Redefining the kilogram and other SI units

Stephan Schlamminger

Chapter 1

Introduction

Measurements of physical properties play a crucial role in our civilization and its evolution through times. At the dawn of human civilization, the need to quantify trade goods and areas of plots of land were essential to ensure fair trades, inheritances, and taxes. Transparent and honest commerce is a necessary underpinning of stable societies. If a product is sold, for example, by the kilogram, the price per kilogram is only meaningful if the buyer and seller agree on what a kilogram is and that its definition is stable. Suppose someone is browsing through a farmers market shopping for tomatoes. Comparing prices makes only sense if the shopper knows that a kilogram is the same from market stand to market stand and that the mass of a kilogram has not changed from the time the market opened to the time it will close. Implicitly knowing that these assumptions are valid, the shopper is comparing prices over space and time and is, hence, able to find the best deal. A rational decision can be reached because of the stability of the unit. Most shoppers do not waste a thought on the stability of the mass unit because experience tells us that the units dont change, or do they? Well, the world is preparing for a significant revision of the international system of units. A vote by the member states of the meter convention is scheduled to take place in November 2018. If the proposed revision of the international system of units is accepted, it will take into effect on May 20th, 2019, precisely twelve dozen years after the meter convention was signed on the same day in 1875. In this short text, the revision of the unit of mass, the kilogram, will be briefly discussed. But, rest assured, the revision will not change the mass of anything by design.

While commerce was undoubtedly the driver behind establishing practical and stable units at the beginning of civilization, Technology and Science put further demands on the units since, at least, the times of the ancient Egyptians. Building the pyramids in ancient Egypt required a tantalizing amount of organizational skill and project management. In addition, proper metrology was essential. Many skilled workers were needed to finish the pyramids. In order to ensure that the pieces made by different people would fit together, a commonly agreed upon length standard and a traceability chain was required. Today, in a global economy, worldwide agreed upon units are more important than ever. A piston manufactured at plant A at location X has to fit in an engine built in plant B at location Y. This requires not only good specifications of the parts but also traceable measurements which need a standard unit of length.

The international system of units abbreviated SI from the French expression Systme international (d'units) traces its root back to the meter convention held in 1875. Some of the units commonly associated with the SI have been used about a hundred years prior to that in the French Republic. The metric system was established following the desire to create a system of units for all times, for all people. The metric system has been very successful over the years, but it has not been without change. Units were added, for example in 1948 the ampere; other units were changed. For example, the definition of the meter changed three times from 1875 to today. While the wording of the definitions could change, great care was and is taken, by the scientists working in the committees responsible for the units that the variations in the units are negligible. The goal, whenever the definition of a unit is revised is to keep the size of the unit the same within the uncertainties that occur when the wording is translated into practice. In metrology, the term realization of the unit describes the process of implementing the theoretical definition of a unit to measure a quantity in this unit.

The revision of the SI planned to take effect in 2019, fundamentally changes the definition of four units, the kilogram, the ampere, the kelvin, the mole. However, the underlying theme behind the revision is more substantial than merely changing the definition of four units. It completes an already started morphing of the system of units from one that rests on base units to one that is built of fundamental constants of nature. The following few pages will explain this new architecture while keeping a focus on the unit of mass, the kilogram.

Chapter 2

Background

The general conference of weights and measures (CGPM, acronym based on the original French term) will meet from 13th to 16th November 2018 in Versailles, France. At this meeting, a vote on a proposal to revise the SI is expected. In case the vote is favorable, the new SI will go into effect on May 20th, 2019. In the following paragraphs, units, measurements, and the SI are discussed. To avoid, confusion, the term pre-2019 SI will be used to the SI valid up to May 20th 2019 and the term new SI for the system of units (hopefully) in place after May 20th, 2019.

To start, lets have a close look at the measurement process. Measuring a quantity is a comparison of this quantity to a multiple of a known unit. The unit acts as a standard and the person performing the measurement tries to figure out, how often the standard fits into the measurand. The way a result is reported is indicative of this process. For example, the length of a table is reported as

$$l = 1.2\,\mathrm{m},$$

where the left part of the equation represents the quantity that was measured, here the Symbol l as an abbreviation for the length of the table. The result of the measurement is reported to the right of the equal sign. It is a product of what is known to be the numerical value, in this case, 1.2 and the unit, here the meter, abbreviated by m. The product on the right indicates that 1.2 meters fit inside the length of the table. As a side note, each measurement comes with uncertainty, which would be added to the right side of the equation as, for example, $\pm 0.01\,\mathrm{m}$. For now, we are not concerned with this measurement uncertainty and focus on the unit. The meter reflects a choice of unit. But, there exist other units that can be used to measure the table length, for example, the length of the table can be measured in feet.

$$l = 1.2\,\mathrm{m} = 4\,\mathrm{ft}.$$

The unit foot fits four times into the length of the table, the unit itself must also be smaller than the unit meter, which only fits 1.2 times into the table.

We see that one quantity, for example, length, can be measured using different units. In order to avoid confusion, one common unit for each quantity should be agreed upon. But, then there are many quantities, that must be measured, not just lengths. This seems to imply that there has to be agreement on an infinite number of units. Here, is where the term system in the international system of units deserves attention. It turns out that most physical quantities that can be measured can be written as a combination of seven units. In reality, one would actually need fewer units, but historically seven units were used in the SI, and this number will be retained after the revision.

A simple example shows, how the units can easily be combined. One could try to measure the area of the table surface of the table discussed above. The area is the product of two lengths, so, if our table is a perfect rectangle, the area can be obtained by multiplying two separate measurements,

$$A = l \times w = 1.2\,\mathrm{m} \times 0.8\,\mathrm{m} = 0.96\,\mathrm{m}^2.$$

The area of the table, in this example, is a little less than a square meter. Alternatively, one could imagine a measurement, where one would try to see how often a square meter fits in the table. One could, for example, use little paper squares of $0.01\,\mathrm{m}^2$ each, and one could count how many can be placed on the table without overlaps or gaps (96 in this case). The critical point of this example is that a new unit that of the area was created by combining two existing units. Here, the two existing units were both the same, the meter. But, it is conceivable that different units can be combined as well.

Figure 2.1: The official logo of the SI as provided by the International Bureau of Weights and Measures (BIPM). The acronym SI, from the French expression of the International Systems (of Units), is surrounded by the seven base units.

The international system of units sets forth seven base units that can be combined to produce a large number of new units, the so called derived units. By convention, some of the derived units have been given special names. Examples are, the Newton, 1 N = 1 kg m s^{-2}, the Watt, 1 W = 1 kg m^2 s^{-3}, and the Volt, 1 V=1 kg $m^2A^{-1}s^{-3}$. Figure 2.1 shows the seven base units in the pre-2019 SI.

In order to understand the spirit of the SI, it is instructive to examine the unit of length, the meter, and its historical evolution. Before the French revolution, the measurement of length in Europe varied from jurisdiction to jurisdiction. The length unit was often posted on administrative buildings (city halls), see Figure 2.2, and, at times, derived from body parts of the local nobility. Obviously, having different length units for each county, hampered trade. Imagine being a traveling salesman for cloth and having to recalculate the price of your product per local unit length for each city. French scientists were discussing a new system of measurements with the desire to design a system for all times and for all people. The idea was to develop a system of units which do not require change either as a function of time or as a function of place. True to this spirit, these thinkers wanted to derive the units in a democratic way, in contrast, to measure the length of a foot of a noble person. One idea that was implemented in the end was to use a 10 millionth part of the quarter meridian. A meridian is a circle on the earth that passes through both poles. An infinite number of such meridians exist, but the meridian running through Paris was chosen. This new definition of length is based on the planet Earth, something all humans share and can identify with.

Since it is entirely impractical to measure a quarter circle spanning the Earth every time a length measurement is needed, an artifact named the meter of the Archive was created using the results from a survey of a part of the meridian carried out by Pierre Méchain (1744 - 1804) and Jean Delambre (1749 - 1822) between 1791 and 1798. Mechain and Delambre surveyed the distance between Barcelona to Dunkirk (both cities at sea level) along the meridian and inferred from this distance the meter. A few years after the convention of the meter in 1875, a new artifact, this time called the international prototype meter, was made using the meter of the Archives as a template.

While the length of the meter of the Archives was defined by the distance between its two ends, the international prototype of the meter was a bit longer than a meter and carried two engraved lines, a meter apart. Transferring the distance between these lines became a leading systematic effect realizing the meter, and hence in 1960, the definition was changed to be a certain multiple of the wavelength of light emitted by a krypton atom, which was beforehand measured using the international prototype of the meter.

Once the meter was established, people were using it for measurements in commerce, technology, and science. One area of research, where the exact definition of the meter was highly relevant, was the measurement of the speed of light in vacuum. Scientists were interested in a precise value for the speed of light ever since Rømer (1644 - 1710) inferred it from his observations of Jupiter's moon Io in 1676. The measurement of the speed of light became more and more accurate over time until the realization of the unit of length, the meter was the limiting factor in measuring the speed of light. At this point, the idea arose to change the definition of the meter by assigning a fixed value to the speed of light. This was adopted in 1983 for the definition of the meter.

It is crucial to understand this idea. By using the definition of the meter and that of the second, the speed of light could be measured in m/s. After 1983, the speed of light was a fixed quantity,

$$c = 299\,792\,458\,\mathrm{m/s}.$$

This is very similar to the equation of the table length above, with one difference. The speed of light has been assigned this numerical value; this means the only free parameter on the right is the meter since the second is fixed using a different definition. So, this defining equation can be used to define the meter. In words, one could say, the meter is the distance that light travels in a vacuum in 1/299 792 458 of a second.

This simple definition of the meter via the speed of light has several advantages:

- It is available to everyone at anytime. While a physical artifact can only be at one place at one time and access most likely must be controlled to avoid vandalism and theft, the speed of light is a property of our universe. By designing an experiment, one can obtain its value. In principle, there are no limits when and where this experiment is performed.
- A fundamental constant of nature does not change its value after it has been used or accessed. The fear of using artifacts is that with each use the artifact may change. This fear is detrimental to the artifacts objective, which is to be used as much as possible so that every measurement is as close to the artifact as possible.
- In the end, units that rely on artifacts are only as stable as the artifacts themselves. So, the stability of the artifact is the ultimate limit on how well a measurement can be linked to the definition of the unit. A unit based on a constant does not have this limitation. As the name implies, the constant does not vary with time and the uncertainty at which the unit can be transferred depending on the experiment that is used to access the constant. Of course, these experiments can be improved over time. So the realization of the unit may be carried out with greater precision over time.

The goal of the new SI is to bring the benefits of the fundamental constant to four more base units. The units that are in used for the quantities mass, current, thermodynamic temperature, and amount of substance.

Figure 2.2: Before the metric system, local weights and measures were customs. In many cities, the measures were displayed. The photo shows three length units displayed at the old city hall of Regensburg, Germany. The units given by the iron bars from left to right are the fathom ($\approx 1.75\,\mathrm{m}$), the foot ($\approx 31\,\mathrm{cm}$), and the cubit ($\approx 83\,\mathrm{cm}$). While similar names were in use, the exact lengths varied from jurisdiction to jurisdiction.

Chapter 3

Current Directions: Fundamental constants instead of units

The meter was the first SI unit that transitioned from an artifact based definition to a definition that is based on a fundamental constant, the speed of light. In this chapter, we discuss the transition for the four more units: the kilogram, the amount of substance, the electric current, and the thermodynamic temperature.

Mass is the property of an object to resist acceleration. The more mass an object has, the more force is required to accelerate the object with the same rate. As Albert Einstein pointed out, acceleration is linked to gravitation via general relativity. Hence, mass can also be seen as the "charge" in the gravitational interaction. This means the gravitational forces between two objects is proportional to the product of their masses. The unit of mass is the kilogram. Historically the kilogram is the mass of an amount of water whose volume is identical to a cube with 0.1 m side length at 4 °C. Similar to the meter, an artifact was made, named the international prototype of the kilogram (IPK). This prototype is stored in a vault at the premises of the BIPM located in the suburbs of Paris. Since its inception in 1899, it has been used for measurements four times. Between these four times, other artifact masses act as a flywheel to keep the international mass scale on track. Each country that has joined the meter convention has a national prototype, see Figure 3.1, that is used to disseminate mass inside the country. The unit of mass is most important for trade since many products are sold by mass. Note, the subtle difference between weight and mass. Weight is the force, measured in units Newton that acts on the mass in the gravitational field of the Earth. In colloquial language, mass and weight are not always correctly distinguished.

The amount of substance measured in the unit mole is a different concept than mass. The assumption of the definition of the mole is that a material is composed of identical identities, atoms or molecules. A mole of stuff is a certain number (precisely $6.022\,140\,76 \times 10^{23}$) of such identities. This number is named the Avogadro constant, after Amedeo Avogadro (1776 - 1856). In the pre-2019 SI the number is not given, but instead, the mole is defined as the number of Carbon-12 atoms in 12 g of ^{12}C (in the ground state). The mass of a mole of a substance can be calculated by the number of entities in a mole times the mass of each one. So, the pre-2019 definition of the mole uses this fact in reverse.

The unit of electric current is the ampere, named after André Marie Ampère (1775 - 1836). The definition of the ampere in the pre-2019 SI takes advantage of the mechanical force that can be generated by currents. The definition of the ampere is very abstract. It says that a current of one ampere flowing in two infinitely thin wires, one meter apart will generate a force of 2×10^{-7} N per meter of length. Clearly, it is impossible to realize the definition in the geometry described above. However, other geometries can be accomplished, for example, nested coils. Consequently, for these geometries, the force per meter length differs from the value of 2×10^{-7} N/m, but its value can be calculated if the exact geometry is precisely known. In the past, the realization of the unit ampere was carried out with devices known as ampere balances. Here, the weight of a known mass was balanced against the force produced by nested coils.

Nevertheless, the experiments to realize the unit of current were and still are tedious and difficult. Advances in quantum electrical standards, see below, provided more convenient units. However, these units are not SI units. More details will be given below.

The final unit discussed here is that of thermodynamic temperature, the kelvin. The kelvin is defined such that the triple point of water is at 273.16 K. The triple point of water is the point, were water exists in all three phases, solid, liquid, and gaseous. The size of the unit of kelvin is basically the difference between the triple point of water and the absolute zero of the temperature scale divided into 273.16 parts.

3.1 Quantum standards, or how the electrical units in the pre-2019 SI became irrelevant

The only electrical unit in the SI is the ampere, which has, as described above, a cumbersome definition. There are of course many other electrical quantities, most prominent the potential (difference) measured in volts and the resistance measured in ohm. How are they defined? The electrical potential is potential energy divided by charge. The usual way in physics courses to introduce the potential is by calculating the work that is required to move a positive test charge near the surface of a charged sphere to infinity. The potential is then given by dividing the work by the amount of charge on the test charge. Work and potential energy are both measured in units joule, the product of meter and newton, which is a product of kg times m/s^2. So, potential energy can be described in the SI using the units of mass, length, and time. The unit of charge can be derived from the unit of current and time because the amount of charge that flows in a capacitor is given by the product of current and time. Hence, the volt can be combined using other units in the SI. Thus, the electrical resistance is also in the SI, since it is the quotient of voltage and current.

In 1962 and 1980, electrical metrology made two quantum leaps. In 1960, Brian Josephson (1940 -), predicted an effect that will revolutionize voltage metrology. He was thinking about a tunneling junction in a superconducting material. Today, we refer to a Josephson junction as a thin barrier made from an insulator or a normal conducting metal between two superconductors. If a current is driven through this barrier, which is possible due to the tunnel effect, and a potential at microwave frequencies is applied across this barrier, a DC potential occurs across this barrier. The Josephson effect can be used to generate a precise, predictable voltage. For a single barrier, the voltage is given by

$$V = ih/2ef,$$

where i is an integer, h is the Planck constant and e is the elementary charge and f is the frequency of microwave radiation applied to the barrier. The quotient $K_J = 2e/h$ is named Josephson constant. The Josephson effect made it possible, for the first time, to generate precise voltages that only depend on fundamental constants.

The second discovery was made by Klaus von Klitzing (1943 -) in 1980. He measured the transverse resistance of a GaAs heterostructure at low temperature at a large magnetic field and found the resistance to be quantized in steps of

$$R = \frac{1}{i}\frac{h}{e^2}. \tag{3.1}$$

The quotient $R_K = h/e^2$ is named the von Klitzing constant.

The effect described above was termed the quantum Hall effect. In contrast, the classical Hall effect describes the potential that develops across a sample that is immersed in a magnetic field with a magnetic flux density B_Z and transversed by an electrical current I_x. The potential difference perpendicular to the current and the magnetic field and is given by

$$V_h = \frac{I_x B_z}{nte},$$

where t is the thickness of the sample, i.e., the dimension of the sample parallel to the magnetic field and n is the charge carrier density. The quantum Hall effect occurs in the limit of the sample thickness converging to zero. In this case, the sample is two dimensional, i.e., the electron motion is confined to a plane. As a consequence, the charge carrier density becomes quantized. This means the charge carrier density is no longer a continuous value that depends on the material properties of the sample. Instead, the charge carrier density can only assume discrete values, which are an integer multiple, i, of the elementary charge, e, times the magnetic flux density and divided by the samples thickness, $n = ieB_z/t$. Substituting the equation for the two dimensional charge carrier for n into the equation for the classical hall effect, leads to cancellation of the samples thickness and the magnetic flux density. Dividing the above equation by I_x on both sides, yields equation (3.1). The discovery of the quantum Hall effect had a huge impact on resistance metrology. Suddenly, there was a fundamental method to generate a quotient of voltage to current, a resistance, with a known value only determined by an integer and two fundamental constants, h and e.

It seems that the Josephson effect and the quantum Hall effect allow the realization of voltage and resistance without having to invoke mechanical experiments. Unfortunately, this is not true. If one were to write the unit of the Planck constant, J s in base units, one finds that it is $\mathrm{kg\,m^2\,s^{-1}}$. The base unit kg is in the Planck constant, and hence one always needs a mechanical setup that can link to the definition of the kilogram to obtain the Planck constant that is required to realize voltage and resistance. The elementary charge, on the other hand, can be determined without mechanical devices. But, once the values of the Planck constant and the elementary charge are obtained, both effects can be used to realize the two units, the volt, and the ohm.

A major advantage of realizing volt and ohm using these quantum effects is that the results are independent of material properties. Hence, these units can be reproduced throughout time identically. Before the quantum effects were known, the volt and the ohm were derived from wire wound resistors and Weston cells (a wet-chemical cell using nasty chemicals such as mercury and sulfuric acid), respectively. These artifacts were very finicky. Small temperature variations, mechanical shock, or electrical current through them could affect their values. These artifacts were linked to the SI using ampere balances. Between such comparisons to the SI, researchers relied on intercomparisons and on ensembles of these artifacts, hoping not all of them would change in the same direction and a changing artifact could be easily spotted and eliminated from the ensemble average.

The reliance on artifacts and their proper handling could be eliminated using the quantum effects. However, there was still a small problem: Scientists continued to measure h and e . The results of these experiments were (and still are) combined by the Task Group on Fundamental Constants (TGFC) of the committee on data for science and technology (CODATA) into recommended values that are published every four years. The process of calculating the recommended values is known as an adjustment. Although this term seems to imply a fine-tuning of published results to obtain the right constants, the truth could not be further from this picture. The values of the fundamental constant are calculated using a mathematical process named least-squares adjustment which finds the most probable value of the constants of nature given the experimental results that were available. Hence, if new values of the Planck constant and the elementary charge became available, the volt and ohm needed to be updated. So, while the underlying physics remains stable, the worlds knowledge of h and e changes as a function of time and requires recalibration of secondary voltage and resistance standards. Of course, the relative changes in h and e over the years were many orders of magnitude smaller than the drifts of the artifacts. Yet, the fact that the Planck constant and the elementary charge had to be changed every four years was a fundamental problem for electrical metrology. A solution was to use fixed values for h and e for the sole purpose of electrical measurements.

In 1988, the international committee for weights and measures (CIPM) agreed to fix the values of the Josephson constant and the von Klitzing constant, which came into effect on the 1st of January 1990. These values were named conventional values of the Josephson and von Klitzing constants to distinguish them from the values of the SI Josephson and von Klitzing constants, which are given by $2e/h$ and h/e^2, using the latest recommendations of the Planck value and the elementary charge available from CODATA. Since 1990 (until hopefully May 2019), most electrical measurements were not carried out with SI units. Instead, conventional units, sometimes abbreviated with a subscript $_{90}$ were used, e.g., V_{90}. For most practical purposes, the conventional units were close enough to the SI units and a distinction between, for example, V and V_{90} was not necessary. Nevertheless, a small difference remained. According to the 2014 recommendation of fundamental constants by CODATA, the conventional volt is relatively larger by 98.3×10^{-9} than the SI volt. While a small difference between the conventional units and the SI units is not a practical problem, it is a philosophical problem. This difference can be compared to a hairline crack in a building foundation. Ideally, the foundation of science, metrology, is sound and coherent down to the smallest differences.

In summary, two main arguments can be made for a revision of the SI in 2019: First, changing the definition of the four units, the kilogram, the mole, the ampere, and the kelvin to a definition based on fundamental constants, will get rid of the artifact based definition of the kilogram and the cumbersome definition of the ampere based on two thin, long wires. Second, this revision allows to bring the electrical units back into the SI, and the conventional units can be abrogated. Taking these steps would end the schism between conventional and SI units and establish a coherent units system across electrical and mechanical quantities.

Figure 3.1: While the international prototype of the kilogram remained at the BIPM, several copies were made and distributed by raffle to the signatory states of the meter convention. These were called the National prototype kilograms and were uses inside the respective nation to establish the mass scales. The national prototyped were transported to BIPM on several occasions and were calibrated against the international prototype or working copies at the BIPM. The prototype shown in the picture is K15, designated to Bavaria.

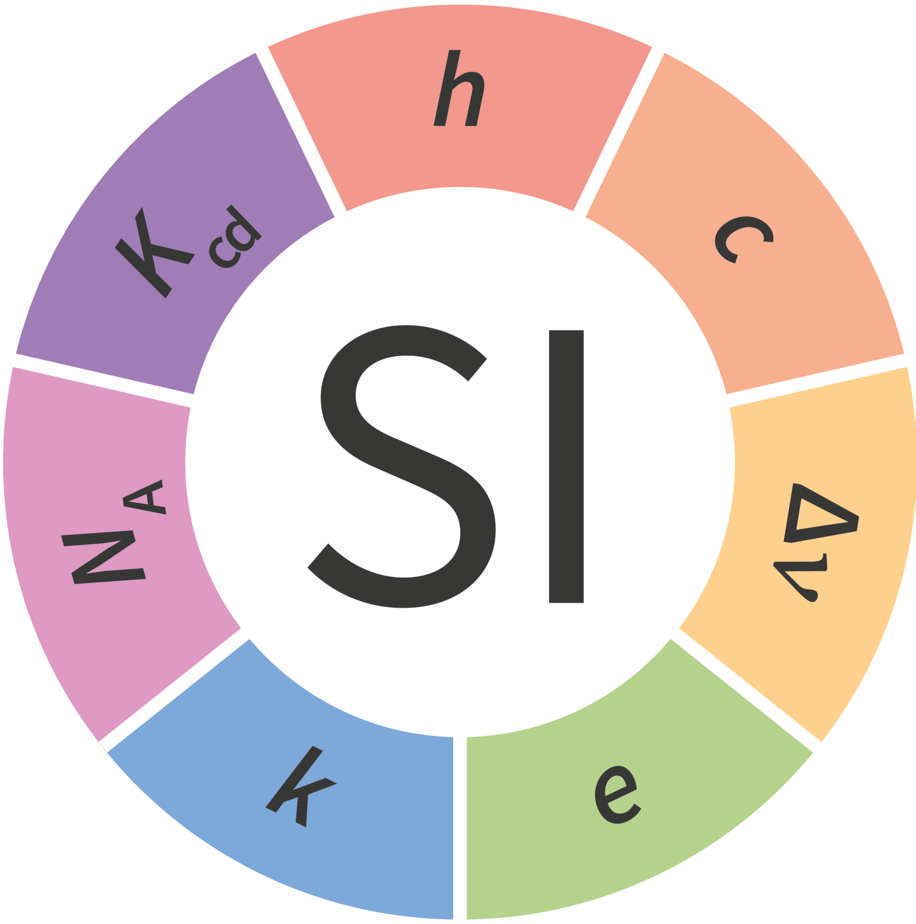

Figure 3.2: Another version of the SI logo showing the seven defining constant of the revised SI. Clockwise from the top position the constants are: the Planck constant, the speed of light, the hyperfine splitting of ^{133}Cs, the elementary charge, the Boltzmann and Avogadro constants, and the luminous efficacy.

Chapter 4

Current Directions: The Kibble balance

In the new SI, there are two principal routes to realize the kilogram of macroscopic masses, the Kibble balance, and the X-ray crystal density method. Here, the Kibble balance is explained in more detail.

The Kibble balance was first proposed in 1974 by Dr. Bryan Kibble (1938 2017), a metrologist at the National Physical Laboratory (NPL) of the United Kingdom. In 1974, the quantum Hall effect was not discovered yet. Hence, the Kibble balance was used for a different purpose, realizing the unit of current, the ampere.

At the heart of the Kibble balance is an electromagnetic device consisting of a coil and a magnet, similar to a motor. The symmetry in the equations describing electromagnetism, Maxwell's equation, allow that each DC motor can be used as a generator. To use the device as a motor, current is injected into the coil and a force is generated. To use it as a generator, the coil is moved (by an external force) and a voltage is generated across the coil. In the Kibble balance the device is used in these two modes, named force mode for the former and velocity mode for the latter. Dr. Kibble realized that combining the modes allows one to cancel a quantity that is really difficult to measure. This will be explained in detail in the following paragraphs.

Figure 4.1 shows a simplified magnet system of a Kibble balance. A circular coil is placed inside a gap of a permanent magnet system. This geometry is similar to that of a loudspeaker. An external current source (not shown in the figure) will force electric current through the wire. Hence, the coil experiences a macroscopic force termed Lorentz force. The design of the magnet system is such that at every point on the coil the current is perpendicular to the magnetic field. Since the Lorentz force is perpendicular to the magnetic flux and the current, it is pointing up or down depending on the direction of the current flow. The Lorentz force is given by a simple equation,

$$F_z = IBl,$$

where I is the current in the coil, B the magnetic flux density (a measure of the strength of the magnetic field), and l is the length of the wire in the magnetic field. This force can now be balanced against the weight (mass times local acceleration, mg) of a mass using a beam balance. The coil is connected to a very sensitive beam balance, see Figure 4.2. On the other side of the beam balance is a tare weight that counteracts the weight of the coil and other hardware. Two measurements are now performed. First, no mass is on the mass pan and the current in the coil is adjusted by a controller such that the balance is servoed to a nominal zero position. Now, the mass is added to the mass pan. The controller will react to this change of force and a new current will be forced through the balance. The difference

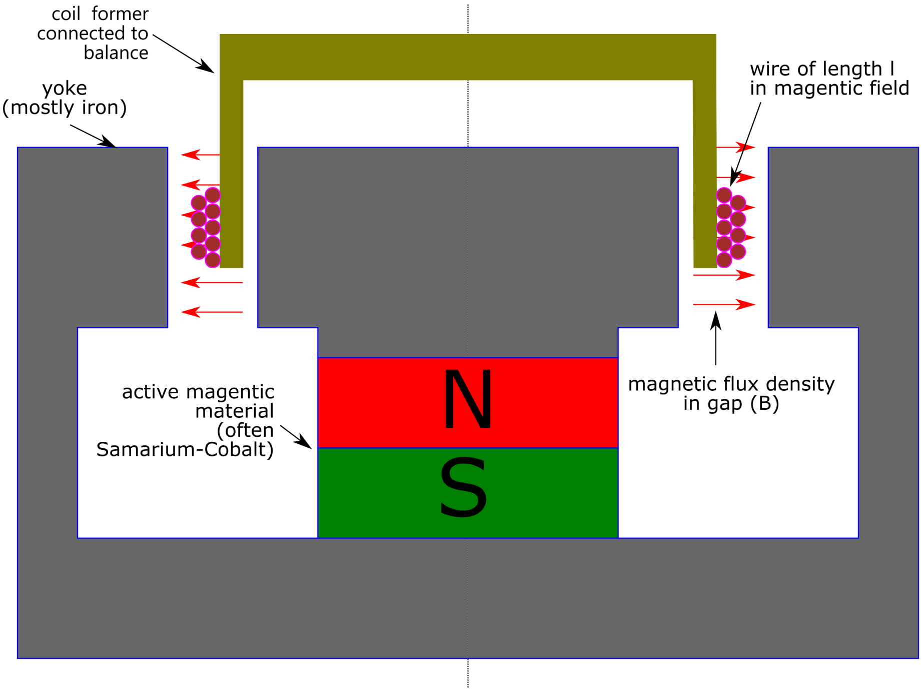

Figure 4.1: A schematic drawing of a possible magnet system for a Kibble balance. The magnet exhibits a cylindrical symmetry around the dotted vertical line. Typical magnetic flux densities in the gap are ranging from 0.4 T to 0.9 T.

of the two currents times Bl corresponds to the weight of the mass, mg.

$$F_{z1} - F_{z2} = mg = (I_1 - I_2)Bl = \Delta IBl \tag{4.1}$$

The advantage of this measurement scheme is that it is independent of the lever arm of the balance because the electrostatic force and the weight will be connected to the balance beam at the same distance from the central pivot. Furthermore, the tare weight can be adjusted such that the two currents are equal with opposite signs. This will remove several nonlinearities in the system, for example, those arising in the voltmeter and the magnet system.

In force mode, the mass of an object can be related to the current in the coil, but the proportionality factor is given by Bl, which is very difficult to determine precisely. Dr. Kibble realized that the quotient of force to current is the same as the quotient of voltage to velocity in velocity mode. Here the coil is swept vertically through the magnetic field, while the induced voltage and the coils speed is measured. Assuming perfectly vertical motion and horizontal field, the induced voltage is,

$$V = Blv.$$

In high precision experiments, both quantities can be measured with relative uncertainties of 10^{-8}.

The expression Bl, that is hard to measure, drops out in the quotient of equation (4.1) to equation (4). After a crosswise multiplication, the so-called watt equation is obtained,

$$\frac{mg}{V} = \frac{\Delta IBl}{vBl} = \frac{\Delta I}{v} \implies mgv = V\Delta I$$

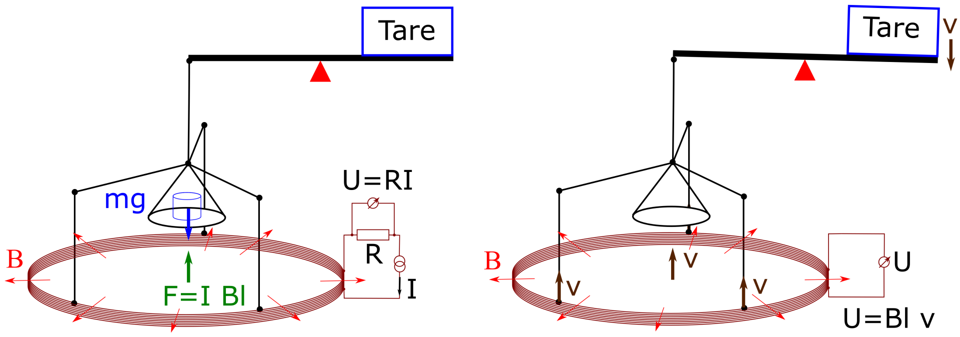

Figure 4.2: The two modes of the Kibble balance experiment. The force mode is shown on the left, the velocity mode on the right. The motor that provides the motion in the velocity mode is not shown.

The watt equation relates mechanical power Fv with $F = mg$ to electrical power VI. The name of the equation stems from the fact, that power, electrical, as well as, mechanical, is measured in the unit watt. The Kibble balance is the tool to make this comparison, see Figure 4.3 for a picture of the NIST Kibble balance. The exact quantity that is determined by the Kibble balance experiment depends on when the measurement takes place. Before 1980 the Kibble balance was used to realize the unit of electrical current, the ampere. Between 1980 and 2019, the Kibble balance was used to determine the Planck constant, see below. After 2019 in the revised SI, the Kibble balance will be used to realize the unit of mass, the kilogram.

So far, the ability of the Kibble balance to make a quantifiable connection between mechanical work and electrical work has been described, but the interrelationship between electrical work and the Planck constant still needs to be established. This link is made by taking advantage of the breakthroughs in quantum electrical metrology described in the previous chapter, the Josephson effect, and the quantum Hall effect.

As is described above, the Josephson effect can be used to measure electrical potential differences in volts. The potential difference is given by

$$V = i_v \frac{h}{2e} f_v.$$

The subscript $_v$ denotes that this is the voltage measurement in the velocity mode. In force mode, the current through the coil needs to be precisely measured. One way to accomplish this measurement is by connecting a resistor in series to the coil. The current passing through the coil is running through the resistor and produces a potential difference (voltage) between the ends of the resistor. Measuring the voltage drop across the resistor is a known problem: Analogous to the measurement of the induced voltage explained above, the Josephson effect can be used. In addition to the voltage measurement, the resistors value needs to be known with high precision. This can be accomplished by comparing the resistor to a quantum Hall resistor using state of the art resistance metrology. This calibration will produce a unit-less ratio r of the measurement resistor and the quantum Hall resistor, hence

$$R = r \frac{h}{e^2}.$$

Combining the voltage measurement and the current measurement yields,

$$I_1 = V_1/R = \frac{i_1}{r}\frac{f_1}{2}e.$$

Here, the subscript one denotes that this is one of two current measurements that have to be accomplished, one for mass on, the second for mass off. It is very gratifying to take a closer look at the equation above. Without looking at the technical details how the measurement is accomplished, the equation basically says that the current is given by a number times the frequency at which the electrons pass by. This clearly indicates that the unit of electrical current has to be electrical charge per second.

Combining the equations for the two currents and the induced voltage yields an expression of electrical power

$$V\Delta I = i_v \frac{h}{4} f_v \left(\frac{i_1}{r} f_1 - \frac{i_2}{r} f_2 \right).$$

If the microwave frequency is chosen to be identical ($f_1 = f_2 = f$), the equation can be simplified to

$$mgv = V\Delta I = \frac{r^*}{4} h f_v f, (16)$$

where the known rational number r^* combines all the known numbers into one. Solving this equation for h and m yields,

$$h = \frac{4}{r^*}\frac{mgv}{f_v f} \quad \text{and} \quad m = \frac{r^*}{4}\frac{h f_v f}{gv},$$

respectively.

Before 2019, the Kibble balance was used to measure the Planck constant (left equation). That required a known mass (derived from the international prototype of the kilogram), the measurement of the local acceleration (g), the velocity of the coil during the sweep and the two frequencies that are used with the Josephson effect. After 2019, the Kibble balance can be used to measure mass, since in the revises SI the Planck constant is a fixed value.

The practical measurement of a mass with the Kibble balance works as follows: First, the coil is swept through the magnetic field, while its velocity and the induced voltage is measured simultaneously. Proper synchronization is essential to cancel external noise and jitter. Typically the Bl factor changes as a function of the coil position, but only the value at the weighing position is important for further processing. The Bl at the weighing position is typically inferred by fitting a curve to measured voltages along the whole path of the sweep, which can range depending on the construction details of the balance from a few millimeters to a several centimeters. This coil sweep is repeated several times to average down statistical fluctuations in the measurement. Then the measurement for the force mode can start. Two measurements are typically carried out in an interleaved pattern. In one measurement the mass pan remains empty while the current is determined to maintain the balance in equilibrium. Then the mass is placed on the mass pan and a second current measurement is performed. This cycle is repeated several times to average statistical fluctuations out. After the force mode is completed another velocity set is measured. Typically the magnetic field is changing slowly over time due to changing temperature and a finite temperature coefficient of the magnet. This requires interleaving the force and velocity modes to reject the effects of the drift on the measurement result.

The measurement procedure above could have been carried out with the original Kibble balance and many Kibble balance throughout the world use this scheme. However, other measurement procedures were proposed and are used. For example, the Kibble balance built at the BIPM can simultaneously measure in force and velocity mode. This is accomplished by using a coil wound with a bifilar wire, which results in basically having two coils that occupy the same space and have therefore nearly identical Bl.

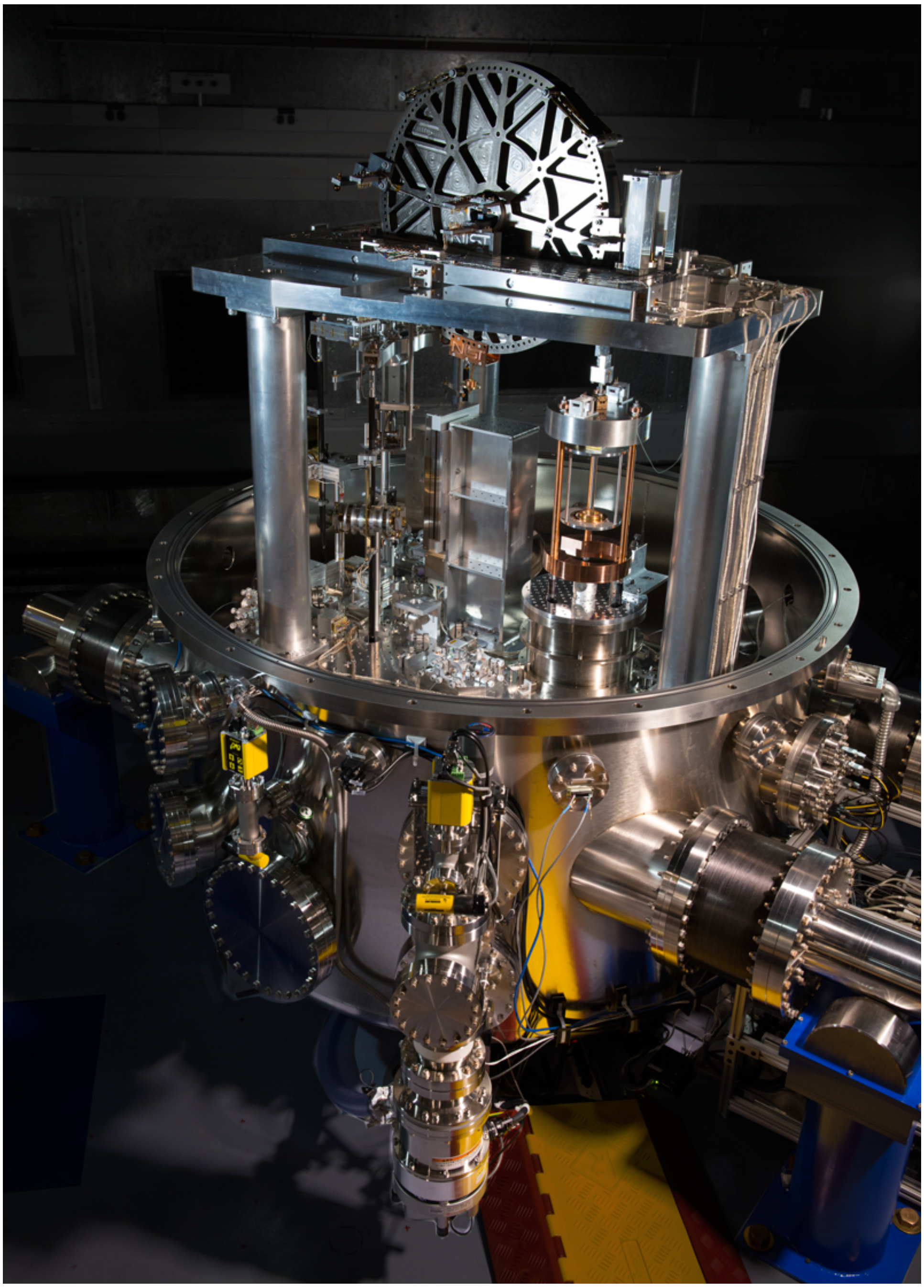

Figure 4.3: A picture of the Kibble balance at the National Institute of Standards and Technology in the USA. The signature component of the NIST Kibble balance is the wheel. It is used as the moving part of the balance instead of a beam. The advantage is that the wheel provides a perfectly vertical motion of the coil. Photo by Jennifer Lauren Lee/NIST.

Chapter 5

Current Directions: The X-ray Crystal Density (XRCD) Method

A second path to realizing the kilogram from the Planck constant is the X-ray crystal density (XRCD) method. This approach has been meticulously refined by the International Avogadro Coordination (IAC), a collaboration of several national metrology institutes and the BIPM (International Bureau of Weights and Measures). The IAC includes the national metrology institutes from Germany, Italy, Japan, China, Canada, and the USA.

As the name implies, before 2019, the IAC was concerned with a precision measurement of the Avogadro constant, N_A. Similar to the Kibble balance the value obtained by this method will change from the measurement of a fundamental constant before 20th May 2019 to the measurement of mass after that day. How does the Avogadro constant link to mass? The numerical value of the Avogadro constant gives the number of entities in a mole. So, it can be used to define the amount of substance, a quantity different from mass. Surprisingly, an equation in atomic physics can be used to obtain a very accurate relationship between the Avogadro constant and the Planck constant.

The equation describes the energy levels in the hydrogen atom. More precisely, if the hydrogen atom transitions from an energy level given by the quantum number n_1 to an energy level with the quantum number n_2 a photon with the wavelength λ is emitted. The reciprocal of the wavelength is given by

$$1/\lambda = R_\infty \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$$

where R_∞ is the Rydberg constant. Note, in reality, there are several corrections to this simple formula, but for an infinitely massive nucleus, indicated by the subscript $_\infty$, it is valid to first order. The measurement of the wavelength of light can be made with a very small uncertainty. Hence it is not surprising that the Rydberg constant is very well known. In 2014, its value was known with a relative uncertainty of 5.9×10^{-12}. On the other hand, the theory of the hydrogen atom with an infinitely heavy nucleus is well understood and the Rydberg constant can be expressed using four other fundamental constants of nature: The Planck constant, the speed of light, the mass of the electron m_e, and the fine structure constant α. The fine structure constant is a unit less constant that determines the strength of the electromagnetic interaction. The Rydberg constant can be written as

$$R_\infty = \frac{\alpha^2 m_e c}{2h}.$$

To obtain a relationship of N_A and h, the numerator and denominator on the right side of the equation are multiplied by the molar mass of the proton $M_p = N_A m_p$. However, the expression M_p is used in the numerator and $N_A m_p$, in the denominator. This yields

$$R_\infty = \frac{\alpha^2 M_p c}{2hN_A m_p/m_e} \implies hN_A = \frac{\alpha^2 M_p c}{2R_\infty m_p/m_e}.$$

The equation provides a path to calculate the product of the Planck and Avogadro constants. The value of this product termed the molar Planck constant, is known with tiny relative uncertainty, for example, 4.5×10^{-10} in 2017. From this product, Plancks constant can be calculated from measurements of Avogadros constant and vice versa without a significant increase in relative uncertainties. For comparison, the typical relative uncertainties of measurements of h and N_A in 2017 were 1.5×10^{-9}, and hence several times the relative uncertainty of the molar Planck constant.

Consequently, if the XRCD method can be used to measure h, it must also be possible to use this method to realize the unit of mass after the redefinition. Of course, one could argue this path via the molar Planck constant. But, a different chain of arguments leads to a result that is closer to how the measurement is actually made. First, from the precise measurement of the Rydberg constant and a fixed value of the Planck constant, the mass of a single electron can be obtained. This can be seen by solving the definition of R_∞ for m_e to obtain

$$m_e = \frac{2hR_\infty}{\alpha^2 c}.$$

Second, the relative masses of any other atom can be measured precisely with respect to the electron. For example, the mass of an atom x can be obtained as

$$m_x = m_e r_x,$$

where r_x is the (unitless) ratio of the mass of atom x to the mass of an electron.

To infer the mass of a macroscopic object, made from a single species of atoms, one needs to count the number of atoms and multiply it by the mass of a single atom. However, counting the atoms in a macroscopic object would take a long time even with a high counting rate, since 1 kg contains 2×10^{25} atoms (using silicon for this example). With a counting rate of 50 million atoms per second, it would take about the age of the universe to count that many atoms.

However, this problem can be solved geometrically. It is a three-dimensional equivalent of a problem the reader may have encountered. Trying to find out how many puzzle pieces are in a complete jigsaw puzzle. One would measure the length and the height of the completed jigsaw puzzle and divide the obtained area by the area of one puzzle piece. This principle works, because the jigsaw puzzle is (almost) periodic and hence the spacings between two adjacent parts are nearly identical throughout the completed jigsaw puzzle. It forms a two-dimensional crystal.

This idea also works in three dimensions, but a perfect crystal is needed. Silicon can be produced as an almost ideal single crystal with very few impurities. The process of growing silicon crystals has been optimized by the semiconductor industry over the past decades. In reference to the jigsaw puzzle, a single puzzle piece corresponds to the unit cell of silicon. The unit cell contains four silicon atoms and has the shape of a cube with a side length of about 5.4×10^{-10} m. The side length of the unit cell is commonly referred to as the lattice constant. The number of atoms in a macroscopic crystal can be obtained by dividing the macroscopic volume by the volume of the unit cell and multiplying by four to account for the fact that there are four atoms per unit cell.

Figure 5.1: A silicon sphere made with enriched ^{28}Si. The sphere shown in the picture was used in a 2010 determination of N_A. Photo: Physikalisch Technische Bundesanstalt (PTB).

The next step is to measure the macroscopic volume of the macroscopic crystal. This volume determination depends on the shape of the crystal. Naturally, the volume is more straightforward to infer for some shapes than for others. A sphere is the chosen shape for this experiment for three reasons: (1) The volume of a perfect sphere only depends on one parameter, the diameter d. (2) In contrast to a cube, which is in principle also given by one parameter, a sphere does not have corners or edges. In reality, corners and edges can not be made infinitely sharp and have to be chamfered. But the volume of each chamfer needs to be subtracted from the volume of the ideal shape. Estimating the volumes of these chamfers is difficult and can lead to uncertainties for the total volume. (3) The sphere has the smallest surface to volume ratio of all solid shapes. Since, the crystal becomes less perfect near the surface, it is beneficial to keep the surface area at a minimum. Figure 5.1 shows a silicon sphere.

For a sphere with diameter d made from silicon crystal with lattice constant a_0, the total number of silicon atoms is given by

$$n = 4\frac{V_{\text{Sphere}}}{V_{\text{unitcell}}} = 4\frac{\pi}{6}\frac{d^3}{a_0^3}.$$

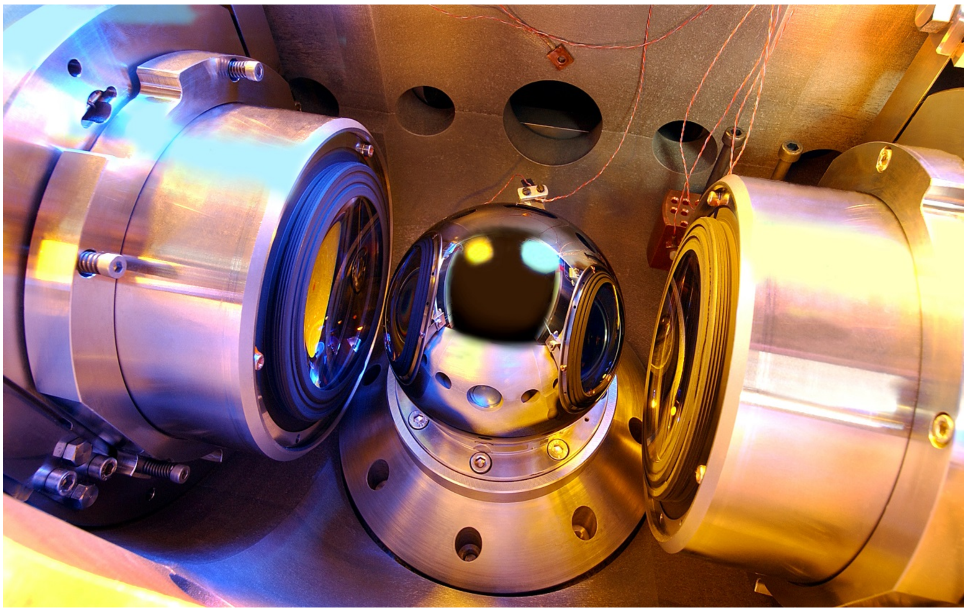

Figure 5.2: A silicon sphere in a sphere interferometer that is used to measure hundreds of sphere diameters with the sphere at different orientations. Photo: Physikalisch Technische Bundesanstalt (PTB).

An ideal sphere is impossible to manufacture. Hence, additional parameters need to be measured to describe the non-sphericity, the deviation of the real object from a perfect sphere. However, this is, in principle, a problem that can be solved by repeatedly applying a known solution. Instead of measuring one diameter, several hundred diameters are measured over different orientations of the sphere. The diameters are measured interferometrically, in custom built sphere-interferometers, see Figure 5.2.

Before the three equations can be put together, a small modification must be made to equation (5). Silicon is an excellent material to make a perfect crystal with one drawback: Natural silicon contains three different isotopes. So, the assumption that all silicon atoms are all identical does not hold. The silicon atoms differ in the number of neutrons in the nucleus. Stable silicon atoms have either 14, 15, or 16 neutrons in the nucleus together with 14 protons. One speaks of the three isotopes ^{28}Si, ^{29}Si, and ^{30}Si. The most important consequence is that the masses of the three atoms differ from each other. Hence, equation (5) must be modified to

$$m_{Si} = (f_{28}r_{28} + f_{29}r_{29} + f_{30}r_{30})\, m_e,$$

where the r_{28},r_{29}, and r_{30} determine the mass ratio of the isotopes ^{28}Si, ^{29}Si, and ^{30}Si to that of the electron and the three abundances f_{28}, f_{29}, and f_{30} determine how likely it is that an atom chosen at random from the crystal is of type ^{28}Si, ^{29}Si, and ^{30}Si. Observe that $f_{28} + f_{29} + f_{30} = 1$.

The relative atomic masses of the three silicon isotopes have been determined with sufficient precision. This is a one time task, since the masses remain constant over time. The relative atomic masses of various isotopes are compiled by the commission on isotopic abundances and atomic weight (CIAAW) by the international union of pure and applied chemistry. The values reported by CIAAW are given as ratios to the mass of Carbon-12 and need to be scaled by m_e/m_{C12} to obtain the r_{28}, r_{29}, and r_{30} mentioned above.

While the relative atomic masses constants of nature, the isotopic abundances of the three isotope varies from crystal to crystal and need to be remeasured. In natural silicon, $f_{29} = 0.047$ and $f_{30} = 0.030$. Both of these fractions are a few percent. Hence in order to obtain, a final result with a relative uncertainty of 10^{-8}, the relative uncertainties in the measurement of f_{29} and f_{30} have to be below 10^{-6} – a very demanding, with todays technology impossible, measurement. At present the, only way to solve this problem is to enrich the isotope ^{28}Si in the silicon. Enriching isotopes is a technique that has been perfected during the era of the cold war for the nuclear arms race, where enriched uranium was required. A similar process, albeit for a peaceful purpose, can be used to enrich ^{28}Si. In the most recently produced crystals $f_{28} \approx 0.999\,95$ has been achieved. Hence, $f_{29} + f_{30} = 5 \times 10^{-5}$. With these numbers, a less aggressive relative uncertainty is required for the measurement of the isotopic abundances. But, yet, the measurement is far from trivial and a new method was invented to accomplish this task.

Combining the equations yields a product of three factors, the number of atoms, the ratio of the mass of the silicon atoms to the mass of the electron and the mass of the electron:

$$m = N \times \frac{m_{Si}}{m_e} \times m_e = 4\frac{\pi d^3}{6a_0^3} \times (f_{28}r_{28} + f_{29}r_{29}f_{30}r_{30}) \times \frac{2hR_\infty}{\alpha^2 c}.$$

The third factor contains fundamental constants that are either fixed (c and h in the new SI) or measured by other experiments R_∞. The measurements required for the second factor has been discussed above. This leaves the first factor, which contains three quantities that need to be measured precisely. (1) The number four, which denotes the number of atoms in the unit cell. A perfect crystal has exactly four silicon atoms per cell. However, this number needs to be slightly modified to reflect the not so perfect reality. In a real crystal, some silicon atoms are missing, effectively lowering the number four, one speaks of voids. The opposite can also occur. Additional atoms (silicon or impurities from other species such as oxygen or carbon) could be at places, where there would not be atoms in a perfect crystal. These places are termed interstitial sites. (2) The length of the unit cell, a_0. The length of the unit cell is measured with X-ray diffraction and these measurements is a tour de force by itself. The apparatus used for this measurement combines an X-ray interferometer with an optical interferometer. The idea behind the measurement is to count how many X-ray fringes, whose period is identical to the lattice constant fit in one optical fringe, whose period is half the wavelength of the light used in the experiment. The strength of this measurement is that it only measures a ratio of numbers. The connection to meter is made via the wavelength of the laser used in the experiment – standard operating procedure for traceable realizations of the meter. (3) The diameter of the sphere d. For this measurement several dedicated sphere interferometers were built, two in Germany and one in Japan. In these sphere interferometers, two measurements are necessary. The optical path length between two lenses is determined with and without the sphere present. The difference is exactly the sphere diameter. By performing many measurements, an average sphere diameter can be obtained. (4) One quantity that is not reflected in the equation are the masses of non-silicon surface layers. Many methods have been developed to estimate the mass of thin surface layers on the spheres.

Several spheres were made and have been measured. The best measurement of N_A has a relative uncertainty of 12×10^{-9}. Figure 5.3 shows an overview of all determinations of h and N_A that were used to calculate a recommended value by CODATA. The figure shows that the measurement of different types agree reasonably well.

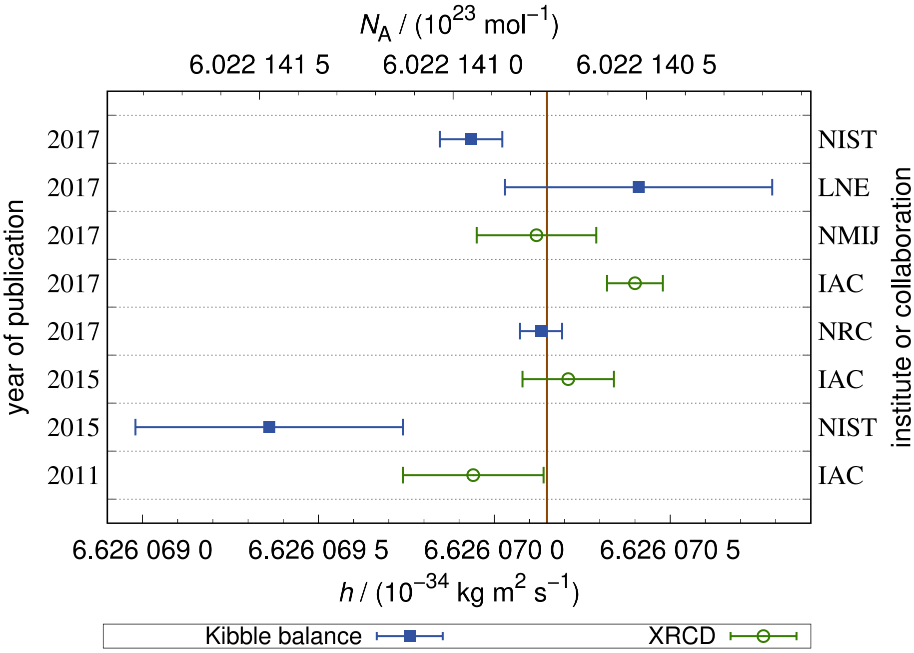

Figure 5.3: Results of high precision measurements of h and N_A that were taken into account to determine the fixed numerical values of h and N_A. The data points shown with empty circles were measured using the XRCD method. The data depicted with solid squares were obtained with Kibble balances.

Chapter 6

Outlook

The redefinition of the International System of Units offers many exciting opportunities for metrology. In the new SI, the unit of mass is defined via a fixed value of the Planck constant and no longer via an artifact. While the old definition gives mass exactly at one point, the kilogram, the new definition is scale independent. Masses ranging from the atomic scale to several kilograms can be realized of a fixed value of the Planck constant.

Before we discuss the benefits of a scale-invariant realization of mass, lets recall how masses with nominal values smaller than 1 kg were calibrated in the old definition. A so-called work down of the masses was required. A work down is a set of comparisons of pairwise equal mass values that allows one to assign a mass value to smaller masses. ,A typical mass set comes with five weights per decade (500 g, two 200 g, two 100 g). To work down one decade about eight measurements are required. Note, in order to work down from a kilogram to a milligram, six decades need to be spanned. The work down requires not only a lot of measurements, it will also increase the relative uncertainties. Since each measurement will add another small component to the uncertainty of the result. For example, for a certain class of weights, the relative uncertainty increases from 83×10^{-9} for a one kilogram weight to 5×10^{-4} for a one milligram weight. In summary, calibrating smaller masses in the pre-2019 system of units has two shortcomings, the work-down is time-consuming, and the relative uncertainties increase for smaller weights.

Both problems can be solved in the revised SI. In principle, Kibble balances can be built for a larger range of masses. Before the redefinition the focus of Kibble balance research was to build 1 kg instruments, after redefinition, Kibble balances with smaller mass range become more attractive. Several groups around the world are building table-top watt balance, with the goal to realize mass values of grams with relative uncertainties of a few parts in 10^6.

The revised SI leaves many other possibilities to measure mass as well. The only condition is that the mass measurement can trace back to the Planck constant. One interesting possibility is the electrostatic balance. In the electrostatic balance, the weight of a mass is compensated with an electrostatic force produced on a movable electrode. Similar to the Kibble balance two measurement modes are required. One is the voltage necessary to compensate a weight; the second is a measurement of the capacitance as a function of the position of the electrode. Half the capacitance gradient multiplied with the square of the applied voltage gives the weight of the mass. Electrostatic forces are typically much weaker than electromagnetic forces. Hence, the electrostatic balances have great potential realizing very small masses of tens of milligram and below.

In the revised SI atomic masses can be measured directly in kilogram using the recoil of an atom while absorbing a photon. These experiments measure the quotient h/m and with h fixed the mass m of an atom can be obtained in kilograms. In 2016, atomic recoil measurements determined h/m with relative uncertainties of a part in 10^9.

Figure 6.1: A third version of the SI logo that shows the defining constants on the inner circle an the base units on the outer. Note, that to realize most base units more than one defining constant is required. For example, to realize the kilogram, the hyperfine splitting of Cs, the speed of light, and the Planck constant is required.

Experiments to measure mass with atomic recoil, electrostatic balances and table-top Kibble balances are just three possibilities to realize mass at different scales. At this point, we may not even have found all the possibilities to realize the kilogram from h. In the future, we might have a new idea that allows to measure mass with unprecedented small uncertainty. As long as a link the h, it will be possible to use this idea to measure mass. So, the new SI fosters innovation, because there is the potential to continuously improve the realization of the units.

This is short essay shows the transformative idea behind the revision of the SI. The units are linked back to fundamental constants of nature and are hence scale invariant. For the first time mass can be measured other than just being compared to yet another mass. As such, the revised SI is a construction of beauty and logic. This beauty is nicely reflected in the logo of the revised SI shown in Figure 6.1.

Additional resources

- A nice book describing the history leading up to the proposed redefinition in 2019 is "From artefacts to atoms The BIPM and the Search for Ultimate Measurement Standards" by Terry Quinn (Oxford University Press).

- "The measure of the world" by Denis Guedj (The University Chicago Press) is a novel about the measurement of meridian from Dunkirk to Barcelona to establish the length of the meter. This is an entertaining and fun read that gives the reader insight to the spirit at the time and the magnitude of the task at hand.

- The website of the BIPM (www.bipm.org) hosts the official SI Brochures: The eighth brochure describes the International system of units before the redefinition. It can be found here: https://www.bipm.org/utils/common/pdf/si_brochure_8_en.pdf

- The ninth brochure (which is currently only available in draft form) describes the new SI. The current draft is located here: https://www.bipm.org/utils/common/pdf/si-brochure-draft-2016b.pdf

- A scientific article on various design aspects of the Kibble balance is available on the open access pages of the journal metrologia. Ian. A. Robinson and S. Schlamminger, The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass, Metrologia 53 A46 (2016). http://iopscience.iop.org/article/10.1088/0026-1394/53/5/A46

- Similarly an open access article is available on the realization of the kilogram by the XRCD method. K. Fujii et al, Realization of the kilogram by the XRCD method, Metrologia 53 A19 (2016). http://iopscience.iop.org/article/10.1088/0026-1394/53/5/A19

- A short report that explains how the final values of the defining constants were determined is also available as an open access article on Metrologia. D.B. Newell et al. The CODATA 2017 values of h, e, k and N_A for the revision of the SI, Metrologia 55 L13 (2018). http://iopscience.iop.org/article/10.1088/1681-7575/aa950a