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REPLY

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Reply

Reply to 'Comment on Relativistic theory of the falling cube gravimeter'*

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Abstract

The comment (Křen and Pálinkás 2017 *Metrologia* **55** 314–5) claims that the paper *Relativistic theory of the falling cube gravimeter* (Ashby 2017 *Metrologia* **55** 1–10) is incorrect. The authors of this comment assert that optical paths in the two interferometer arms of an absolute gravimeter shift only the absolute phase difference between interferometer arms and therefore cannot affect the measured value of g, and that the only needed relativistic correction is the commonly applied 'speed of light correction'. Neither claim stands up to scrutiny.

Keywords: gravimeters, relativity, acceleration, retroreflectors

The value of δg quoted in [1] results from assuming reasonable values of D, d, n, and γ, since actual values were not known. Such numerical values are needed as inputs to fitting routines, should depend on the specific situation and will affect results.

The falling cube receives a signal at its face, shifted downwards in wavelength due to motion. The wavelength in the glass is diminished due to the refractive index. In the cube's rest frame the signal frequency chirps upwards and the wavelength is further slightly diminished, thus many wavefronts become entrained in the glass and unavailable for interference until they emerge later from a lower position. Contrary to what the authors of [2] assert, this time-dependent dynamical effect on the phase of the emerging signal is not compensated for elsewhere in the apparatus. Whereas one might think at first that the constant term in the non-relativistic phase difference (equation (69) in appendix B of [1]) could be cancelled by contributions from the reference beam, this term arises from analysis of the signal that passes through the falling cube *only*. The reference beam could be produced by a reflector of different properties (e.g. a hollow cube with $n = 1$ and $D \approx d$) and its phase may be modified by a constant depending on distance from the beamsplitter, passage through windows, etc, but such a constant cancels in the differences computed in equation (46) of [1]. Further, the relativistic terms separately satisfy the same conditions, (46) in [1].

In the cube rest frame, a wavefront emerges from the face after spending proper time $2Dn/c$ in the glass. Coordinate transformations between the laboratory and cube rest frame show the delay in the laboratory frame is also equal to $2Dn/c$ to order c^{-2}, but the frequency that emerges later from the face differs from the entering frequency by additional time-dependent contributions, than if reflection occurred at the face. The coordinate transformations provide a means for analyzing wavefront propagation in two ways—either in the cube rest frame or in the laboratory frame. The two approaches give the same results; the simplest parts are described in the publication [1].

In a gravitational field g with a gradient, the net force acts at the center of mass (CM) so the CM falls in the same way that a point mass at the CM would fall. The comment dismisses the details of the actual physical processes taking place in the gravimeter, and denies the relevance of parameters d, the distance of the cube face from the CM, and n, the refractive index of the glass. If this were true, a given software data fitting package would give the same result for g independent of n, whether or not the prism is large and solid or small and hollow, whereas software based on the model in [1] would account for such properties and give differing results.

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The comment states 'The counting interferometer ... does not measure differences of measuring arm with respect to reference arm'. The meaning of this comment is unclear since the measured quantities of interest are related to phase differences between the two arms.

Also, the comment states 'the second term γd is irrelevant too, because the position of the testing body is standardly related to its CM (i.e. it is not shifted to the face by d)'. Actually the paper [1] always relates the position of the cube to its CM and never states its position is shifted to the face. It should be standard to relate the cube's position to the CM, but it is untenable to compute the delay with speed c from cube to detector from the CM since the CM is inside the glass.

The comment states that 'only the finite "speed of light", that is already commonly applied by accounting for the variable time delay in the measuring arm of the interferometer, is the necessary "relativistic" correction for the absolute gravimeters with macroscopic objects'. This statement misses one of the main points explained in [1], namely that the expression for retarded time must be iterated twice, to obtain relativistic corrections to order c^{-2}, rather than once as appears to be the case normally [4, 5]. Suppose a plane wave source at moving position $Z_{\rm face}(T_T)$ in the lab emits a wave that propagates downward at speed c in vacuum, the observable being the time T at which the wave arrives at a detector at a fixed position $Z = 0 < Z(T_T)$. The emission time T_T is a retarded time and is given by an implicit equation [3]: $T_T = T - Z_{\rm face}(T_T)/c$. This equation can be solved by iteration, as discussed in [1]. Alternatively, a Taylor expansion to order c^{-2} gives the approximation

$$T_T = T - \frac{Z_{\rm face}(T)}{c} + \frac{Z_{\rm face}(T)V_{\rm face}(T)}{c^2}, \qquad (1)$$

which has non-negligible time-dependent contributions to the phase difference between the arms, of order c^{-2}. These were incorporated into derivations in [1], where they play a role in arriving at the stated conclusions. The 'speed of light correction as commonly stated' [4, 5] assumes light propagates from the CM with speed c, whereas part of the optical path is certainly through the glass. Further, the commonly used expression is incapable of providing all terms of order c^{-2} correctly. In summary, no credible grounds are given for denial of the simplified explanation in appendix B of [1], and the comment [2] does not provide any helpful arguments that would show that the derivations in [1] are flawed.

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[5] Nagornyi V D, Zanimonskiy Y M and Zanomonskiy Y Y 2011 Correction due to the finite speed of light in absolute gravimeters *Metrologia* **48** 101–13 (equations (2) and (3))