Random-Deadline Missing Probability Analysis for Wireless Communications in Industrial Environments

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***Abstract*—The flexibility of wireless networks, and the need for more data to be transferred in modern industries have motivated the industrial wireless communications. Strict industrial requirements including the time-sensitive delivery of data have led to developing appropriate wireless technologies to satisfy these requirements. Various industrial wireless protocols employ Time division multiple access (TDMA) because of preventing data collision between various transmissions and the easiness of employment. In this work, we study the scheduling problem of multiple wireless data flows for industrial applications. The data flows have random strict deadlines following a given probability distribution. In modern industries, these flows may be sensing data routed from a sensor to the controller or control commands directed from the controller to an actuator. A randomized frame-based scheduling scheme is analyzed where each time slot in the frame is assigned to a data flow randomly. We derive a method to calculate the average number of packets missing their deadlines per frame that can be used for flow admission control or optimization of the scheduling algorithm. Finally, we study the effects of various system parameters on the ratio of the average number of packets missing their deadlines to the average total number of packets generated.**

I. Introduction

Meeting the requirements of industrial communicantions and improving control capabilities have motivated enhancing various industrial networking protocols. Recently, industrial wireless technologies have been recognized as attractive alternatives to the typically employed wired networks due to increased networking flexibility, and the ease of installation and maintaince. Wireless Highway Addressable Remote Transducer Protocol (WirelessHART) and ISA100.11a are examples of these wireless technologies that have been compared in the literature [1], [2].

Strict deadlines are commonly enforced for data packets in industrial networks due to the importance of timing in industrial applications. Thus, the lack of reliability of wireless transmissions, in the form of considerable bit errors resulting in packet loss or delivery past their deadlines, is the main challenge facing industrial wireless networks. Hence, optimizing transmission control policies has been widely discussed for networks with hard deadlines in [3]-[6].

In many industrial wireless technologies, time division multiple access (TDMA)-based medium access control (MAC) protocols are used to achieve data reliability. The use of TDMA-based MAC protocols eliminates the liklihood of intra-network interference possibilities and hence the data packets have a better hhigher reliability. Typically, the main two advantages of using a TDMA-based MAC protocol is achieving bounded transmission delay and improving network energy efficiency by controlling the number of transmissions at every time slot [7].

Scheduling plays a key role in enhancing wireless network reliability by ensuring that only one network node transmits at any given time. Moreover, scheduling allows

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transmitting the same data muliple times to increase the chance of reaching the destination correctly before the deadline. Scheduling in wireless sensor networks (WSN) has been studied widely in the literature such as [8]-[10]. The network parameters and the corresponding performance measures are studied in [8] while TDMA-based scheduling is applied. Moreover, a survey pf various scheduling algorithms is introduced in [9]. A comparisoin of various algorithms in IEEE 802.15.4-based networks his presented in [10].

In this work, we consider a randomized frame-based scheduling policy to schedule multiuple data flows with strict deadlines. In the proposed scheduling policy, a time slot is assigned to a flow following some random distribution. In existing industrial wireless communications protocols, the schedule is commonly evaluated once every transmission frame composed of many time-slots. The distribution of the transmission probabilities and the schedule may be re-evaluated at the beginning of each new frame. The route of each of the data flows is assumed to be known before the schedule is evaluated. The data flows are not assumed to be periodic. Instead, the packets generation and their deadlines are assumed to be probabilistic with some defined probability mass functions. A similar model is previously considered in [11] for the case of deterministic scheduling policy which is obtained by solving a Markov decision problem.

The consideration of data flows with random deadlines is motivated by having event-based signals that may be affected by random events or random processing delays. In [12]-[15], the concept of having data flows with random deadlines is discussed. In this paper, we use random deadlines in a different setting compared to the existing literature and we also consider the effects of the wireless channels.

The average number of packets missing their deadlines per frame is derived for a randomized scheduling policy. The schedulability of a set of flows for a given average number of packets missing their deadlines per frame is discussed. Numerical evaluation of the obtained results is conducted to assess the performance of the system for various parameter settings.

The rest of the paper is organized as follows. The system model is presented in Section II. The average number of packets missing their deadlines is analyzed in Section III. Numerical results are presented in Section IV. Finally, Section V presents the concluding remarks.

II. System Model

We consider the case of M flows to be scheduled over one wireless frequency band. The flows are denoted by $\mathcal{F} = \{F_1, F_2, ...F_M\}$. The route of the flow F_m is predefined and denoted by ϕ_m. The number of hops of the route is h^*_m and each packet in F_m has to be successfully delivered over all the hops before it gets to its destination. A new packet arrives in F_m when the deadline of the previous packet expires. The fixed frame length of the schedule is T time-slots which is commonly denoted in indutrial wireless protocols as the hyper-period. The hyper-period in the literature is commonly defined as the least common multiple of the packet generation periods of the field devices [16]. This definition is valid only in the case of periodic data flows.

At each time-slot, one transmission at most occurs as we assume one wireless frequency band. The wireless link between two nodes i and j in the route of F_m is modeled as a binary erasure channel. The state of the wireless channel is independent of the packet generation process. The wireless link between the two nodes is characterized by the success transmission probability $\rho_{i,j}$. The success probability value is generally driven by the channel impulse response as well as the wireless system's parameters such as required error rate, the transmission power, and the modulation and coding schemes. We assume that $\rho_{i,j} = \rho$, for all (i,j), for the sake of simplicity while the proposed algorithms can still be analyzed in the more general case.

The deadline of each packet in F_m is modeled by a positive discrete random variable D_m which has integer values. These values are denoted by $d_m \in \mathcal{B}_m$ where the random varibale space $\mathcal{B}_m = \{h^*_m, h^*_m+1, \ldots, D^*_m\}$, where D^*_m can be infinity. The probability mass function of D_m is $f_m(.)$ with mean and variance of μ_m and σ^2_m, respectively. Each packet in F_m has a strict deadline and is discarded if not successfully received by the destination prior to its deadline. We assume that the following packet in a flow is generated and released as the deadline of the previous packet expires.

The schedule is generally evaluated at the network manager where the schedule for each frame is evaluated at the beginning the frame. The hyber-period in this work is defined as the least common multiple of the nearest integer numbers to the average deadlines of the considered flows.

In this work, we analyze the performance of a randomized scheduling policy. The considered performance metric is the ratio of the average number of packets missing their deadlines to the average number of packets generated in a hyper-period. The analysis can be used for schedulability testing or admission control by comparing the performance of a scheduling policy to a preset threshold.

III. Performance Analysis

In this section, we derive the average number of packets missing their deadlines in a hyper-period where a randomized scheduling policy is employed. We start by studying the probability of a certain packet in a flow to miss its deadline. Then, we define the packets arrival random process and study its behavior which is then used to calculate the average number of packets missing their deadlines.

A. Probability of a Packet Missing Its Deadline

The randomized policy is determined by the transmission probabilities of the flows. The probability p_m is defined to be the probability of F_m to be scheduled at a time slot. These probabilities are set based on the average values of various system characteristics. Note that, at a single time slot, a single flow is scheduled and hence the transmission decision is to pick an index of a flow using the probability mass function p_m such that $\sum_{m=1}^{M} p_m = 1$.

We then define $q_m(t_m, h_m, p_m)$ as the probability of a packet in F_m to miss its deadline if it has currently h_m hops remaining in its route and t_m time slots remaining before its deadline expires given that the probability for a packet to be scheduled for transmission is p_m. At any time slot, the three events that may occur to a packet in F_m are i) the packet is transmitted and successfully received, ii) the packet is transmitted but failed to reach the following node in its route, and iii) the packet is not scheduled to be transmitted. The probability $q_m(t_m, h_m, p_m)$ is evaluated as the probability not to have h_m successful transmissions in the following t_m time slots as follows

$$q_m(t_m, h_m, p_m) = \sum_{i=0}^{h_m-1} \binom{t_m}{i} (p_m\rho)^i (1 - p_m\rho)^{t_m-i}, \quad \text{for } h_m \le t_m \tag{1}$$

where this probability is calculated only for the packet which have not missed their deadlines yet such that $h_m \le t_m$. Moreover, the boundary and initial conditions are not considered in this analysis because of their negligible effect on performance. The used policy is randomized and the hyper-period is long enough compared to the packets deadlines such that the couple of packets at the beginning and the end of the observation interval are too small compared to the total number of missed packets.

Finally, we evaluate the average probability $\bar{q}_m(p_m)$ of a packet in F_m to miss its deadline given that the probability for a packet to be scheduled for transmission is p_m. We use a frame-based scheduling transmission and hence the exact states of flows are not known during the schedule calculation. Thus, in order to obtain the average probability, we calculate the average probability of a packet to miss its deadline at the arrival instant of the packet. The value of the deadline at the arrival instant is not known and follows the random distribution $f_m(.)$. Hence, the average probability is defined as follows

$$\bar{q}_m(p_m) = \sum_{D_m=h_m^*}^{D_m^*} f_m(D_m) q_m(D_m, h_m^*, p_m). \tag{2}$$

B. Average Number of Packets Missing their Deadlines

In order to calculate the average number of packets missing their deadlines per flow during the hyper-period T, we introduce the random variable X_m to represent the number of packets of F_m that are generated within T. Also, we define the random sequence $\mathcal{T}_{m,x}$ containing x elements to represent the sequence of deadlines of x packets of F_m within the hyper-period. The ith element of this random sequence is denoted by $\mathcal{T}_{m,x}(i)$ which represents the random deadline of the ith packet. Moreover, the summation of the elements of this random sequence is denoted by

$$\Sigma_{m,x} = \sum_{i=1}^{x} \mathcal{T}_{m,x}(i). \quad (3)$$

The limiting values of the random variable X_m are calculated using the limiting values of the deadline random variable, D_m. The minimum value of X_m occurs when all the packets have the maximum allowable deadline and it equals 1 when D_m^* is infinity. It can be expressed as follows

$$X_m^{(\min)} = \begin{cases} \left\lceil \frac{T}{D_m^*} \right\rceil & h_m^* \leq D_m^* < \infty \\ 1 & D_m^* = \infty \end{cases}, \quad (4)$$

where $\lceil . \rceil$ is the ceiling function. The maximum value of X_m occurs when all packets of F_m have the minimum allowable deadline which equals to h_m^*. Hence, it is evaluated as follows

$$X_m^{(\max)} = \left\lceil \frac{T}{h_m^*} \right\rceil. \quad (5)$$

We next calculate the probability distribution of the random variable X_m. The event $\{X_m = x_m\}$ occurs when the sum of the deadlines of the first $x_m - 1$ packets in the sequence $\mathcal{T}_{m,x_m}$ is below T while the sum of the deadlines of the first x_m packets is greater than or equal to T. Hence, the expression of $\Pr(X_m = x_m)$ is stated as follows

$$\Pr(X_m = x_m) = \Pr(\Sigma_{m,x_m-1} < T, \Sigma_{m,x_m} \geq T). \quad (6)$$

Then, we calculate the sum of the probabilities of all the events for which Σ_{m,x_m-1} takes values between 0 to $T-1$ and the value of the deadline of the x_mth packet is greater than or equal to $T - \Sigma_{m,x_m-1}$. The sum of these probabilities represents having the x_mth packet as the last packet of the F_m in the hyper-period. The expression is rewritten as follows

$$\Pr(X_m = x_m) = \begin{cases} \sum_{l=(x_m-1)h_m^*}^{T-1} \Pr(\Sigma_{m,x_m-1} = l, \mathcal{T}_{m,x_m}(x_m) \geq T - l), \\ \qquad\qquad \text{for } X_m^{(\min)} \leq x_m \leq X_m^{(\max)}, \\ 0, \qquad\qquad \text{otherwise,} \end{cases} \quad (7)$$

while the condition for the first case guarantees the possibility of having x_m packets of F_m within T.

By independence of the deadlines of the packets of the same flow, we can multiply the probability of last packet deadline by the probability of the summation of all the previous deadlines to obtain their joint probability expression. We first calculate the expression for the last packet deadline to be greater than or equal to $T - l$ as follows

$$\Pr(\mathcal{T}_{x_m}(x_m) \geq T - l) = \sum_{\mathcal{T}_{x_m}(x_m)=T-l}^{D_m^*} f_m(\mathcal{T}_{x_m}(x_m)). \quad (8)$$

Then, the expression in (7) can be evaluated, using the independence of last packet deadline and the sum of the previous packets deadlines, as follows

$$\Pr(X_m = x_m) = \sum_{l=0}^{T-1} \left(\Pr(\Sigma_{m,x_m-1} = l) \sum_{\mathcal{T}_{x_m}(x_m)=T-l}^{D_m^*} f_m(\mathcal{T}_{x_m}(x_m)) \right). \quad (9)$$

Moreover, the value $\Pr(\Sigma_{m,x_m-1} = l)$ is evaluated using the deadline distribution of F_m as follows

$$\Pr(\Sigma_{m,x_m-1} = l) = \sum_{\mathcal{T}_{x_m-1}|_{\Sigma_{m,x_m-1}=l}} \left(\prod_{x=1}^{x_m-1} f_m(\mathcal{T}_{x_m}(x)) \right), \quad (10)$$

where $\mathcal{T}_{x_m-1}|_{\Sigma_{m,x_m-1}=l}$ is any random sequence of length $x_m - 1$ such that the sum of all the deadlines equals l to include all the combinations of the deadlines leading to this value.

Thus, the average number of packets missing their deadlines in F_m is calculated by summing the probabilities of all the packets to miss their deadlines over the distribution of X_m. All the events of packets missing their deadlines in F_m are independent of each other with average probability of $\bar{q}_m(p_m)$. Hence, the average number of packets missing their deadlines, which is denoted by $\bar{N}_m$, is evaluated as follows

$$\bar{N}_m = \sum_{x=X_m^{(\min)}}^{X_m^{(\max)}} \Pr(X_m = x) \sum_{i=1}^{x} \bar{q}_m(p_m). \tag{11}$$

By rearranging the terms in the summations, the expression is evaluated as follows

$$\bar{N}_m = \bar{q}_m(p_m) \sum_{x=X_m^{(\min)}}^{X_m^{(\max)}} x\Pr(X_m = x). \tag{12}$$

Then, the average number of packets missing their deadlines in all the M flows over the hyper-period is evaluated as follows

$$\bar{N} = \sum_{m=1}^{M} \bar{N}_m. \tag{13}$$

Moreover, the average number of sent packets by all the flows during a single hyper-period is evaluated as follows

$$\bar{N}_{\mathrm{T}} = \sum_{m=1}^{M} \sum_{x=X_m^{(\min)}}^{X_m^{(\max)}} x\Pr(X_m = x). \tag{14}$$

Finally, the ratio of the average number of packets missing their deadlines to the average number of sent packets is defined as follows

$$R_{\text{Missed}} = \frac{\bar{N}}{\bar{N}_{\mathrm{T}}} = \frac{\sum_{m=1}^{M} \bar{q}_m(p_m) E[X_m]}{\sum_{m=1}^{M} E[X_m]}, \tag{15}$$

where $E[X_m]$ is the expected value of the random variable X_m.

This ratio can be used for flow admission control when a randomized scheduling policy is employed. Suppose $M-1$ flows have already been admitted into the network and the value of R_{Missed} is less than or equal to a prescribed threshold, such as 10%. Upon arrival of the Mth flow, R_{Missed} is computed again. If it does not exceed 10%, the new flow is admitted. Otherwise, it is rejected.

In addition, once R_{Missed} is computed for a given choice of $\{p_m : m = 1, ..., M\}$, it serves as an upper bound to the value of R_{Missed} for the optimal randomized schedule, which in principle can be found by minimizing R_{Missed} over all possible distributions $\{p_m : m = 1, ..., M\}$ through numerical exhaustive search. However, the upper bound may be loose.

IV. Numerical Results

In the following, we assess the performance of a randomized scheduling algorithm when transmission of packets with random deadlines is considered. The measured objective function is R_{Missed}. We will compare the performance of the optimal randomized strategy for various system parameters.

In the following results, we assume symmetric flows such that all the M flows have the same value for h_m^* and the same value for D_m^*. The deadlines D_m follow a discrete uniform distribution over the range $h_m^*, ..., D_m^*$. The performance results are obtained by simulating the system using the optimal scheduling transmission probabilities, which are $p_m = 1/M$, $\forall m$ due to the use of symmetric flows. Although the simulations are done over multiple hyper-periods, occasional dips in the curves are observed that are due to the finite time duration of simulations.

In Fig. 1, we consider R_{Missed} against the number of hops per flow for different settings of ρ and D_m^*. The relation between R_{Missed} and h_m^* is monotonically non-decreasing with higher slope at lower values of h_m^* and the slope decreases as h_m^* increases. As expected, having higher values of D_m^* and ρ enhance the performance significantly.

In Fig. 2, we consider the performance against the number of flows for different settings of ρ. A monotonically non-decreasing relation is found between R_{Missed} and M with higher slope at lower values of M and the slope decreases as M increases. This figure illustrates the use of the deadline missing probability analysis in

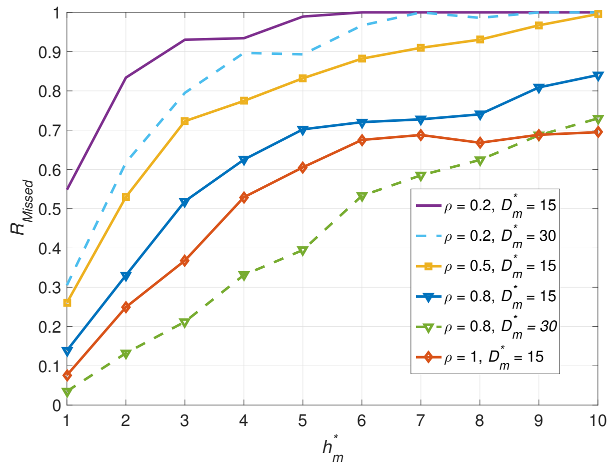

Fig. 1. The ratio R_{Missed} vs. h^*_m for different values of ρ and D^*_m with $M = 3$

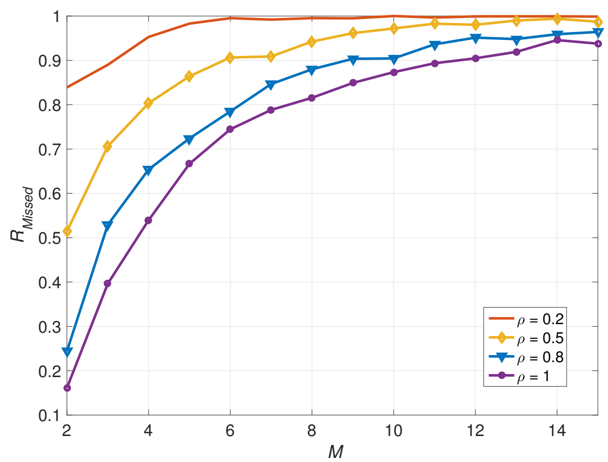

Fig. 2. The ratio R_{Missed} vs. M for different values of ρ with $h^*_m = 3$ and $D^*_m = 15$

admission control in which a required ratio threshold is set. Hence, the maximum number of admitted flows can be determined and exploited. Moreover, in the more general case of asymmetrical flows, similar analysis can be used to check the schedulability of a set of flows or admitting a new flow to the network in addition to the existing flows while keeping R_{Missed} below a prescribed value. Furthermore, in the case of asymmetrical flows, a performance criterion R_{Missed} can be computed for individual flows and different benchmarks enforced for different flows. Hence, flow admission control or optimization of transmission probabilities for various flows can be carried out to meet these requirements.

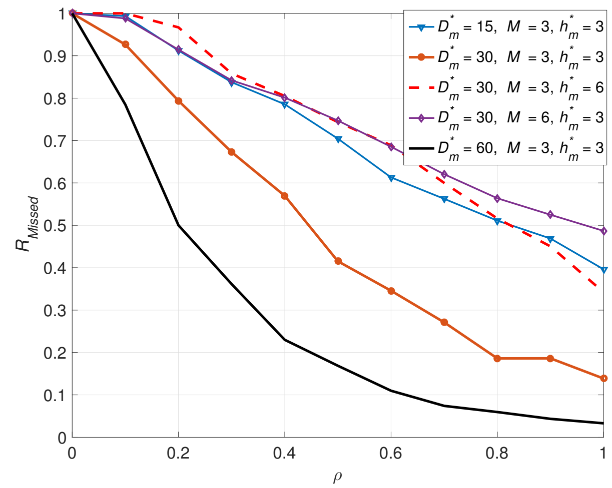

Fig. 3. The ratio R_{Missed} vs. ρ for different values of D^*_m, M, and h^*_m

In Fig. 3, a comparison of the performance against the channel success probability is shown. A monotonically non-increasing relation is found where the largest variation in the curve slope is found at the widest range of random deadlines, i.e. for $D^*_M = 60$. The figure shows that the importance of the channel quality is more pronounced for networks with a larger number of data flows, a larger number of hops on the routes of the flows, or tighter deadlines ranges. For $D^*_m = 15$, the curve is almost linear and hence any change in the value of ρ leads to a corresponding change in the performance. While for $D^*_m = 60$, the performance improved significantly for small values of ρ and the rate improvement decreased for the higher values of ρ.

Finally, in Fig. 4, the performance against D^*_m is shown. The value of D^*_m determines the range of the random deadlines such that the range is wider for a higher D^*_m. As expected, it is observed that R_{Missed} decreases with D^*_m or with ρ. The figure also shows the importance of channel quality for the case of a tight deadlines range.

V. Conclusions

In this work, we have analyzed the performance of randomized frame-based scheduling for industrial wireless networking. The network has multiple data flows

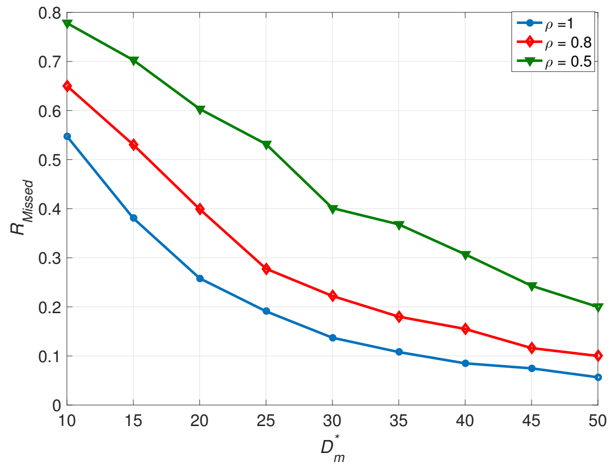

Fig. 4. The ratio R_{Missed} vs. D_m^* for different values of ρ with $h_m^* = 3$ and $M = 3$

with random packet deadlines. Each flow is assigned a transmission probability where the frame schedule is composed at the beginning of each frame. We have derived the expression for the probability of a packet to miss its random deadline. Also, we derived the expression for the ratio of the average number of packets missing their deadlines to the average number of packets generated by all the flows per frame. Then, we studied the performance of the system using the optimal transmission probabilities to minimize that ratio. We have shown that the optimal policy is robust to the changes of the number of route hops when the random deadline range is relatively large. Moreover, a good wireless channel is needed for the more constrained networks, i.e. networks that have a larger number of data flows, a larger number of hops on the routes of the flows, or tighter deadlines ranges. We have also shown the way to use the derived expressions for flow admission control and schedulability. In this work, the optimal policy is obtained numerically and hence more efficient algorithms and heuristic alternatives are to be studied in future work.

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