

# Problems with Stirred Immunity Measurements in a Reverberation Chamber

## Corrections to DO-160G Section 20.6

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**Abstract**—When DO-160G, Environmental Conditions and Test Procedures for Airborne Equipment [1], Section 20.6, was modified in 2010 to simplify electromagnetic susceptibility measurements in a reverberation chamber and switch from a stepped paddle (mode-stepped or mode-tuned) to a continuously moving paddle (mode-stirred), an error was introduced which overestimates the test level by approximately 1.4 to 3 dB, and possibly more (the error should increase with exposure frequency). We offer a possible explanation for the cause of the error, and suggest several possible ways to give a more accurate estimate of the exposure field.

**Keywords**— Airborne; field uniformity; immunity testing; mode-stirring; mode-tuning; reverberation chamber; susceptibility testing

### I. INTRODUCTION

When RTCA (Radio Technical Commission for Aeronautics) published version G of DO-160, Environmental Conditions and Test Procedures for Airborne Equipment, in 2010 [1], there were a few apparently minor changes to section 20.6. That section describes a reverberation chamber test method that can be used for electromagnetic susceptibility testing as an alternative to anechoic chamber methods. The principal change between DO-160F [2] (Rev. F) and DO-160G (Rev. G) relates to how metal paddles are moved during the measurement. The rotating paddles change the electromagnetic boundary conditions and essentially randomize the fields in the chamber. In Rev. F, the paddles were stepped to a small number (on the order of 12) of fixed positions. A stepped measurement is also referred to as “mode tuned”. The same number of paddle steps was required in both the chamber calibration and subsequent measurements, because the expected peak field generally increases with the number of paddle steps. In Rev. G, the paddles are moved continuously. This is also referred to as stirred or “mode stirred”. The change was made because stirred measurements are perceived to be faster, simpler, more thorough, more accurate, and provide higher test levels. In this paper, we will not address the validity of these perceptions, or the relative merits of the two methods. Instead, we focus on the validity of the procedures and equations as given in Rev. G.

To make the change from stepping to stirring, a new method for estimating the test field was required. In Rev. F, the field is determined based on the *average* power received by an antenna in the chamber and a correction factor (which is a function of the number of paddle steps used in the calibration and measurement). However, there is a requirement that this method gives results consistent with measurements based on field as determined by a field probe.

In Rev. G, the field is determined based solely on measurement of the *peak* power received by an antenna in the chamber. Field uniformity evaluations are still performed with probes, but there is no requirement that a comparison be made between results using a probe and results using received power. However, in a companion user guide [3], there is a recommendation that the chamber performance be verified using data taken during the uniformity evaluation. The new stirred approach makes a significant assumption that the peak total field is directly related to the square root of the peak received power. This assumption is incorrect (a reasonably good assumption is that the peak linear or Cartesian field is directly related to the square root of the peak received power [4], but not the peak total field). This can result in undertesting by 1.4 to 3 dB, and possibly more. It is possible that such errors are considered to be within typical uncertainty bounds of immunity testing, but should be corrected to avoid a consistent bias resulting in undertesting of all systems.

In this paper, we discuss these results in greater detail. In Section II we give a summary of the electromagnetic and statistical theory of reverberation chambers and explain the error of the assumption above. Section III discusses possible remedies for these problems, followed by our conclusions.

### II. BACKGROUND

The fields in a reverberation chamber are best described with respect to a three-axis field probe with three electrically small orthogonal dipoles. The individual dipoles measure a linear or Cartesian component of the electric field at a given point inside the chamber. These components are generally referred to as the x-, y-, and z-components of the electric field for an arbitrary Cartesian coordinate system inside the chamber. The *total* electric field is computed by taking the

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square root of the sum of the squared magnitudes (RSS) of the three individual linear components.

Assuming a fixed static paddle configuration, the fields should also be static (neglecting vibrations or thermal changes in the chamber or paddles). If the paddle position is changed by a very small amount (fractions of a degree to a few degrees, depending on frequency), the measured field will be essentially unchanged and can be estimated reasonably well from measurements at the previous position. If the paddle is moved sufficiently, the field can change significantly (tens of dB) and the new sample is effectively uncorrelated with the previous sample. Procedures for estimating the minimum rotational distance  $\Delta\phi_{\min}$  required to generate uncorrelated samples are still being developed due to the complexity of the problem [5], but rough estimates are available. If the samples are approximately uncorrelated, it is assumed that they can be treated as if they are independent. Any step size greater than or equal to  $\Delta\phi_{\min}$  is assumed to yield independent samples. The number of possible independent samples  $M_{ind}$  that can be generated by a single paddle is  $M_{ind}=360^\circ/\Delta\phi_{\min}$ .

The linear field components are assumed to be the sum of many plane-waves, rays, or modes, each with “randomized” amplitudes, phases, path lengths, etc. which implies that a central limit argument can be used to describe the fields, with the real and imaginary components of  $E_x$ ,  $E_y$ , and  $E_z$  each independent and normally distributed with 0 mean and identical variances  $\sigma^2$ . From this, the squared magnitude of any rectangular component, which we will refer to generically as  $E_{Rect}$ , can be described as the sum of the squares of two iid (independent and identically distributed) zero-mean Gaussian random variables, so therefore has a  $\chi_2^2$  or chi-squared distribution with two degrees of freedom, also known as an exponential distribution. Similarly, the squared magnitude of the total electric field  $E_T$  can be described as the sum of the squares of six iid zero-mean Gaussian random variables so therefore has a  $\chi_6^2$  or chi-squared distribution with six degrees of freedom. We can take the square root of the results, giving the magnitude of a rectangular component or total field as having a  $\chi_2$  or  $\chi_6$  (chi distribution with two or six degrees of freedom), respectively. These results are assumed to be independent of position within the working volume of the chamber or the orientation of the Cartesian coordinate system used for describing  $E_x$ ,  $E_y$ , and  $E_z$ . Measurements have been consistent with these assumptions [4].

Hill has proposed a plane-wave-spectrum model for describing the fields inside a reverberation chamber [6] that requires fewer assumptions. Two important results are related to the power  $P_{Rec}$  received by a perfectly matched lossless antenna. First,  $P_{Rec}$  should have a  $\chi_2^2$  distribution (the same as the squared magnitude of a rectangular component of the electric field), and second, a relationship between average squared magnitude of the total electric field  $|E_{Total}|^2$  and the average power is

$$\langle P_{Rec} \rangle = \frac{1}{2} \frac{\langle |E_{Total}|^2 \rangle \lambda^2}{\eta_0 4\pi} = \frac{\langle |E_{Total}|^2 \rangle \lambda^2}{8\pi\eta_0}, \quad (1)$$

where  $\lambda$  is the wavelength of the excitation frequency in free space,  $\eta_0$  is the wave impedance of free space, and  $\langle \cdot \rangle$

represents the expected value or ensemble average over all paddle positions. Equation (1) can be rearranged to give  $\langle |E_{Total}|^2 \rangle$  as a function of  $\langle P_{Rec} \rangle$ ,

$$\langle |E_{Total}|^2 \rangle = \frac{\eta_0 8\pi \langle P_{Rec} \rangle}{\lambda^2} \approx \frac{377 \cdot 8\pi \langle P_{Rec} \rangle}{\lambda^2}, \quad (2)$$

where the approximation  $\eta_0 \approx 377$  is used. Equation (2) is useful because measurements of received power are generally easier, faster, and more accurate, than measurements of electric field. However, for the susceptibility tests in DO-160, we need the maximum total electric field, not the average squared magnitude of the field. It is tempting to take the square root of both sides of (2) to get field instead of squared field, and to replace the average measured received power with the maximum measured received power to estimate the maximum fields, and it appears that this was done to derive the equation in section 20.6.2.b.iii of Rev. G. Unfortunately, both steps are incorrect.

As shown in [4], order of operations is important, and the square root of the average of a quantity is not the same as the average of the square root. Specifically,  $\sqrt{\langle |E_{Total}|^2 \rangle} \neq \langle |E_{Total}| \rangle$ . This error is relatively small, and taking  $\sqrt{\langle |E_{Total}|^2 \rangle}$  overestimates  $\langle |E_{Total}| \rangle$  by about 0.36 dB. A more significant error is introduced by assuming that, if an equation gives a valid relationship for the expected values of two random variables, then the same relationship holds for the extreme values of those random variables. This may be a reasonably good approximation *if both sides have the same statistical distribution*. In this case, however, the square root of received power has a  $\chi_2$  distribution, and the total field has a  $\chi_6$  distribution. The equation in Rev. G essentially assumes that the expected maximum total field is simply  $\sqrt{3}$  times the expected maximum of a rectangular component of the field. Unfortunately, the relationship is not that simple. The expected maximums increase with the number of independent samples for each parameter, but the expected maximum of the total field increases more slowly than the expected maximum of a rectangular component. This means that the equation given in Rev. G overestimates the total field in all cases where  $M_{ind} > 1$ , and this overestimate increases with  $M_{ind}$ . To estimate the bias, we can take the mean values from Table 7 of [4] and compare it to  $\sqrt{3}$  times mean values from Table 4 of [4]. The mean values from those tables, along with the resulting bias, are summarized in Table I for a few select values of  $M_{ind}$ . For larger values of  $M_{ind}$ , the bias will continue to increase, but by only a few tenths of a dB per order-of-magnitude increase in  $M_{ind}$ .

TABLE I. MEASUREMENT BIAS

$M_{ind}$	Normalized Expected Max Rect. Field	Normalized Expected Max Total Field	Bias (dB)
10	2.370	3.472	1.5
100	3.199	4.253	2.3
1,000	3.857	4.869	2.7
10,000	4.415	5.394	3.0

A better approach is to give a relationship between received power and a squared rectangular component of the field. Assuming an isotropic environment, the squared total field is simply the sum of the three rectangular components,  $\langle |E_{Total}|^2 \rangle = 3\langle |E_{Rect}|^2 \rangle$ , so

$$\langle |E_{Rect}|^2 \rangle = \frac{\eta_0 8\pi \langle P_{Rec} \rangle}{3\lambda^2} = \frac{120\pi \cdot 8\pi \langle P_{Rec} \rangle}{3\lambda^2}, \quad (3)$$

where we use  $\eta_0 = 120\pi$ . Here, both sides have the same expected value, and also have the same distribution, so we can now say

$$|E_{RectMax}|^2 \cong \frac{120\pi \cdot 8\pi P_{RecMax}}{3\lambda^2}, \quad (4)$$

where we use  $\cong$  to indicate that this relationship is somewhat loose. Measurements of  $P_{RecMax}$  do not tell us the maximum field at the antenna or anywhere else in the chamber. It is more precise to say that measurements of  $P_{RecMax}$  allow us to estimate  $|E_{RectMax}|^2$ . A better interpretation is that the terms on either side of the  $\cong$  in (4) have essentially the same statistical distribution. Taking the square root of both sides of (4) and manipulating the result gives

$$|E_{RectMax}| \cong \frac{8\pi}{\lambda} \sqrt{5P_{RecMax}}. \quad (5)$$

In deriving (4) and (5), we avoided the order-of-operations problem where  $\sqrt{\langle |E_{Rect}|^2 \rangle} \neq \langle |E_{Rect}| \rangle$  by first switching from average to maximum, and then computing the square root, since  $\sqrt{|E_{RectMax}|^2} = |E_{RectMax}|$ .

The approximation given in (5) is not the best estimate of maximum field, since measurements of maximum received power can have relatively large uncertainties. A better approach is to estimate the maximum field based on the average received power and the number of independent measurements based on the equations and tables given in [4].

Equation (5) should be used with caution at lower frequencies (less than about 1 GHz), since the statistical link between received power and field can get distorted if chamber losses are very low [4]. For example, with 10,000 independent samples, maximum received power should be approximately 10 dB greater than the average received power, but if the losses in the chamber are much less than 10 dB, then the statistical model for received power is trumped by conservation of power and the power distribution will be compressed. For most stepped measurements with small numbers of samples, this problem will not be significant. For large numbers of samples, however, this can lead to a significant underestimate of the field

It is possible to generate an equation similar to (5) for the total field, but this requires simultaneous measurements of the power received from three different antennas and summed together (possibly using a calibrated power combiner). If we refer to the power received by a single antenna generically as  $P_{Rec}$  and the summed power as  $P_{Total}$ , we have  $\langle P_{Total} \rangle = 3\langle P_{Rec} \rangle$ , so this gives us

$$\langle |E_{Total}|^2 \rangle = \frac{\eta_0 8\pi \langle P_{Total} \rangle}{3\lambda^2} = \frac{120\pi \cdot 8\pi \langle P_{Total} \rangle}{3\lambda^2}, \quad (6)$$

which is the same equation as (3), but with  $P_{Total}$  and  $E_{Total}$ , instead of  $P_{Rec}$  and  $E_{Rect}$ . Since  $P_{Total}$  and  $E_{Total}$  have the same distribution, we can also write

$$|E_{TotalMax}| \cong \frac{8\pi}{\lambda} \sqrt{5P_{TotalMax}}. \quad (7)$$

This approach should be verified experimentally, but this is beyond the scope of this paper.

Equations (5) and (7) should be useable for stirred measurements as well as stepped measurements, with the caveats mentioned above for both increased uncertainty due to estimating field based on maximum power rather than average, and also the compression issue at lower frequencies. However, this should also be verified experimentally before it is used in practice.

### III. REMEDIES

Given the error described here, there are severable possible ways to address it, each with different degrees of expense, difficulty, and accuracy. None of the remedies are likely to be popular, since they all require more power for performing the tests. The simplest approach is to just ignore the issue and accept the error as being within typical uncertainty bounds of an immunity test. However, regulators are unlikely to accept this option, because it gives a consistent bias to the tests. An equally simple remedy is to decrease the claimed test level by a fixed correction factor. Since the bias tends to increase with frequency, a correction factor consistent with a large number of independent samples, such as 2.7 to 3 dB is recommended. Another possibility is to modify the standard so that the test level is defined in terms of  $E_{Rect}$  rather than  $E_{Total}$  since  $E_{Rect}$  is much more closely related to the square root of the received power than  $E_{Total}$ . This has the significant advantage of making DO-160 consistent with the reverberation chamber test standard IEC 61000-4-21 [7], but will increase power requirements for the test even beyond the 2.7 to 3 dB suggested above. Finally, the power from three antennas could be combined to give  $P_{Total}$  as described above. This has the advantage of leaving the standard mostly untouched except for some notation and an explanation on how to combine the signals and calibrate whatever power combining option is used. Of all of these options, the most accurate and defensible are the last two, switching the type of electric field used in the measurement from  $E_{Total}$  to  $E_{Rect}$  or using a calibrated power combiner to determine  $P_{Total}$ . However, whatever method is chosen, it should be verified experimentally before being accepted in the standard.

### IV. CONCLUSION

We have shown that RTCA DO-160G, section 20.6 has an error resulting in a consistent overestimate of the test field. The test field is generally overestimated by 1.4 to 3 dB. We also pointed out that measurements based on maximum received power can have larger uncertainties than measurements based on average received power, and that there are potential problems below 1 GHz due to power

compression. We presented several different methods for addressing the error, from ignoring it to changing the measurement to give better results.

#### V. REFERENCES

- [1] Environmental Conditions and Test Procedures for Airborne Equipment, RTCA/DO-160G, RTCA Inc. December 8, 2010.
- [2] Environmental Conditions and Test Procedures for Airborne Equipment, RTCA/DO-160G, RTCA Inc. December 6, 2007.
- [3] User Guide Supplement to DO-160G, RTCA Inc. December 16, 2014.
- [4] Ladbury J M, Koepke G H, Camell D G, Evaluation of the NASA Langley research center mode-stirred chamber facility: Technical Note 1508, National Institute of Standards and Technology, Gaithersburg, Maryland USA, 1999
- [5] Pfennig S, "A General Method for Determining the Independent Stirrer Positions in Reverberation Chambers: Adjusting the Correlation Threshold." IEEE Trans. Electromagn. Compat., vol. 58, no. 4, pp. 1252–1258, July, 2016.
- [6] D. Hill, "Plane wave integral representation for fields in reverberation chambers", IEEE Trans. Electromagn. Compat., vol. 40, no. 3, pp. 209–217, Nov. 1998.
- [7] IEC 61000-4-21, Electromagnetic compatibility (EMC): Testing and Measurement Techniques - Reverberation Chamber Test Methods, 2003.