Design of a cell-based refractometer with small end-effects

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***Abstract*—In cell-based laser refractometers, interferometer pathlength uncertainty introduced by deformation and stress in the windows through which the beams pass can be the chief factor limiting measurement accuracy. The fractional contribution of pathlength uncertainty to our recent determination of the Boltzmann constant was 9.8×10^{-6}, and more than two times larger than the next largest uncertainty component. We briefly describe the error, and propose a design in which cell window effects contribute less than 3×10^{-6} fractional error to the measurement of helium refractivity; performance that would be competitive with state-of-the-art primary thermometry and barometry.**

***Index Terms*—Interferometry, length metrology, refractive-index gas thermometry.**

I. Introduction

In a recent experiment [1], we used a cell-based laser refractometer called MIRE to determine the Boltzmann constant with 12.5×10^{-6} relative standard uncertainty, via measurements of helium refractivity [2] and the equation of state. Error arising from changes in window pathlength, induced by deformation and stress caused by the change in gas pressure, proved a debilitating uncertainty component, and produced a fractional error of 580×10^{-6} in our 25 cm-long cell. We attempted to cancel the error by measuring helium refractivity in cells of different lengths, and with almost identical material properties, window geometry, and location of beam transmission through all pairs of windows (i.e., cancellation of a common-mode error). After cancellation, the error contribution (9.8×10^{-6}, fractional) remained more than two times larger than the uncertainty in pressure and temperature measurements of the helium gas.

An additional drawback of the MIRE was the choice of crown glass for windows and spacer. This glass was chosen mainly for ease of manufacture, but its high and relatively unknown coefficient of thermal expansion meant that accurate refractivity measurements could only be carried out close to the temperature at which the cell length was measured (i.e., 20 °C). Thus, the uncertainty in thermodynamic temperature was the second largest uncertainty component in our determination of the Boltzmann constant, and our original choice of glass has precluded MIRE from performing as an accurate refractive-index gas thermometer over a broader range.

II. Designs for small end-effects

As discussed by Shelton [3], the change in pathlength through a window exposed to pressure $d_w \cdot p = (n_i - 1) \cdot (w_f - w_i) + w_f \cdot (n_f - n_i)$ consists of two terms: the first arising from a change in the geometric thickness of the window from initial w_i to final w_f, and the second term from a change in the refractive index of the glass from initial n_i to final n_f. For a pressure increase inside the cell, the first term is negative and the second term is positive (in general) and smaller than the first term. In other words, an increase in pressure inside the cell decreases the optical pathlength through the window. In our case, we estimate the change in geometric thickness from finite-element analysis, and the change in refractive index from photoelastic coefficients and applied stress [4], [5], where the estimate of stress also comes from finite-element analysis.

MIRE was designed around a triple-cell of Fig. 1(a), as this seemed an expedient way of passing two beams through a center cell at the same gas pressure and temperature, and two beams through the outer cells at vacuum. However, this arrangement meant the diameter of the bores were 19 mm, and $d_w = -23.75$ fm/Pa; furthermore, the design had beams passing near the edges of the bores where the sensitivity of window pathlength to changes in beam location was $\frac{\mathrm{d}}{\mathrm{d}x} d_w = 3.0\,\frac{\text{fm/Pa}}{\text{mm}}$. This sensitivity coefficient is important when considering how well the error can be corrected via measurements of refractivity with cells of different lengths. We note two things: (1) our previous effort with alignment achieved four

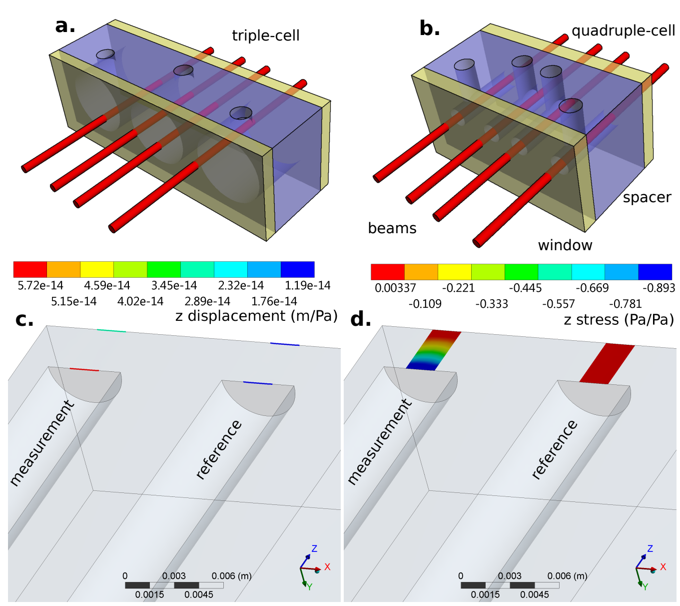

Fig. 1. (a) Sketch of the original MIRE 1.8 cm triple-cell. (b) A quadruple-cell. (c) Distortion and (d) stress on a fused-silica window at the point of transmission for one pair of measurement and reference beams [i.e., a quarter-section of the geometry shown in (b)].

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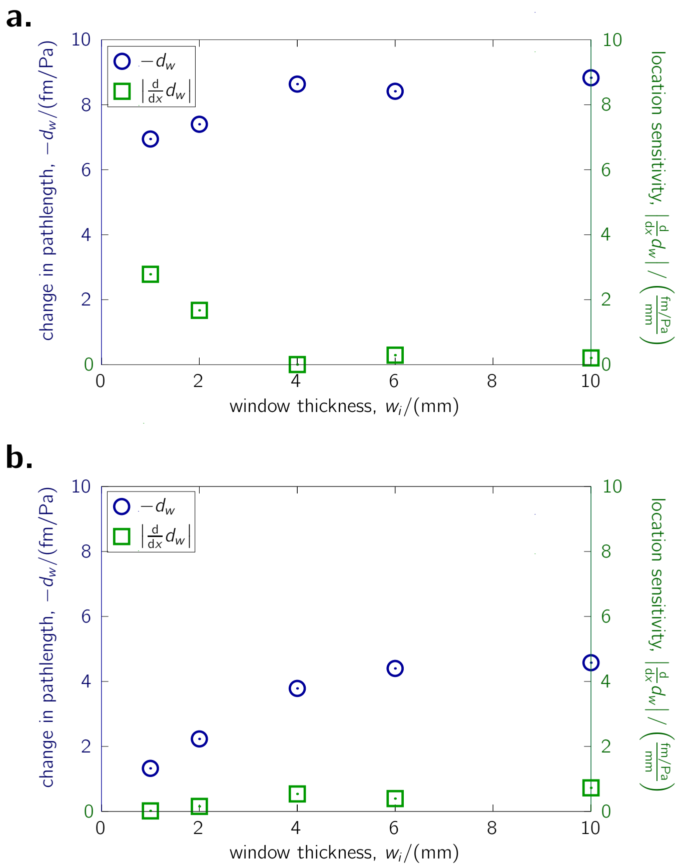

Fig. 2. Change in optical pathlength through (a) fused-silica and (b) sapphire windows. Sensitivity of the window pathlength to change in beam location shown on second y-axis.

beams parallel to $\pm 85\,\mu$rad, and thus beam location through long and short cells differed by as much as $74\,\mu$m on one window; and (2) a variable-length cell would use the same windows for long and short cell lengths, and could achieve a very effective cancellation, but here we wish to avoid the complication of motion and external metrology, and consider cancellation with two or more cells of very different lengths.

There are two obvious ways to reduce the larger $w_f - w_i$ term: (1) reduce the area upon which the pressure acts, and (2) increase the elastic modulus of the window material. A quadruple-cell design can reduce the exposed area and also center the beam location in the bore. An all fused-silica design is shown in Fig. 1(b–d), with one 5 mm diameter hole for each of the four 2 mm diameter beams, passing through 4 mm thick windows. The results of finite-element and photoelastic calculations are shown in Fig. 2(a), where the change in window pathlength is plotted as a function of window thickness. A window thickness of between 4 mm and 6 mm would be optimum in ensuring $d_w \approx -8\,\mathrm{fm/Pa}$, and this thickness range also exhibits low sensitivity to changes in beam location $\frac{d}{dx} d_w < 0.5\,\frac{\mathrm{fm/Pa}}{\mathrm{mm}}$. In an all fused-silica quadruple-cell refractometer, we thus expect a factor of 3 or more reduction in the uncertainty of the window pathlength change compared to our previous triple-cell design. A few more points are worth noting: (1) an additional benefit of the small diameter bore is the possibility to reach pressures above 1 MPa where errors in refractivity measurements due to uncertainties in cell length and interferometer phase are proportionally smaller; (2) the thermal expansion of fused-silica is low and well-known enough that uncertainty in the length of the cell would not be a dominant component if used as a refractive-index gas thermometer in the range (293 ± 100) K; and (3) fused-silica is available in precision-bore tubing, which could simplify cell manufacture and allow cell lengths of 50 cm and longer, more than twice as long as our previous design (i.e., a further reduction of 2 in the window pathlength uncertainty contribution).

The alternative way to reduce $w_f - w_i$ would be to use a quadruple-cell design with sapphire windows and a spacer of low thermal expansion glass-ceramic. In Fig. 2(b) we plot pathlength change and sensitivity to beam location for sapphire windows, where the optical axis is parallel to the c-axis of the crystal, and geometry is the same as Fig. 1. For a window thickness of 2 mm, the change in pathlength is about four times smaller than fused-silica, and the sensitivity to beam location is similarly low. However, the large difference in thermal expansion between window and spacer leads to a temperature-dependent change in pathlength of $d_w = 9.6\,\mathrm{pm/mK}$ for $w_i = 2\,\mathrm{mm}$, and $19.0\,\mathrm{pm/mK}$ for $w_i = 4\,\mathrm{mm}$. This effect would likely be the limiting factor toward achieving sub-3×10^{-6} relative uncertainty in the measurement of helium refractivity. In our past experiments, a fill from vacuum to 367 kPa increased cell temperature by 40 mK, and after 4 h cell temperature was still almost 10 mK above its initial temperature (i.e., the point at which the initial interferometer phase was measured). It thus seems likely that during gas measurements the change in interferometer phase would be uncertain at the 100 pm-level.

III. Conclusion

We have proposed a design for a laser refractometer that employs a quadruple-cell. We expect an all fused-silica cell with 5 mm diameter bore-holes should reduce uncertainty due to changes in window pathlength by a factor of 3 compared to our previous design [1]. The design should make possible realizations of either the pascal or kelvin to within 3×10^{-6} relative standard uncertainty, via measurement of helium refractivity and the equation of state [2]. An alternative design with sapphire windows and a low thermal expansion spacer would have lower pressure-induced changes in pathlength, but mismatch in thermal expansion and the resulting temperature-induced changes in pathlength are a major concern.

References

[1] P. F. Egan, J. A. Stone, J. E. Ricker, J. H. Hendricks, and G. F. Strouse, "Cell-based refractometer for pascal realization," *Optics Letters*, vol. 42, no. 15, pp. 3945–3948, 2017.

[2] M. Puchalski, K. Piszczatowski, J. Komasa, B. Jeziorski, and K. Szalewicz, "Theoretical determination of the polarizability dispersion and the refractive index of helium," *Physical Review A*, vol. 93, 032515, 2016.

[3] D. P. Shelton, "Lens induced by stress in optical windows for high-pressure cells," *Review of Scientific Instruments* vol. 63, no. 8, pp. 3978–3982, (1992).

[4] H. Bach and N. Neuroth, eds., *The Properties of Optical Glass*, Springer–Verlag, 1998, ch. 2.

[5] B. D. Guenther, *Modern Optics*, 2nd ed. Oxford University Press, 2015, ch. 13 & 14.