

# Importance of Preserving Correlations in Error-Vector-Magnitude Uncertainty

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**Abstract**—Correlations between measurement records, e.g. frequencies or time records, are an important consideration in the uncertainty analysis of high-frequency electronic systems. To study the importance of preserving correlations, we introduce a method to scramble the correlations of a correlated uncertainty analysis and develop a software tool to do this as part of the NIST Microwave Uncertainty Framework. We then compare the results of a correlated uncertainty analysis and the corresponding scrambled analysis in estimating the uncertainty in the Error-Vector-Magnitude of a modulated signal. This comparison shows that preserving these correlations is critical to accurately assessing system performance and uncertainty.

**Index Terms**—measurement uncertainty, Monte Carlo methods, digital modulation, error-vector-magnitude.

## I. INTRODUCTION

Uncertainty analysis is an essential element in designing and deploying microwave and millimeter-wave electronic systems. At high frequencies, as electronic components become less ideal, correlations between records of system response can become significant. These correlations can occur across frequencies, time points or, for modulated signals, symbols. Correlated uncertainty analyses can incorporate physical errors and accurately estimate the resulting effects on the system. This is apparent in frequency or time records as these data are transformed between the frequency and time domains where, for example, a frequency domain signal with an oscillatory correlated uncertainty profile may correspond to a pulse-like uncertainty profile in the time domain [1].

Error-Vector-Magnitude (EVM) is a common figure of merit for evaluating the accuracy of digitally modulated signals [2]. Recently, [3] a covariance-based correlated uncertainty analysis was performed for modulated signals from a precision source at 44 GHz using the NIST Microwave Uncertainty Framework (MUF) [4] to determine the uncertainty in EVM.

We develop a technique and software tool to allow us to test the effect of preserving correlated uncertainties and show, in a simple example, how scrambling these correlations, while preserving the variances but not the covariances, can substantially modify the results of uncertainty analyses.

## II. CORRELATED UNCERTAINTIES

Covariance-based uncertainty analyses as outlined in [1] and implemented in the MUF [4], are capable of propagating uncertainties through complex transformations while preserving correlations. These correlations can be attributed to systematic errors in underlying physical components or from a distribution of realized measurements. For example, if we measure the length of a line in a calibration standard the error in the

measurement will induce a correlated uncertainty profile in all corrected measurements that use this standard. In addition to accurately estimating the overall uncertainty, these correlations constrain the realization of deviations from a nominal value. In order to test the importance of preserving correlations, we developed a method to scramble these correlations, modifying the covariances while preserving the variances.

### A. Sensitivity Analysis

A sensitivity analysis of a measurement can be defined as a collection of deviations from a nominal value, with each deviation attributed to a unique mechanism. Following the notation in [1], we can write the covariance matrix provided by a sensitivity analysis as  $\Sigma_{SA} = (\mathbf{J}_\sigma)(\mathbf{J}_\sigma)^T$ , where  $\mathbf{J}_\sigma$  is a  $K \times N$  matrix where the  $N$  columns are defined as sensitivity analysis vectors  $\{\mathbf{S}_n\}_{n=1}^N$  of multivariate dimension  $K$ . Here the  $n$ th mechanism is perturbed by its standard uncertainty, transformations are applied (e.g., transforming between the time and frequency domains), and the resulting deviation from the nominal value of the measurement defines  $\mathbf{S}_n = (S_{1,n}, S_{2,n}, \dots, S_{K,n})^T$ .

To change the covariances but preserve the variances of the sensitivity analysis, we “scramble” the correlations of the sensitivity analysis vectors by resampling these vectors at each multivariate component (i.e. permuting the columns of each row of  $\mathbf{J}_\sigma$ ) and multiply by a randomly chosen  $\pm 1$  to create a new collection of sensitivity analysis vectors  $\{\mathbf{S}_n^{CB}\}_{n=1}^N$ . Thus at each multivariate index,  $i = 1, \dots, K$ ,

$$S_{i,n}^{CB} = \gamma_{i,n} S_{i,\tilde{n}_{i,n}}, \quad (1)$$

where each  $\tilde{n}_i = \{\tilde{n}_{i,n}\}_{n=1}^N$  is an index set drawn randomly without replacement from the original set of  $n = 1, \dots, N$  and  $\gamma_{i,n}$  is randomly drawn from the set  $\{-1, 1\}$ . The  $\gamma_{i,n}$  terms are added as the sensitivity analysis assumes symmetric distributions and produces identical results in either case.

### B. Monte Carlo Analysis

A Monte Carlo uncertainty analysis consists of a sample of Monte Carlo replicates  $\{\mathbf{M}_q\}_{q=1}^Q$ , where each  $\mathbf{M}_q = (M_{1,q}, M_{2,q}, \dots, M_{K,q})^T$  corresponds to a realization of all of the underlying error mechanisms propagated through the required transformations. An innate feature of this type of uncertainty analysis is that each of the Monte Carlo replicates has preserved the correlation with the underlying mechanisms through these transformations. The mean of the Monte Carlo sample  $\bar{\mathbf{M}}$  represents a physically realistic estimate of the expected value of the measurement while the distribution

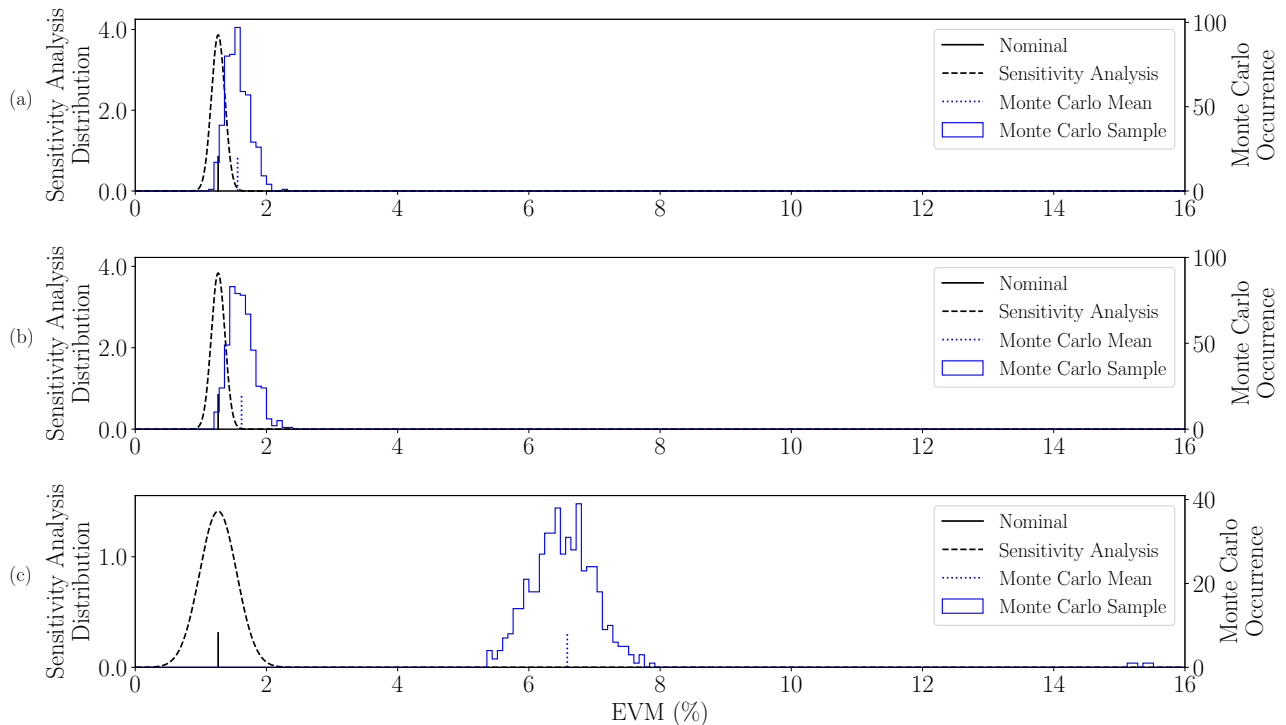


Fig. 1: EVM uncertainty analysis results (following [3]) using (a) a model for an ideal cable (b) the actual cable measurement with correlated uncertainties and (c) the actual cable with a scrambled uncertainty analysis. The mean of the Monte Carlo sample goes from 1.56% and 1.62% in the correlated analyses to 6.59% in the scrambled analysis. Statistics from these plots are shown in Table I.

of the sample about the mean defines a coverage interval corresponding to the likelihood that a value falls in this range. Additionally, the standard deviation of the distribution can be used as an estimate of the standard uncertainty [5].

To modify the covariances but preserve the variances, we obtain a new Monte Carlo sample  $\{M_q^{CB}\}_{q=1}^Q$  by resampling at each multivariate component,  $i = 1, \dots, K$ ,

$$M_{i,q}^{CB} = M_{i,\tilde{q}_i,q}, \quad (2)$$

where each  $\tilde{q}_i = \{\tilde{q}_{i,q}\}_{q=1}^Q$  is drawn randomly without replacement from the original set of  $q = 1, \dots, Q$  as above.

### C. Conservation of Variance

Equations (1) and (2) show how to scramble correlations across a multivariate sensitivity analysis and Monte Carlo sample. We now show that these resampled distributions preserve variances as calculated from the original distribution.

For multivariate component  $i$ , the variance corresponding to the sensitivity analysis, i.e., the  $i$ th diagonal entry of the covariance matrix  $\Sigma_{SA}$ , can be written  $\sum_{n=1}^N S_{i,n}^2$ . The variance corresponding to the scrambled sensitivity analysis vectors can be shown to be identical to those of the original analysis as

$$\sum_{n=1}^N (S_{i,n}^{CB})^2 = \sum_{n=1}^N \gamma_{i,n}^2 S_{i,\tilde{n}_i,n}^2 = \sum_{n=1}^N S_{i,n}^2.$$

This relationship also holds for the two Monte Carlo samples where we obtain the same variances in the distribution about the mean ( $\bar{M}$ ).

Note that although the resampled distribution preserves variance, transforming the data (say from the frequency domain to the time domain) may lead to a change in the variance between the two distributions.

### D. MUF “Correlation Buster”

The above methodology for scrambling correlations has been introduced as a tool, the “Correlation Buster”, in the NIST MUF to facilitate investigations into the importance of preserving correlations in uncertainty analyses. The MUF represents measurements with uncertainty as a collection of the nominal value, sensitivity analysis vectors, and Monte Carlo replicates. This collection can be used to define a new measurement with the same overall uncertainty but with significantly different correlations by resampling the sensitivity analysis vectors and Monte Carlos replicates, as in (1) and (2).

## III. CORRELATED UNCERTAINTIES IN EVM

EVM is an important metric for characterizing the accuracy of a received modulated-signal waveform transmitted and received in a nonideal system [2]. EVM is calculated by comparing the relative difference between a received demodulated waveform and the symbols of the corresponding modulation scheme.

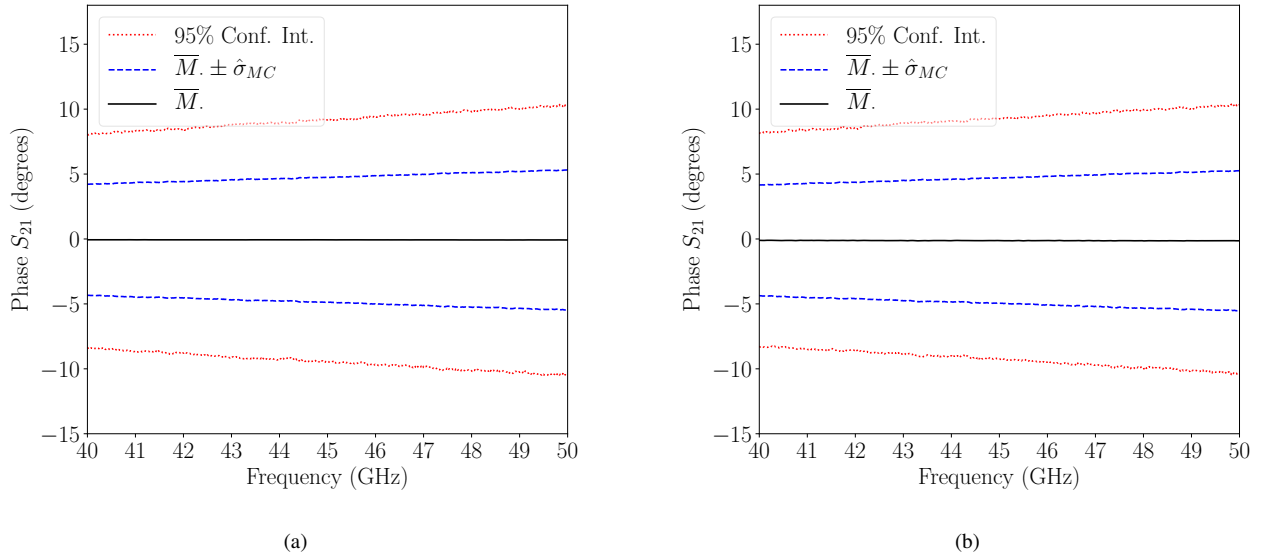


Fig. 2: Monte Carlo analysis of phase uncertainty corresponding to (a) the correlated MUF uncertainty analysis and (b) the scrambled uncertainty analysis using the Correlation Buster. The mean of the Monte Carlo sample  $\bar{M}$ ., the range within the standard deviation of the sample  $\hat{\sigma}_{MC}$ , and the 95% confidence interval of the sample is shown. We see that the frequency-dependent uncertainties are nearly identical and the standard uncertainty in phase is approximately five degrees.

TABLE I: EVM statistics for the MUF analysis (MUF) and the scrambled analysis using the Correlation Buster (CB) using an ideal through (Ideal) or a cable measurement with uncertainty (Cable). We show the nominal value, the mean of the Monte Carlo sample  $\bar{M}$ ., the standard uncertainty as calculated by the sensitivity analysis  $\hat{\sigma}_{SA}$  and the standard deviation of the Monte Carlo sample  $\hat{\sigma}_{MC}$ .

	Nominal	$\bar{M}$ .	$\hat{\sigma}_{SA}$	$\hat{\sigma}_{MC}$
MUF - Ideal	1.26	1.56	.103	.178
MUF - Cable	1.26	1.62	.104	.191
CB - Cable	1.26	6.59	.283	.910

One of the first components of any EVM algorithm is to time shift the received signal to determine the optimal sampling times. The time-shifted, demodulated, and sampled signal creates an I/Q trajectory on a constellation diagram. EVM is the normalized sum of the squared distance between ideal symbols and the received I/Q samples.

Several studies have investigated methods for estimating uncertainty in EVM measurements [6], [7] and the IEEE P1765 standards development working group is developing a recommended practice for estimating the uncertainty of EVM in modulated signals for wireless communications. Recently, [3] performed a covariance-based correlated uncertainty analysis using the MUF of a 1-GSymbol/sec, 64-state quadrature-amplitude-modulated (64-QAM) signal at 44 GHz including errors in the source and the receiver. The uncertainty analysis also included a characterization of the cable connecting the source and receiver under multiple bends.

In Figure 1(a) and (b) we reproduce the results of [3] modeling the cable with an ideal cable and with the actual cable including correlated uncertainties, respectively. We see the nominal EVM for both cases is 1.26%, while the mean of the Monte Carlo sample is 1.56% for the ideal case and 1.62% when including the cable and its uncertainties. Next, we applied the NIST Correlation Buster to scramble the correlations of the uncertainty analysis of (b) before converting to time domain, aligning the signals and calculating the EVM. The results of this scrambled analysis are shown in Fig. 1(c) and all of these results are tabulated in Table I. In the scrambled analysis we see a dramatic increase in EVM for the Monte Carlo sample which has a mean of 6.59%. Additionally, we see large increases in both the standard uncertainty, as calculated by the sensitivity analysis, and the standard deviation of the Monte Carlo sample.

#### IV. DISCUSSION

As shown in Fig. 1, scrambling the correlations has a significant effect on the EVM of the entire Monte Carlo sample. To analyze a potential cause for this, we look at the uncertainty of the scattering parameters of the cable due to bending. Figure 2(a), shows the Monte Carlo uncertainty analysis of the phase of the  $S_{21}$  transmission coefficient of the cable averaged over ten different bending states as in [3] where we see a standard deviation of about five degrees. The uncertainty in phase of  $S_{21}$  of this cable after running the Correlation Buster is shown in Fig. 2(b) where we see an almost identical uncertainty at each frequency. However, the underlying uncertainties have a different covariance structure.

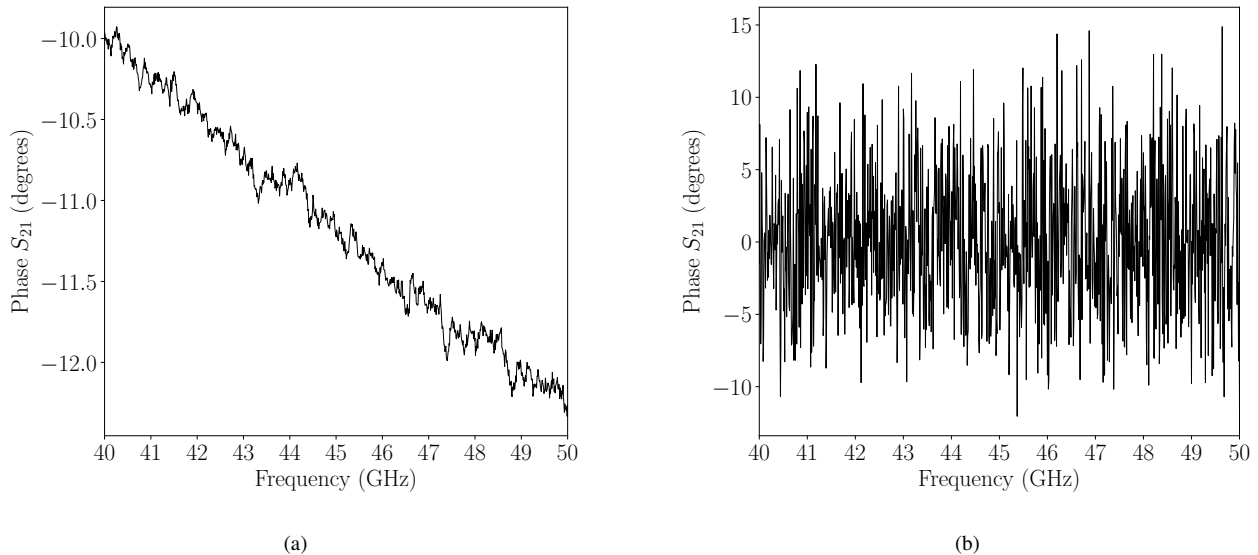


Fig. 3: A typical Monte Carlo replicate for the transmission ( $S_{21}$ ) of the cable for (a) the original MUF uncertainty analysis and (b) the MUF uncertainty analysis where the correlations have been scrambled using the Correlation Buster. We see that a linearly correlated trend in the original MUF analysis is resampled to provide a quasi-random phase variation by the Correlation Buster.

Although the total uncertainty is preserved, the cross-frequency correlations are modified by the Correlation Buster. We plot a representative Monte Carlo replicate from both the MUF correlated analysis and the scrambled analysis in Figs. 3(a) and (b) respectively. Here we see a linear trend as a function of frequency in the Monte Carlo replicate from the original analysis, while the replicate from the scrambled analysis has random phase errors. The linear frequency domain trend in Fig. 3(a) corresponds to a time offset in the time domain. The time shift of the signal in the EVM algorithm corrects for such a time offset. However, the distortion caused by the random phase errors in the scrambled analysis is not corrected.

This shows that the uncertainty analysis which preserves cross-frequency correlations can constrain the uncertainties so that the effect of time offsets can be corrected. Prescribing an *a priori* uncertainty bound without such constraints can introduce unphysical distortion and either underestimate (as is the case here) or overestimate the measured performance and uncertainty of a system.

## V. CONCLUSION

We have highlighted the importance of preserving correlations in the uncertainty analysis of microwave and millimeter-wave systems. We used the NIST MUF to reproduce the covariance-based correlated uncertainty analysis of a modulated signal as in [3]. We introduced a method which scrambles correlations while preserving overall uncertainty and introduced a tool in the MUF to create scrambled analyses.

We showed that the original correlated phase uncertainty in the transmission of a coaxial cable due to cable bending led

to a time offset which is typically corrected by time shifting in an EVM algorithm. However, scrambling these correlations led to distortion of the signal which was not corrected and significantly increased EVM and the associated uncertainty.

This analysis presented a clear example of why tracking correlated uncertainties is important and how correlated uncertainty analysis can significantly impact measures of system performance and uncertainty.

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