Portable High-Frequency Electrodynamic Force Standards

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***Abstract* — We describe high-frequency electrodynamic force standards. Such standards can be calibrated voice coils, or high-frequency Kibble (watt) balances. We discuss the design of such devices and present example operating parameters for a 10 N Kibble balance force standard designed for operation to frequencies up to 20 kHz.**

***Index Terms* — dynamic force, electrodynamic, force calibration, force metrology, force standard, kibble balance, watt balance.**

I. INTRODUCTION

Accurate dynamic force standards are required for dynamic calibration of force transducers and for characterizing mechanical systems more broadly (e.g., modal testing and model validation). In many force measurement applications, the force is time-varying, such as in automobile crash testing, fatigue testing, and measurements of machining forces. The frequency ranges of interest wherein dynamic calibration is called for varies greatly among force measurement applications; the range up to approximately 30 kHz covers the majority of industrial applications. There are a number of different relations upon which a primary force standard might be based. Perhaps the most commonly used is Newton's 2nd law, $\boldsymbol{F}(t) = \int \rho(\boldsymbol{r})\boldsymbol{a}(\boldsymbol{r},t)d^3\boldsymbol{r}$, where $\boldsymbol{F}$ is the force, ρ is mass density, $\boldsymbol{a}$ is acceleration and $\boldsymbol{r}$ and t are space and time coordinates respectively. We describe here primary dynamic force standards based on the Lorentz force law.

In dynamic force metrology, a challenge is that the transducer is often required to be integrated into a host mechanical setting, which can greatly affect the dynamic response of the transducer. In other words, the dynamic response responsible for the transducer output signal is a property of the coupled transducer-host system. This is exemplified by the fact that while force transducers with a lowest resonance frequency of several tens of kilohertz are available, this lowest resonance frequency may be shifted down to 1 kilohertz or below in an application setting. A solution to this challenge is *in-situ* calibration, that is, using a portable standard to calibrate the integrated transducer-host system as assembled for the application. The standards we describe here are designed for such *in-situ* calibration: traceable sources that can be taken to the application setting for calibrating the force measurement system, or built into the application apparatus. They are more broadly useful for providing known dynamic forces in characterizing mechanical systems.

II. ELECTRODYNAMIC DYNAMIC FORCE STANDARDS

A. Calibrated voice-coil actuator

The Lorentz force law applied to a current flowing in a wire yields

$$\boldsymbol{F} = -I \oint_L \boldsymbol{B}(\boldsymbol{r}) \times \boldsymbol{dl}(\boldsymbol{r})\, d^3\boldsymbol{r} \;\equiv -I\, \boldsymbol{\beta}, \qquad (1)$$

where I is the current flowing in the wire of length L and differential length element $\boldsymbol{dl}$, immersed in magnetic field of flux density $\boldsymbol{B}$. The wire is commonly in the form of a coil, attached to a support cylinder, and immersed in a radial magnetic field. Current flowing in the coil results in an axial force, transmitted to external structures connected to (or in contact with) the armature structure (coil & support). This is well established as a method of static and low frequency force measurement. The force determined according to (1) will deviate from the true force exerted by the actuator due to the inertia of the armature and to parasitic forces on the armature due e.g. to supports, wires and air drag.

For a high-frequency dynamic force standard, such an actuator should be designed with a low-mass high-stiffness armature, and with minimal parasitic forces. Transverse translation and angular motions of the coil in the magnetic field will be significant sources of uncertainty, and must be minimized. A stiff air- or magnetic-bearing guide for the armature will help. The uniformity of the magnetic field in the region of the coils should be maximized.

A practical method of determining the integral $\boldsymbol{\beta}$ is to calibrate it as parameter by measuring the static force generated by a DC coil current. Any time variation of this parameter, for example due to change of the magnetization with temperature, will contribute to the uncertainty budget.

B. High-frequency Kibble balance

An alternative design is a high-frequency Kibble (or watt) balance. The value of $\boldsymbol{\beta}$ is related to the induced voltage U generated across the wire when it is moved at velocity $\boldsymbol{V}$ through the magnetic field [1]

$$U = \boldsymbol{V} \cdot \oint_L \boldsymbol{B}(\boldsymbol{r}) \times \boldsymbol{dl}(\boldsymbol{r}) d^3\boldsymbol{r} \equiv \boldsymbol{V} \cdot \boldsymbol{\beta}. \qquad (2)$$

Thus the force amplitude is derived from (1) and (2) as

$$F = UI/(V\cos\theta)\,, \qquad (3)$$

where θ is the (nominally-zero) angle between the velocity vector and $\boldsymbol{\beta}$, and all quantities are time-dependent. This

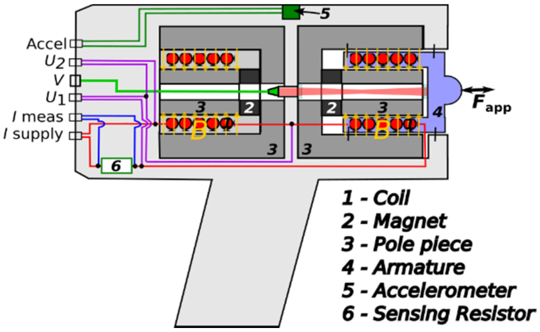

Fig. 1. A design for a hand-held portable Kibble balance type dynamic force standard.

measurement can be done simultaneously with the force application and current measurement, similar to the Kibble balance design in [2]. The uncertainty due to motion of the armature within the magnetic field and due to time variation of $\boldsymbol{\beta}$ are thereby attenuated. In such a simultaneous measurement scheme large voltage drops due to the coil resistance and inductance are superposed on the induced voltage U. A dual coil differential design with nominally identical coils and matched magnetic fields allows cancellation of most of the inductive and resistive voltage drops. Such a design is shown in Fig. 1. For this design the induced voltage is related to the measured voltage by

$$U = U_{\text{meas}} - I\,\Delta Z, \tag{4}$$

where ΔZ is the (frequency-dependent) impedance difference between the two coils. This difference can be measured by holding coil 1 fixed (in addition to coil 2) so that $U = 0$ V, for which case $\Delta Z = U_{\text{meas}}/I$. A bifilar coil, such as described in [3], would eliminate the resistive voltage drop but not the inductive voltage drop.

In the design shown, current is measured using a calibrated sensing resistor, voltage is measured using a calibrated external voltmeter, and velocity is measured using a fiber-coupled Fabry-Pérot interferometer. Additionally, an accelerometer is used to measure the absolute body acceleration, which in turn is used in calculating the absolute armature acceleration and thereby the correction for its inertia. The device is designed for applying the force to an exposed location on the target system. In Table 1 we give design parameters and operating characteristics of a 10 N device.

The uncertainty of the force values generated by the device depend on the values of dynamic variables that occur during use and thus have some dependence on the target system to which the force is being applied. An uncertainty evaluation [4] based on a prototypical target system shows that uncertainties below 1 % can be realized up to frequencies exceeding 10 kHz, excluding frequencies in the near vicinity of antiresonances. At these antiresonant frequencies the uncertainty can be improved by the approach of determining $\boldsymbol{\beta}$ from the measurements of current, velocity and voltage at other frequencies and then using (1) to determine the force.

Table 1. Device parameter values and operating dynamic variables, over the frequency range 10 Hz to 10 kHz.

Quantity	**Value**
DEVICE PARAMETERS	
$\boldsymbol{\beta}$ Integral	25 N/A
Armature Mass	41 g
Body Mass	1.5 kg
Armature Stiffness	1.7×10^{8} N/m
Armature Resonant Frequency	9.4 kHz
Flexure Stiffness	3×10^{4} N/m
Current Sense Resistance	10 Ω
Coil Wire Resistance (R)	17 Ω
Resistance Difference Between 2 Coils	$0.005\,R = 0.085$ Ω
Coil Inductance ($\mathcal{L}$)	1×10^{-5} H $+ (2\pi/\omega)^{0.6}\ 0.05$ H·s
Inductance Difference Between 2 Coils	$0.01 \times \mathcal{L}$
Mechanical Damping Ratio	0.01
DYNAMIC VARIABLES	
Deployment Force Amplitude (F)	10 N
Current Amplitude (I)	0.4 A
Body Acceleration Amplitude	0.33 m/s^2
Armature-Body Rel. Velocity Ampl. (V)	4.4×10^{-6} m/s to 0.11 m/s
Armature-Body Rel. Displacement Ampl.	4.6×10^{-10} m to 8.3×10^{-5} m
Armature Back EMF (U)	1.1×10^{-4} V to 2.8 V
Total Armature Voltage Drop (U_{meas})	0.034 V to 2.8 V
Resistive Voltage Amplitude Per Coil	0.68 V
Differential Resistive Voltage Drop Ampl.	0.034 V
Current Sense Voltage Amplitude	4 V
Inductive Voltage Amplitude Per Coil	0.1 V
Differential Inductive Voltage Drop Ampl.	1.3×10^{-3} V to 0.06 V

III. CONCLUDING REMARKS

We are undertaking assembly and testing of devices such as those described here. We note that for the purposes of calibrating force transducers, an alternative to the use of such a device as a separate standard can be to integrate the standard into the transducer. Lastly, although we have focused on macroscopic forces up to kilohertz frequencies, such devices can be fashioned for higher frequencies and smaller forces, with a more or less inverse relationship between frequency range and force amplitude.

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