Optimal Dynamic Spectrum Access Scheme for Utilizing White Space in LTE Systems

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***Abstract*—In this study, we design and implement an algorithm for optimal dynamic spectrum access (DSA) in a shared spectrum system where the primary user (PU) is a Long Term Evolution (LTE) system. The cumulative hazard function from survival analysis is used to predict the remaining idle time available in each channel for secondary user (SU) transmission subject to a probability of successful completion. Optimal allocation of physical resource blocks (PRBs) for the SU is shown to be a variation of the unbounded knapsack problem. We evaluate the algorithm performance using three data sets collected from real LTE systems. The algorithms achieve good white space utilization and have a measured probability of interference around the target threshold.**

I. Introduction

Dynamic spectrum access (DSA) has been proposed as a means of making more efficient use of available spectrum. In a common DSA scenario, there are one or more primary users (PUs) operating in a given band with priority access. One or more secondary users (SUs) can opportunistically transmit during times that the PUs are idle. When the PUs want to use the band again, the SUs must stop transmitting. In general, the SUs know whether the band is occupied by the PUs at a given time, either by querying a central coordinator (i.e., scheduler) or by sensing the band. However, the SUs do not know future activity of the PUs. Hence, DSA can be thought of as a prediction scheme in which an SU scheduler has to predict, with a certain probability, that the SU can complete its transmission before a PU reappears.

LTE cellular systems provide various services using one or more frequency bands comprising multiple channels. LTE is slated to be the first technology to be deployed in the Citizens Broadband Radio Service (CBRS) band, which is based on the *shared spectrum* paradigm [1]. Although opportunistic DSA is not supported in the CBRS band at present, its semi-dynamic three tier priority-based spectrum sharing is a positive step towards that. The first tier (highest priority) is the federal incumbent, which operates infrequently and only in limited geographic areas. The second tier is licensed LTE; the third tier is unlicensed. Thus, it is appropriate to start thinking about DSA in a network where an LTE system acts as the PU and the SUs communicate by opportunistically accessing the spectrum.

Although an LTE eNodeB could in theory be tasked with scheduling the SUs along with its own PU UEs, we believe there are reasons why this is not desirable and, therefore, not likely. In general the SUs will not be subscribers to the PU service. Thus, it is better to deploy a separate and independent SU system whose operation is transparent to the primary LTE system, thereby requiring no changes to the LTE system. This approach also enables legacy LTE systems, i.e., those not implementing our scheme, to participate in the DSA system. The only interaction between PU and SU systems is at the operator level, where a Service Level Agreement (SLA) may be agreed upon that specifies various operational policies, e.g., an upper bound on the probability of interference.

Resources in LTE are allocated in chunks called physical resource blocks (PRBs). Each PRB is 0.5 ms (one slot) long in time and 180 kHz wide in frequency. In our scheme, therefore, DSA becomes a matter both of predicting spectrum occupancy by the PU and allocating PRBs for the SU transmission accordingly.

The contributions of this paper are as follows. We present a DSA algorithm that allocates PRBs to SUs in a system with a statistical guarantee on the probability of interference to the PU LTE system. We frame the allocation of PRBs across time and frequency as a variation of the unbounded knapsack problem. Our algorithm is simple and efficient, which makes it easier to implement in a real system. We show the effectiveness of our algorithm using LTE uplink datasets collected from real deployed LTE systems at the time scale used by LTE systems (1 ms).

II. Related Work

Two state time-inhomogeneous Discrete-Time Markov Chain (DTMC) [2], semi-Markov DTMC models [3], Alternating Renewal Process [3], [4] and semi-Markov Continuous-Time Markov Chain (CTMC) based models [5], [6], [7] are some of the models proposed in the literature to represent spectrum occupancy.

There have been few models proposed for *predicting* spectrum occupancy, which is critical to allocating spectrum to the secondary users. The Partially-Observable Markov Decision Process (POMDP) model [8], prediction based on expected remaining OFF time [9] and the two state semi-Markov model [3] are some of the methods used for this purpose. In [10], the transmission duration of an SU is constrained based on the maximum bound on probability of interference to the PU. Residual idle time of an Alternating Renewal Process is used in [4] to indirectly predict reappearance of the PU. Pattern mining of spectrum occupancy data has been used to predict channel availability [11], [12]. In [13], [14], the authors used the cumulative hazard function from survival analysis to design DSA algorithms that allocated resources (time duration) in a single dimension. In this work, we consider allocation of resources in two dimensions, namely time and frequency.

Long Term Evolution (LTE) systems employ packet schedulers that schedule users in the time domain and then allocate physical resource blocks (PRBs) among them according to criteria such as channel quality and service rate [15], but usually no distinction is made between different classes of User Equipments (UEs). Application-aware schedulers where the allocation of PRBs and power is based on UE real-time requirements, along with a guaranteed quality of experience (QoE) for all users, has been researched in [16]. However, these do not consider the requirement for an SU in a DSA system where the scheduler must account for predicting future PU activity.

III. Allocation Algorithms

We begin this section with a more precise statement of the problem we are solving. Then we briefly explain how we use the cumulative hazard function from the field of *survival analysis* [17], [18] to limit the probability of interference caused by an SU transmission to the PU system. This is a key element of our DSA algorithm. We then present the optimization formulation for our problem, followed by the algorithm that optimally allocates PRBs for an SU.

A. Problem Statement

We use an LTE system operating in a given band as the PU. The width of the band in *frequency* is ($N \times 180$) kHz, i.e., the width of N PRBs. We refer to each 180 kHz range of frequency as comprising a channel to aid conceptual understanding. SUs may transmit on PRBs that are idle, but interference to the PUs should be limited. Hence in this DSA application, an SU requests $\mathcal{W}$ PRBs, and the goal is to devise an algorithm that allocates up to $\mathcal{W}$ PRBs, maintaining the probability of interference on each channel below a specified threshold. Hence, this is a two-dimensional resource allocation problem in which the number of allocated PRBs is maximized, subject to the constraint that the probability of interference on each channel is below a threshold.

B. Use of Cumulative Hazard Function

A communication channel alternates between idle and busy states. Let $I_1, B_1, I_2, B_2, \ldots$ represent the successive idle and busy states of a channel. We assume these states are independent and the lengths of all the idle states have the same distribution, say $F(t)$. Our algorithm makes use of the hazard function associated with the distribution $F(t)$. The hazard function at time t, $h(t)$, measures how likely an idle period of unknown length I will end in the next instance given that it has lasted for t units of time and is given by

$$h(t) = \lim_{dt \to 0} \frac{Pr[t \leq I < t + dt \mid I \geq t]}{dt} = \frac{f(t)}{1 - F(t)} \quad (1)$$

where $f(t) = dF(t)/dt$. Given a specific channel to be shared, the algorithm allows a request to transmit for τ units of time only if the probability that the current idle period I will last for additional duration τ given that it has been idle for duration t (when the SU request arrived) is more than a set threshold p. That is, the SU request is granted if the following condition is satisfied.

$$Pr[I \geq t + \tau \mid I \geq t] > p \quad (2)$$

This threshold $p, 0 < p < 1$, is the probability of successful transmission by the SU. It can be shown that [13]

$$P[I \geq t + \tau \mid I \geq t] = \exp(-[H(t + \tau) - H(t)]) \quad (3)$$

where $H(t) = \int_0^t h(s)ds, t \geq 0$ is the cumulative hazard function. However, in practice, $H(t)$ needs to be estimated from the idle time data for which a large sample $I_1, I_2, \ldots, I_n$ of n idle durations is collected. Let $I_{(1)} \leq I_{(2)} \cdots \leq I_{(n)}$ be the ordered $I_i, i = 1, \ldots, n$. Then it can be shown that a non-parametric estimate $H_n(t)$ of $H(t)$ is given by [13]

$$H_n(t) = \sum_{i: I_{(i)} \leq t} \frac{1}{n - i + 1} \quad (4)$$

From (2) and (3), using $H_n(t)$ in place of $H(t)$, it is easy to deduce that transmission is allowed by the SU if the following inequality is satisfied.

$$H_n(t + \tau) - H_n(t) < (-\ln p) \quad (5)$$

C. Optimization Formulation

Let x_i be the number of PRBs allocated on channel i, w_i be the weight of the channel i and N be the number of channels. Let $H_n^i(\cdot)$ be the non-parametric estimate of the cumulative hazard funtion of channel i. Let $\theta_{th} = (-ln\, p)$, where p is the set threshold for probability of successful transmission and t_i be the amount of time that channel i has been idle when the SU request for $\mathcal{W}$ number of PRBs arrives. Then the two dimensional resource allocation problem can be formulated as a variation of the well-known *unbounded knapsack problem* [19] given by

$$\textit{maximize} \sum_{i=1}^{N} w_i x_i, \quad \textit{subject to} \quad (6)$$

$$\sum_{i=1}^{N} x_i \leq \mathcal{W}, \; x_i \in \{0, \mathbb{Z}^+\} \quad (7)$$

$$w_i \geq 0, \; i = 1, \ldots, N \quad (8)$$

$$\max_{1 \leq i \leq N} \theta_i \leq \theta_{th} \quad (9)$$

where $\theta_i = H_n^i(t_i + x_i) - H_n^i(t_i)$.

The above optimization formulation maximizes the total weighted value of the allocated PRBs subject to the constraints. Constraint (7) limits the allocation to a maximum of $\mathcal{W}$ PRBs requested by an SU, whereas constraint (9) makes sure that the probability of interference on any channel is below the specified threshold.

To keep our analysis simple, in this study, we have assumed that all channels have equal weight by setting $w_i = 1$, $1 \leq$

$i \leq N$. In general, the weights can be assigned based on (for example) channel quality or priority.

D. Definition of Algorithm

The unbounded knapsack problem is known to be NP-Hard [19]. Hence, we use a *pseudo-polynomial* algorithm using *dynamic programming* to solve our two dimensional resource allocation problem [20]. Let t_i be the length of the current idle period for channel i and $H_n^i(\cdot)$ be the non-parametric estimate of cumulative hazard function of channel i. The algorithm for finding the optimal allocation of resource blocks is presented in Algorithm Fixed_PRB, which outputs x_i's, the number of PRBs allocated to the SU in channel i. Essentially the algorithm uses the SU transmission constraint given in (5) across each channel and solves the optimization formulation presented in (6).

Algorithm Fixed_PRB : Request Grant of $\mathcal{W}$ PRBs

input:
$\mathcal{W}$ = number of PRBs requested
parameters:
$\mathbf{H} = \{H_n^1, H_n^2, \ldots H_n^N\}$: non-parametric estimate of cumulative hazard function for channels 1 to N
$\mathbf{t} = \{t_1, t_2, \ldots t_N\}$: length of current idle period for channels 1 to N in terms of PRBs
p - the probability of successful transmission threshold
output:
$\mathbf{x} = \{x_1, x_2, \ldots x_N\}$: number of resource blocks allocated in channels 1 to N

$\theta_{th} := -\ln p$
$\theta := 0$
$\mathbf{x} := \mathbf{0}$
while $\sum_i x_i \leq \mathcal{W}$ **do**
  $\theta_{min} = \min_i \; H_n^i(t_i + x_i + 1) - H_n^i(t_i)$
  $i_{min} = \text{argmin}_i \; H_n^i(t_i + x_i + 1) - H_n^i(t_i)$
  if $\theta_{min} \leq \theta_{th}$ **then**
    $x_{i_{min}} = x_{i_{min}} + 1$
    $\theta = \theta_{min}$
  else
    break
  end if
end while
return $\mathbf{x}$

For some applications, an SU may want to request the maximum possible number of PRBs subject to the constraints. A minor modification to Algorithm Fixed_PRB can achieve that. The only change required is to remove the condition in the *while* loop and make it an infinite loop. The algorithm terminates when no more PRBs can be allocated on any of the N channels, i.e., the *if* statement inside the while loop fails. We call this *Algorithm Max_PRB*.

E. Example

Figure 1 illustrates a possible allocation using Algorithm Fixed_PRB with 5 channels. In the figure, the SU requests

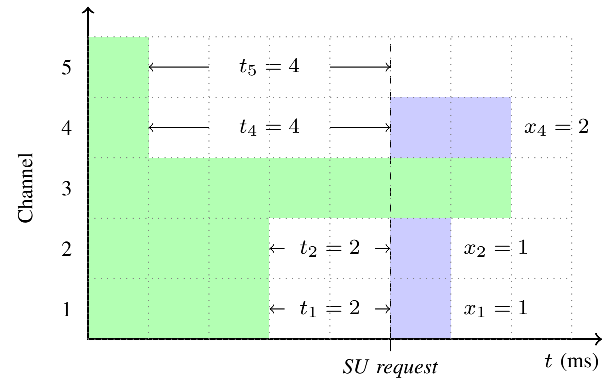

Fig. 1. Allocation of resource blocks for the SU. PU transmissions in green, SU transmissions in blue

$\mathcal{W} = 4$ PRB pairs at time shown. The idle time lengths of the channels are shown as $t_i,\ i = 1, 2, 4, 5$ (Channel 3 is busy at the time of request). The optimal allocation is shown as the set of $\{x_i : \sum_i x_i = \mathcal{W}\}$ that satisfies the interference constraint across all the channels.

IV. Evaluation

A. Spectrum Occupancy Data

We use three sets of collected data to model PU activity. The first set was collected using an indoor antenna on the National Institute of Standards and Technology (NIST) campus in Gaithersburg, MD. The second and third datasets were collected using outdoor antennas in the city of Philadelphia, PA. The locations were chosen to test our algorithms in indoor and outdoor settings in an urban environment. Data were collected in Band 17, a 10 MHz uplink (UL) LTE band centered at 709 MHz at all the three locations.

The NIST campus data collection system consisted of a 10.78 cm "rubber duck" antenna connected to an Ettus Universal Sofware Radio Peripheral (USRP) [1]. A 56 point power spectrum (in dB) for each 1 ms period (after aggregating I/Q samples every 80 ns) was computed and the middle 50 values used as power values of the 50 LTE channels. Since LTE allocates PRBs in pairs, our collected dataset consisted of an integer power value, in dB, for a pair of PRBs (1 ms in time). A noise power threshold was then applied to produce a binary occupancy sequence for each of the 50 channels. The noise power threshold was chosen 3 dB above the value that produced a probability of false alarm (PFA) below 1 %.

Data was collected for two different one hour periods:

- 3:00 PM to 4:00 PM local time, Monday, August 28, 2017 (1^{st} day)
- 3:00 PM to 4:00 PM local time, Tuesday, August 29, 2017 (2^{nd} day)

[1]The identification of any commercial product or trade name does not imply endorsement or recommendation by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.

We chose these so that we could compare the same time period on two separate days.

The second and third datasets were collected at two locations in the metro Philadelphia area on the CityScape spectrum monitoring system [21] with additional processing of the I/Q samples to produce output files in the same format as the data collected at NIST. We converted the CityScape datasets into binary occupancy sequences using the noise threshold to which the CityScape USRPs were calibrated.

B. Simulation

In our experiments, we used occupancy data of LTE uplink channels from channel 15 to 24 (total of 10 channels) as PU traffic. This ensures that the SU does not transmit over the control channels, *Physical Random Access Channel (PRACH)* and *Physical Uplink Control Channel (PUCCH)*. Idle and busy periods for each channels were built based on a sampling interval (1 ms) and noise threshold. Then a non-parametric estimation of the cumulative hazard function of each channel is computed. The threshold for probability of successful transmission is set to 0.9, which implies the threshold of probability of interference is 0.1.

We evaluate the performance of our algorithms in different configurations. The configurations are denoted with a concatenation of three text strings separated by underscores in the form *station_train_run*, where *station* $\in$ {*lab, upenn01, upenn02*} is the location where the data was collected, *train* $\in$ $\{1^{st}, 2^{nd}\}$ represents the data used for *training* the algorithm, i.e., data used for building the cumulative hazard function and *run* $\in \{1^{st}, 2^{nd}\}$ denotes the data that represents the PU spectrum occupancy while *running* the algorithm. For lab station, values of 1^{st} and 2^{nd} represent one hour data from 1^{st} and 2^{nd} day respectively. For upenn01 and upenn02 station, 1^{st} and 2^{nd} represent the first and second half of the one hour data collected at those stations respectively. For example, in configuration *lab*_1^{st}_2^{nd} an algorithm is trained using one hour data collected in NIST lab from the 1^{st} day and run using one hour data from the 2^{nd} day. An algorithm trained using the 1^{st} half-hour data collected at upenn01 station and run using the 2^{nd} half-hour data is denoted as configuration *upenn01*_1^{st}_2^{nd}.

The SU traffic is modeled as a Poisson arrival process. For Algorithm Fixed_PRB, an SU requests a fixed number of PRBs and the algorithm allocates up to that many PRBs while ensuring that the probability of interference is below the specified threshold. For Algorithm Max_PRB, an SU requests the maximum possible number of PRBs, and the algorithm allocates the maximum possible PRBs subject to the given interference probability constraint. The requests and allocations in our simulation are always in terms of 1 ms time lengths, but we use the term PRBs rather than "pair of PRBs" for simplicity.

C. Metrics

Performance of the two algorithms was measured with the following metrics.

- White Space Utilization (WSU): Given the spectrum occupancy of a set of channels, the WSU by an SU is defined as the fraction of total idle PRBs used by the SU for its own transmission. In other words, it is the ratio of number of idle PRBs used by the SU for its own transmission to the total idle PRBs present in the spectrum occupancy of the set of channels.
- Probability of Interference (PoI): The PoI of the secondary user across a set of channels is defined as the probability that a transmission of the SU collides with that of the PU on those channels. Thus, it is the ratio of the number of times an SU transmission collides (or runs into a busy period) with a PU transmission across those set of channels to the total number of SU transmissions over a long observation period.
- Percentage Overlap of SU Transmission (POST): This is the number of PRBs used for SU transmissions that overlaps with PU transmissions across a set of channels expressed as a percentage of total PRBs used for SU transmissions across the same set of channels.

V. Results

A. Performance of Algorithm Max_PRB

Figures 2, 3 and 4 show the performance of Algorithm Max_PRB in terms of WSU as the average request inter-arrival duration increases for lab, upenn01 and upenn02 datasets respectively. As average request inter-arrival duration increases, the offered load to the algorithm decreases, leading to a decrease in WSU for all the datasets. However, for the lab dataset, WSU is much higher than for the upenn01 and upenn02 datasets. For the lab dataset, all ten channels are idle for about 95 % of the time (for 1^{st} day), whereas for upenn01, it varies between 89 % to 90 % and for upenn02, it varies between 57 % to 89 % (see Table I). Further, for the lab dataset, maximum idle duration across the ten channels (for 1^{st} day) vary between 35 345 ms to 51 532 ms. But the maximum idle durations in the upenn01 dataset lie between 136 ms and 191 ms. For upenn02 those numbers are 34 ms to 154 ms. Thus, the lab data has idle durations two orders of magnitude greater than those of upenn01, and upenn01 idle durations are significantly higher than those of upenn02. Also, the idle durations of the lab data were not continuous, causing long *flat* periods in the $H(\cdot)$ function, leading to allocations of more PRBs than in the upenn01 and upenn02 datasets. This also causes the slope of $H(\cdot)$ function of the lab data to be lower than that of upenn01 and upenn02 datasets. Since a lower slope of $H(\cdot)$ function leads to higher number of PRB allocation (see Eqn (5)), WSU for lab dataset is much higher compared to those of upenn01 and upen02. Comparing WSU between upenn01 and upenn02, we notice that the WSU of upenn01 is higher. This is attributed to the fact that fraction of idle time and the maximum idle duration of channels are much higher in upenn01 dataset than those of upenn02.

In Figure 2, we observe that the WSU for lab_1^{st}_1^{st} is the highest and for lab_2^{nd}_2^{nd} is the lowest. This is largely because there are larger idle durations on the 1^{st} day compared

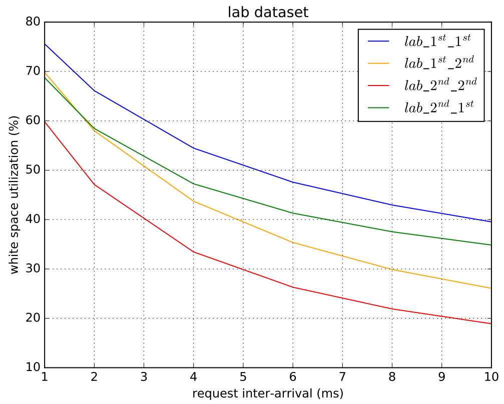

Fig. 2. WSU vs inter-arrival time for lab dataset running Algorithm Max_PRB

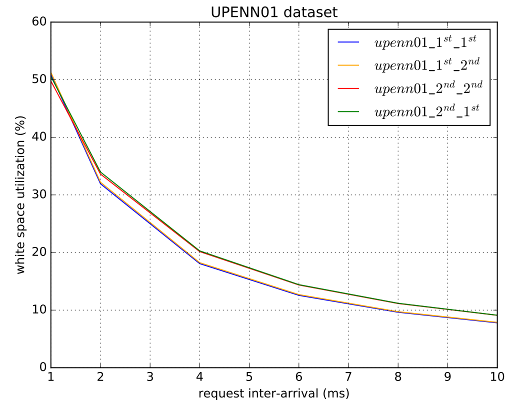

Fig. 3. WSU vs inter-arrival time for upenn01 dataset running Algorithm Max_PRB

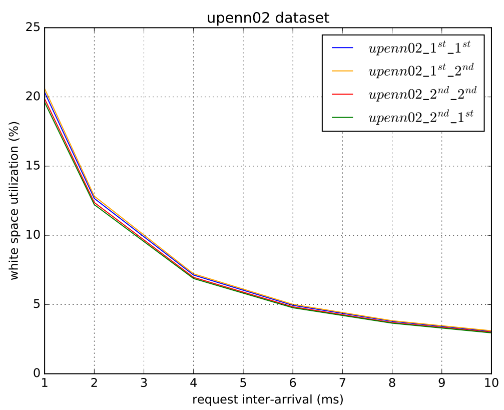

Fig. 4. WSU vs inter-arrival time for upenn02 dataset running Algorithm Max_PRB

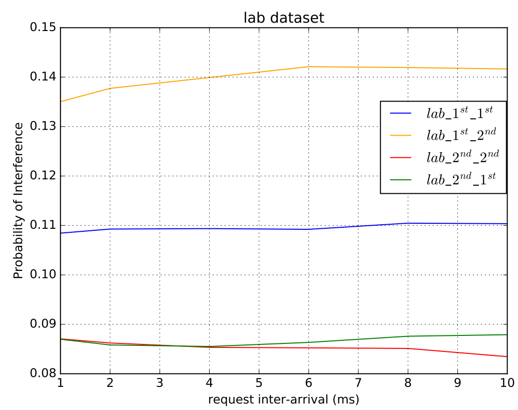

Fig. 5. Prob. of Interference vs inter-arrival time for lab dataset running Algorithm Max_PRB

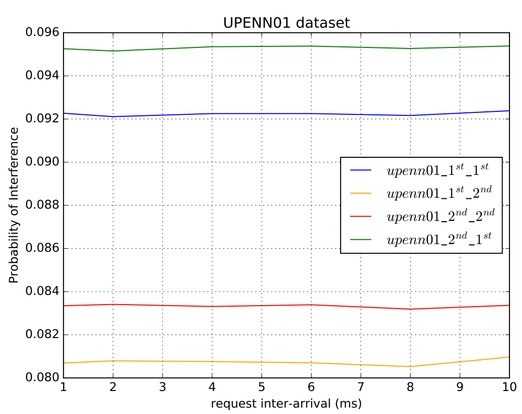

Fig. 6. Prob. of Interference vs inter-arrival time for upenn01 dataset running Algorithm Max_PRB

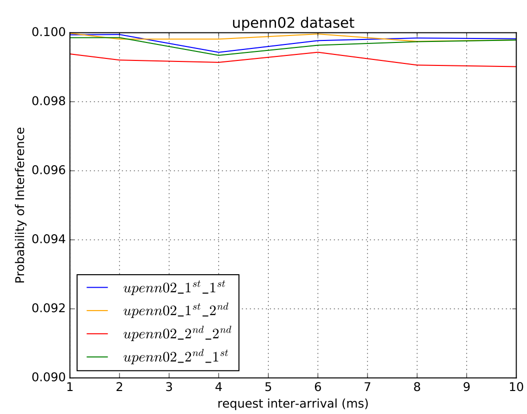

Fig. 7. Prob. of Interference vs inter-arrival time for upenn02 dataset running Algorithm Max_PRB

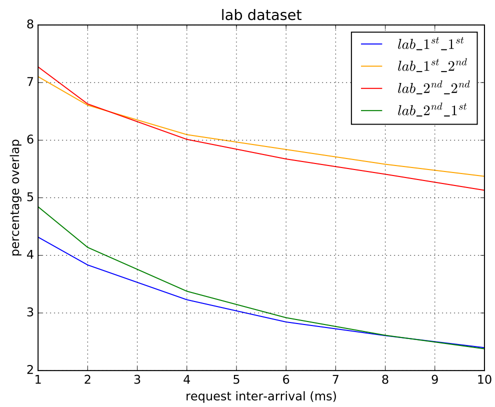

Fig. 8. Percentage overlap vs inter-arrival time for lab dataset running Algorithm Max_PRB

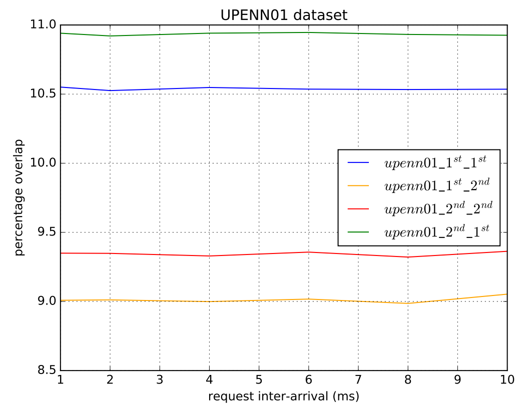

Fig. 9. Percentage overlap vs inter-arrival time for upenn01 dataset running Algorithm Max_PRB

to the 2^{nd} day. WSU of the other two configurations lies between these two.

For the upenn01 and upenn02 datasets, the relative performance of WSU among different configurations is close to each other. Note that configurations with $1^{st}\_1^{st}$ and $2^{nd}\_2^{nd}$ suffixes are not practical, since the algorithm is run on the same data as the training data. In practice, the algorithm will train (i.e., build the $H(\cdot)$ function) on the first half hour (or some fixed duration) data and then allocate PRBs to SU using the built $H(\cdot)$ function. So, $1^{st}\_2^{nd}$ represents such a scenario. $2^{nd}\_1^{st}$ represents a scenario where the PU traffic may be swapped between the two half hour periods (in case such traffic occurs in practice). Since the relative performances are close to each other, it shows that a system deploying Algorithm Max_PRB can perform as well as the theoretical system (e.g., $1^{st}\_1^{st}$) when it trains on first half hour data and then runs on the next half hour.

Figures 5, 6 and 7 present the measured PoI as average SU request inter-arrival time increases for the three datasets. Measured PoI for the upenn01 and upenn02 datasets is always below the set threshold (0.1) for all configurations. For the lab dataset, the measured PoI is below the threshold for the $lab\_2^{nd}\_2^{nd}$ and $lab\_2^{nd}\_1^{st}$ configuration. However, for the other two configurations ($lab\_1^{st}\_1^{st}$ and $lab\_1^{st}\_2^{nd}$) the measured PoI is slightly higher than the threshold. The first lab dataset has longer idle time durations and is idle for a higher fraction of time. So, when the algorithm is trained using this data, the PRB grants are more generous ($H(\cdot)$ function is more relaxed) and hence when the algorithm runs on the second dataset it encounters more interference. When the algorithm is trained and run on first dataset (configuration $1^{st}\_1^{st}$), the PoI slightly exceeds the threshold mostly due to estimation error (of $H(\cdot)$ function). Measured PoI remains almost constant in all configurations of all datasets as the inter-arrival time between SU requests increases. Hence, the performance of the PU system remains almost the same regardless of the load on the SU system.

Figures 8, 9 and 10 show POST values for the lab, upenn01 and upenn02 datasets respectively. The percentage overlap is below 7.5 % for all configurations for the lab dataset. For upenn01 and upenn02 the maximum we see are 11 % and 13 %, respectively. The lab dataset has a higher fraction of idle duration, and some idle periods are very long, which leads to a lower POST value.

chan num	lab (1^{st} day)		lab (2^{nd} day)		upenn01		upenn02	
	% Idle duration	max Idle duration (ms)	% Idle duration	max Idle duration (ms)	% Idle duration	max Idle duration (ms)	% Idle duration	max Idle duration (ms)
15	95.8	43431	92.3	28892	90.6	170	89.4	127
16	95.4	43471	92.1	28892	90.8	184	89.9	132
17	95.6	43432	92.4	28935	91.0	163	89.2	154
18	95.8	35345	92.7	28980	90.8	191	87.8	106
19	95.7	44151	92.8	28980	89.8	166	82.0	68
20	95.6	51532	92.8	28980	88.9	149	77.8	63
21	95.9	51532	92.9	28980	88.7	142	77.0	66
22	96.0	49255	93.1	19401	89.8	136	57.2	34
23	95.9	51529	93.1	31681	90.3	155	70.6	36
24	95.8	51529	93.2	30841	90.3	142	83.4	79

TABLE I
IDLE TIME DURATION INFORMATION FOR THE DATASETS

B. *Performance of Algorithm Fixed_PRB*

Figures 11, 12 show how WSU varies when the number of requested PRBs increases for Algorithm fixed_PRB over the lab and upenn01 datasets, respectively, when the average SU request inter-arrival time is 1 ms. WSU increases as more PRBs are requested, since the algorithm exploits more white space before reaching a limiting value. The algorithm is limited by the available white space and the interference constraint. Hence, beyond a certain point, the algorithm cannot grant the requested number of PRBs and instead allocates the maximum possible within the constraints. In fact, the values at which WSU saturates match the WSU values of Algorithm Max_PRB corresponding to average request inter-arrival time of 1 ms.

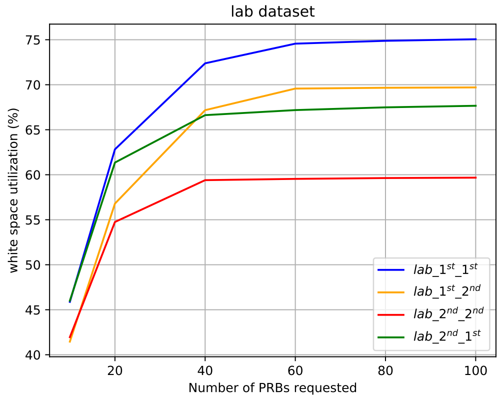

Fig. 11. WSU vs number of Requested PRBs for lab dataset running Algorithm Fixed_PRB

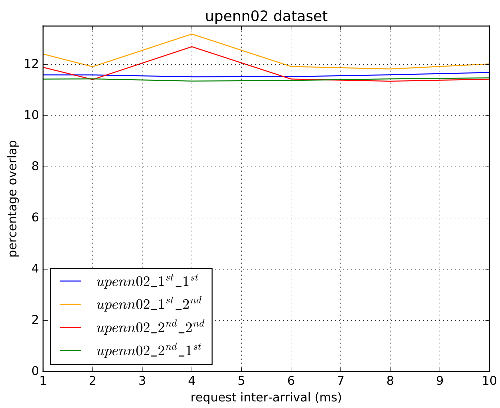

Fig. 10. Percentage overlap vs inter-arrival time for upenn02 dataset running Algorithm Max_PRB

Similar to Algorithm Max_PRB, the WSU values are close to each other for various configurations in the upenn01 dataset. For the lab dataset the WSU for $lab\_1^{st}\_1^{st}$ is the highest and for $lab\_2^{nd}\_2^{nd}$ is the lowest, and WSU for the other two configurations lies between these two. The reasons for these relative performances are the same as those given for Algorithm Max_PRB.

Figures 13, 14 show a measured PoI for the lab and upenn01 datasets that initially increases as the number of requested PRBs increases and then stays constant. These constant (saturated) PoI values match the corresponding values for Algorithm Max_PRB when the average request inter-arrival time is 1 ms. For all the datasets and all configurations, except for $lab\_1^{st}\_2^{nd}$, measured PoI is less than the set threshold. The anomaly in $lab\_1^{st}\_2^{nd}$ is due to the same reason given for Algorithm Max_PRB.

Performance of the algorithm in terms of POST as the requested number of PRBs increases is shown for the lab and upenn01 datasets in Figures 15, 16. Overall, the POST values are low. Similar to WSU and PoI, the saturated values match

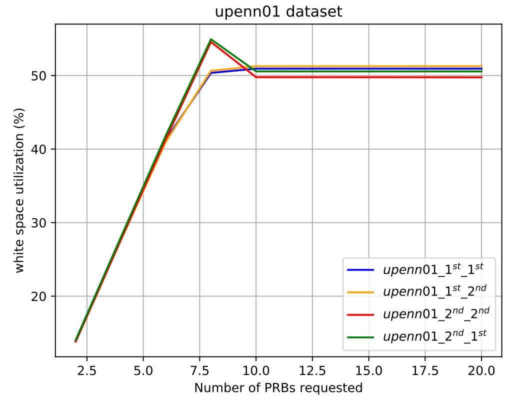

Fig. 12. WSU vs number of Requested PRBs for upenn01 dataset running Algorithm Fixed_PRB

with those of Algorithm Max_PRB.

Due to space limitations, we have not provided the performance graphs of Algorithm Fixed_PRB over upenn02 dataset. But they are similar to those of upenn01 dataset.

VI. Conclusion and Future Work

We presented an optimal DSA algorithm for allocation of PRBs to the SU. The algorithm is a form of the unbounded knapsack problem that maximizes the number of allocated PRBs such that the PoI across all the channels remains below a threshold using a non-parametric estimate of the cumulative hazard function. The algorithm was tested on real LTE datasets, one collected in our laboratory and two collected in center city Philadelphia. Our results show that the algorithm is able to make good use of the available white space while keeping the overall PoI around the desired threshold. Hence, our algorithm can be readily implemented in a practical LTE network.

In this study, we used equal weights for the channels. Our scheme should be studied when the channels have different weights due to different priority or channel quality. The SU scheduling model assumed ideal knowledge of the PU activity at the time of the SU request and the ability to schedule the SU transmission within one subframe. Further work should investigate imperfect sensing of the band and realistic scheduling overhead.

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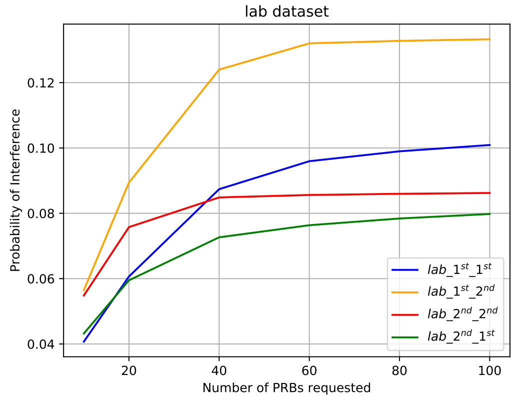

Fig. 13. Prob. of Interference vs number of Requested PRBs for lab dataset running Algorithm Fixed_PRB

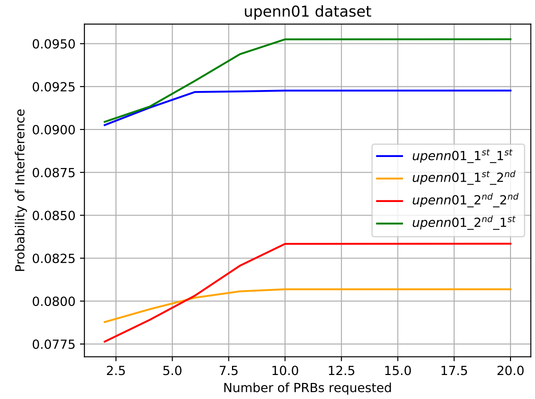

Fig. 14. Prob. of Interference vs number of Requested PRBs for upenn01 dataset running Algorithm Fixed_PRB

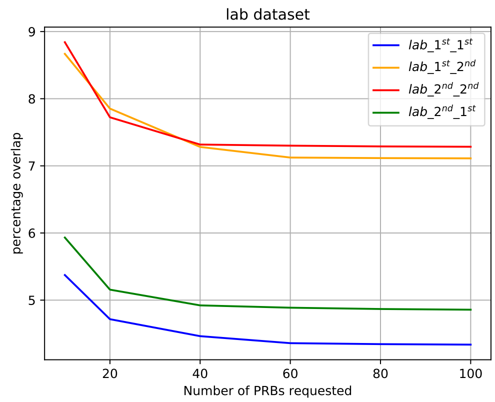

Fig. 15. Percentage overlap vs number of Requested PRBs for lab dataset running Algorithm Fixed_PRB

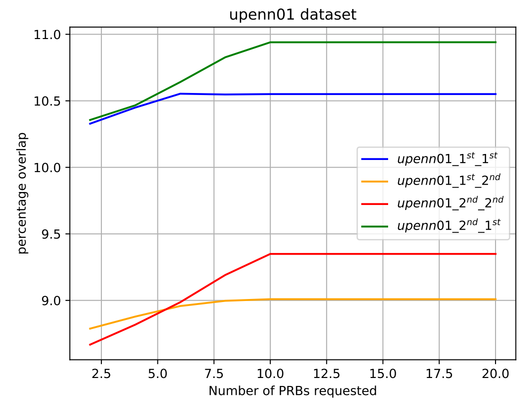

Fig. 16. Percentage overlap vs number of Requested PRBs for upenn01 dataset running Algorithm Fixed_PRB

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