Persistence of Electromagnetic Units in Magnetism

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***Abstract* — The centimeter-gram-second system of electromagnetic units (EMU) has been used in magnetism since the late 19th century. The International System of Units (SI), a successor to Giorgi's 1901 meter-kilogram-second system, was adopted by the General Conference on Weights and Measures in 1960. However, EMU remains in common use for the expression of magnetic data. The forthcoming revision of the SI provides an excellent reason for magnetics researchers to abandon the use of EMU.**

***Index Terms* — International System of Units, electromagnetic units, permeability of vacuum.**

I. Introduction

The International System of Units (SI), established in 1960 by the General Conference on Weights and Measures, has been generally accepted by researchers in most scientific disciplines. Magnetics is a notable exception, where the centimeter-gram-second (CGS) system of electromagnetic units (EMU), as formulated by Maxwell in 1873, remains in common use. The coexistence of SI and EMU in magnetics, and the conversion from one to the other, has been a source of confusion and error for students and practitioners.

Although the use of SI in magnetics has some inconveniences, they are minor compared to the ambiguities in the EMU system. The case for the SI remains compelling for the reasons first articulated by Giovanni Giorgi at the beginning of the 20th century. Importantly, the expected revision of the SI will make it incompatible with EMU.

II. Advantages and Disadvantages of SI and EMU

One of the advantages of SI is that it unifies magnetic and electrical units, whereas CGS bifurcates into EMU and electrostatic units (ESU). A possible disadvantage in SI is that two constitutive relations are recognized: $B = \mu_0(H + M)$, the Sommerfeld convention, where B is magnetic flux density, μ_0 is the permeability of vacuum, H is magnetic field strength, and M is the magnetization; and $B = \mu_0 H + J$, the Kennelly convention, where J is the magnetic polarization. Because M and J have different units, confusion is averted.

A disadvantage of SI is that the units for H, amperes per meter, are too small. Researchers have the urge to use tesla for H (which they could use if they instead wrote $\mu_0 H$), or they mistakenly refer to B instead of H.

Turning to EMU, the disadvantages are more serious. The unit for magnetic moment m is often expressed as "emu"; however, "emu" is not a unit, but is simply an indicator of electromagnetic units. The actual units for m are ergs per gauss or ergs per oersted.

As in SI, volume susceptibility is dimensionless, but its value in EMU is smaller than in SI. It may be appreciated that values of volume susceptibility, reported in the literature as dimensionless in both SI and EMU, might be difficult to compare. In EMU, volume susceptibility is often expressed in "emu," "emu per cubic centimeter," or "emu per cubic centimeter per oersted," a state of confusion originating from the misuse of "emu" as the unit for magnetic moment.

Magnetization (magnetic moment per unit volume) is commonly expressed either as M in "emu per cubic centimeter" or as $4\pi M$ in units of gauss. They are dimensionally equivalent, but they differ numerically by 4π. This double definition often leads to misunderstandings and mistakes.

Care is required when electrical and magnetic quantities are combined: the EMU unit of current is the abampere. Some researchers are reluctant to abandon the ampere and use mixed units in equations that do not balance dimensionally.

III. The Permeability of Vacuum in the Revised SI

The forthcoming revision of the SI [1], in which fixed values will be assigned to the Planck constant h and the elementary charge e [2], will accentuate the philosophical differences between EMU and SI: μ_0 is fixed at unity in the former but will be measurable, in principle, in the latter [3], and quantities will no longer be strictly convertible by factors of 4π and powers of 10 [4].

In the revised SI, μ_0 will be derived from fixed constants h, e, and the speed of light c, and the experimentally determined fine structure constant α [3]:

$$(\mu_0)_{\text{experimental}} = (2h/ce^2)_{\text{fixed}} \cdot (\alpha)_{\text{experimental}} \, . \tag{1}$$

The value of μ_0, initially equal $4\pi \times 10^{-7}$ H/m to 9 significant figures, may change slightly over time as better measurements of α are made [3]. Magnetics researchers will have to choose to work and publish in one of two incompatible systems: EMU, which has a long tradition, or SI, which unifies all metrology and which has been adopted by international convention.

Of course, the adoption of SI by magneticians accustomed to working in EMU will require not only relatively straightforward conversions of units, but the conversion of equations (e.g., insertions and deletions of μ_0 and 4π).

Fortunately, it is much easier to verify the dimensional consistency of equations in SI than in EMU.

IV. Giorgi's Rationalized MKS System

The demotion of μ_0 from its immutable value of $4\pi \times 10^{-7}$ H/m within the SI might seem to violate the sanctity of a fixed constant. However, when a rationalized, four-dimensional system was first proposed by Giorgi [5], both μ_0 and the permittivity of vacuum ε_0 were regarded as subject to experimental refinement, with $\mu_0 \approx 1.256 \times 10^{-6}$ H/m and $\varepsilon_0 \approx 8.842 \times 10^{-12}$ F/m, "free from any unnecessary 4π," and both subject to the condition $(\mu_0 \varepsilon_0)^{-1/2} = c \approx 3 \times 10^8$ m/s [6].

That same condition applies in EMU, with $\mu_0 = 1$ and $\varepsilon_0 = c^{-2}$, and in ESU, with $\mu_0 = c^{-2}$ and $\varepsilon_0 = 1$. In 1905, Giorgi noted that his rationalized system "is neither electrostatic nor electromagnetic, because neither the electric nor the magnetic constant of free ether [vacuum] is assumed as a fundamental unit" [6].

V. Is *B* the Same as *H* in Vacuum?

One of the appealing aspects of EMU is that, in vacuum, *B* is equal to *H* in value and dimensions, despite their different unit names (gauss and oersted). Whether the fields *B* and *H* in vacuum are physically the same in EMU (and in the present SI) is historically controversial [7].

In the present SI, *B* and *H* in vacuum are mutually convertible through the fixed constant μ_0 despite their different values and dimensions. This, too, is appealing. It is similar to the conversion of *B* and *H* in ESU (if one assumes *c* is a fixed constant in CGS; no one really knows because no international standards organization maintains the CGS system).

In the revised SI, *B* and *H* will not be similarly convertible. In vacuum, one could, in principle, measure either *B* or *H,* depending on the experiment, and calculate its counterpart using the most recent value of μ_0. Or one could, in principle (if not in reality), measure both *B* and *H* and calculate a new value of μ_0. Thus, it seems implicit in the revised SI that *B* and *H* in vacuum are physically different.

Before the value of *c* was fixed in the SI in 1983, the same considerations applied to (1) the electric flux density (electric displacement) *D* and the electric field strength *E* in SI and (2) *B* and *H* in ESU: they were mutually convertible through ε_0, a constant whose value depended on the measured value of *c.*

VI. Conclusion

In introducing his rationalized meter-kilogram-second system in 1901 [5], which later evolved into the meter-kilogram-second-ohm [6] and then the meter-kilogram-second-ampere systems, Giorgi wrote, *"Il sistema CGS, con questo, perde ogni ragione di esistere; ma non credo che il suo abbandono sarà lamentato da alcuno."* ("With this, the CGS system loses every reason to exist; but I do not think that its abandonment will be lamented by anyone.")

He may be correct, eventually.

References

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