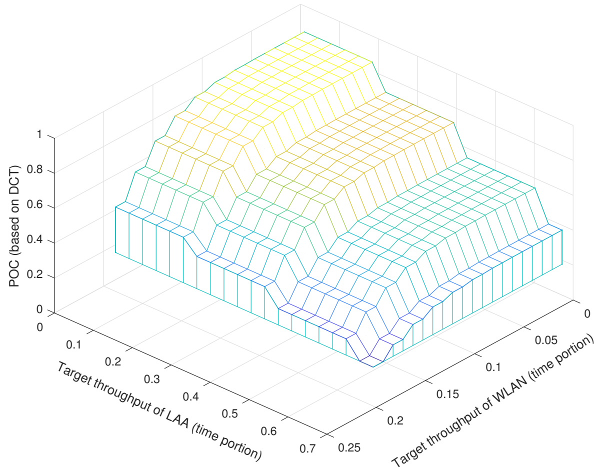

Fig. 5: DCT-based probability of coexistence of LTE-LAA and WLAN systems vs. target throughput when the delay threshold is 40 ms.

4 shows that as the permitted delays for LAA and WLAN system increase, the PoC increases monotonically towards unity. The target throughput of LAA (or WLAN) systems in Fig. 5 ranges from 0 to its maximum throughput in the coexistence scenario. We observe from Fig. 5 that as the target throughput values decrease, the PoC increases, and vice versa. This is a tradeoff between threshold throughput and PoC. Furthermore, to achieve the same (or very close) joint PoC metrics, there exist multiple pairs of target rates for LAA and WLAN systems. Therefore, there is another tradeoff between the two systems based on DCT thresholds and achieved PoC.

V. Conclusion

In this paper, we have defined and evaluated original probability of coexistence metrics to quantify spectrum sharing performance of LAA and WLAN systems based on delay constraints. We have analyzed the delay-related performance metrics, including DOP and DCT which are practical and novel, but not well addressed due to technical difficulties in modeling and analyzing these metrics. We have implemented LTE-LAA and WLAN MAC scheme programming and extensive computer simulations, which have verified the accuracy of our analytical results. Numerical results show that there are a few tradeoffs between delay and throughput requirements, and between achieved performance of LTE-LAA and WLAN systems. These results fill a major technical gap on defining and analyzing meaningful PoCs to quantify success or failure of wireless coexistence performance, which incorporate effects of practical delay constraints. This method may be combined with the application layer and physical layer information to properly set threshold values of delay and throughput, and achieve diverse PoC targets, for LAA and other systems. In future work, we will implement hardware based experiments to validate the performance analysis.

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