Scaling of Mass and Force Using Electrical Metrology

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***Abstract* — The impending redefinition of the International System of Units (SI) allows realization of the kilogram using electrical metrology. It can be shown that electrical metrology can also be used to scale mass using electrostatic forces. An approach to mass metrology at the milligram scale using an Electrostatic Force Balance (EFB) is described. The relative uncertainty in EFB mass measurements is found to be substantially lower than those obtained with the current method based on subdivision of the artifact kilogram.**

***Index Terms* — capacitance, force, metrology, standards, voltage. .**

I. Introduction

In the near future, the redefinition of the International System of Units (SI) will allow realization of the base units from fundamental physical constants. One implication of this change is that the kilogram can be realized using electrical metrology, as in the Kibble balance. The kilogram is only a single value of mass, and must still be scaled if calibrations are to be obtained for smaller masses. The traditional methods for accomplishing this involve subdivision from a kilogram mass artifact [1]. As mass is subdivided to smaller values, however, relative uncertainty increases. This problem can be circumvented if mass is realized directly at the scale of interest. One approach to doing this involves the use of an electrostatic force balance (EFB) that uses electrical metrology to determine force, F, from

$$F = \frac{1}{2}\frac{dC}{dz}(V - V_s) \qquad (1)$$

where C is capacitance between two electrically separate conductors, z is separation between the conductors, V is applied voltage between the conductors, and V_s is a surface potential due to patch effect or adsorbed contaminants. Each of the terms in Eq. 1 can be derived from fundamental physical constants. Voltage from a Josephson junction array calibration, C from a quantum Hall device or a calculable capacitor standard calibration, and z based on the frequency of the light used in a laser interferometer measuring displacement. As a result, it can be shown that the electrostatic force from a capacitor and the gravitational force from a mass artifact can be compared on the EFB, and the mass value determined from a measurement of electrostatic force and local gravitational acceleration [2]. The mass value determined from electrostatics can be shown to arise from Planck's constant in the same fashion as the Kibble (Watt) balance, and a comparison with mass derived from kilogram subdivision shows the two methods yield identical results, within the uncertainty of the measurement at the milligram scale. The EFB method is therefore compatible with the SI in both its current and redefined forms, and a special test at the U.S. National Institute of Standards and Technology (NIST) has been established to disseminate the measurement.

II. Experimental Methods

The EFB is an electromechanical balance mechanism that uses a 4-bar linkage to rigidly translate the inner cylinder of a concentric cylinder capacitor through a well-defined trajectory. The electrostatic force measurement used to determine mass is carried out in two steps.

In the first step, the inner capacitor cylinder is translated to fixed points along the trajectory defined by the 4-bar linkage by applying a force to an auxiliary electrode on the opposite side of the balance from the main electrode. At each point the relative displacement is monitored using the laser interferometer, and the capacitance between the inner and outer electrodes measured using a 3-terminal capacitance bridge. This data allows determination of the capacitance gradient, dC/dz, in Eq. 1. Although the motion is aligned to be nearly collinear with the gravitational acceleration vector, it is more accurately characterized as arcuate motion. This is an important consideration, as it introduces higher-order nonlinearities into the capacitance gradient as described in previous work [3]. The mass measurement is carried out at a null position so higher order terms in the capacitance gradient drop out, but their influence on the first order term is important to characterize.

In the second step, a null position is held by applying a voltage to the main electrode while masses are placed onto and removed from the balance. The null position is held using a digital servo based on feedback from the laser interferometer position measurement. The change in applied voltage yields the weight of the mass artifact. The measurement is performed at both positive and negative voltage polarity to subtract the effect of V_s. All measurements are performed at vacuum using an automated mass exchange system. Further information on balance alignment, traceability, measurement methods, and uncertainty analysis can be found elsewhere [2].

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systematic uncertainties dominate which are approximately independent of scale. Because the EFB has a finite force resolution, statistical noise becomes more important at lower mass.

Uncertainty components	Relative Magnitude (k=1)
Type B uncertainties	1 mg artifact
transfer of length	$1x10^{-7}$
transfer of Voltage	$4.0x10^{-6}$
transfer of capacitance	$1.2x10^{-7}$
alignment of capacitor cylinders	$1.2x10^{-7}$
corner loading	$6.0x10^{-7}$
stray capacitance	$8.2x10^{-7}$
hysteresis	$1.4x10^{-6}$
alignment of balance travel	$7.3x10^{-7}$
temperature dependence of d*C*/d*z*	$4.4x10^{-6}$
Type A uncertainties	
Weighing	$3.8x10^{-6}$
Combined relative standard uncertainty	$7.3x10^{-6}$
Mass Value (mg)	0.9983245(73)

Table 1. Uncertainty components in EFB mass measurement [2].

III. Results

The measurement uncertainties produced by the NIST EFB mass measurements are shown in Fig. 1 for the mass range of 50 micrograms to 20 milligrams in comparison to those available with kilogram subdivision. The trendline plotted extrapolates information from currently available calibration data to estimate the likely uncertainties achievable for mass less than 1 milligram. The EFB mass measurements exhibit uncertainties 1 to 2 orders of magnitude lower than the current subdivision methods, and comparisons between the two methods at the 1 milligram and 20 milligram level show the two methods have produced results with a relative difference of $5x10^{-5}$ or less over the course of 10 years, despite multiple changes in operators and instrumentation [2].

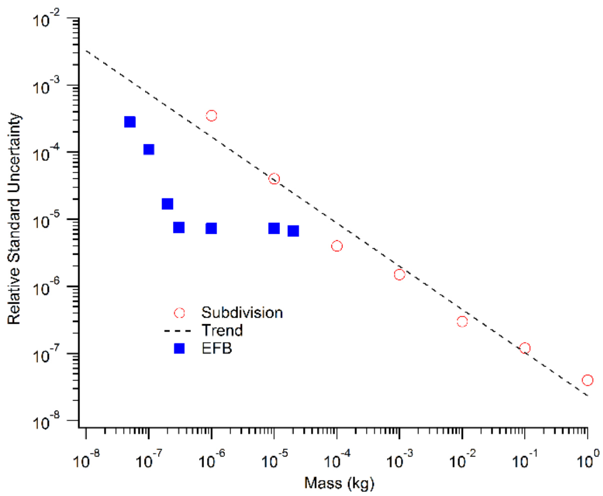

Fig. 1. Relative combined standard uncertainty of EFB and subdivision mass measurements. The dashed line is a linear fit to the subdivision data to estimate uncertainty less than 1 milligram for that method.

A representative uncertainty analysis for the EFB weighing of a 1 milligram mass artifact is shown in Table 1. Currently, the largest contributor to the uncertainty is the temperature dependence of the capacitance gradient, which is caused by thermal expansion of the balance mechanism. An additional correction for surface adsorbed water layers is given in other work [4], and has been verified to provide an accurate correction by measurement of a 10 milligram stainless steel mass artifact.

Figure 1 shows a distinct discontinuity where the relative uncertainty increases below 500 micrograms. This can be understood as the mass value at which the balance's statistical (type A) uncertainty becomes dominant. Above this level, systematic uncertainties dominate which are approximately independent of scale.

IV. Summary

The EFB method for mass measurement uses electrical metrology to scale the unit of mass at milligram to microgram scales. The method is compatible with both current SI mass methods based on subdivision of the kilogram artifact and with the redefined SI based on the value of Planck's constant. Scaling mass in this fashion allows for a reduction in mass uncertainty of 1 to 2 orders of magnitude compared to kilogram subdivision.

References

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