Modeling and Simulation Analysis of the Physical Sidelink Shared Channel (PSSCH)

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***Abstract*—This paper examines the performance of the Long Term Evolution (LTE) Physical Sidelink Shared Channel (PSSCH) in out-of-coverage (OOC) device-to-device (D2D) communication scenarios. We develop a closed form expression for the distribution of the number of User Equipments (UEs) that successfully decode a message sent on the PSSCH, given the number of UEs that received the transmitter's Sidelink Channel Information (SCI) message over the Physical Sidelink Control Channel (PSCCH). We validate our results using Monte Carlo simulations of the PSSCH and network simulations in ns3, and discuss some of the effects of system parameters on performance.**

I. Introduction

Device to device (D2D) communications was developed by the 3rd Generation Partnership Project (3GPP) to provide Proximity Services (ProSe) for LTE networks, and was added to the LTE standard in Release 12 [1]. Communications between D2D User Equipments (UEs) go over a *sidelink* rather than from the source UE to a base station via an uplink and then via a downlink from the base station to the destination UE. D2D communications will be used by network operators to provide new services and to offload intra-cell traffic, reducing load on the base station.

D2D communications is also an important component of the Nationwide Public Safety Broadband Network (NPSBN) [2]. The ProSe standard includes support for out-of-coverage (OOC) D2D communications, although it is for public safety agencies only. A motivation for OOC communications is the clear need for public safety personnel to be able to communicate when no base station is available. Example cases include operations in remote areas, loss of network infrastructure (e.g., due to hurricanes or wildfires), or operating inside buildings with severe structural penetration loss.

ProSe defines various sidelink channels that use resource pools consisting of groups of Physical Resource Blocks (PRBs); these channels carry data and control messages to support various D2D functions. Resource pools do not have to be contiguous in either the time or frequency domains, but they recur periodically in the time domain. Such pools exist to support device discovery, synchronizing clocks among of groups of devices, and communication between devices.

A. Background

A UE that intends to send data to other UEs over the sidelink uses the Physical Sidelink Shared Channel (PSSCH). The UE must first advertise the pending transmission using the Physical Sidelink Control Channel (PSCCH) to send a Sidelink Control Information (SCI) message, which tells other UEs which PSSCH resources the transmission will occupy, in addition to other information such as the Modulation and Coding Scheme (MCS) that will be used [3, Clause 5.14]. OOC UEs choose PSCCH resources randomly; each PSCCH resource corresponds to a pair of PRBs. The ProSe standard specifies the mapping from resource index numbers to PRB locations in the control channel resource pool [4, Clause 14.2.1.1]. If two or more UEs choose the same PSCCH resource index, their SCI messages will interfere with each other and will be unintelligible[1]. An additional source of message loss is the half-duplex nature of UE transmissions. A UE can miss an SCI message from another UE if it sends its own SCI in the same pair of subframes. In previous work, we modeled the PSCCH and we showed that if the PSCCH resource pool is properly dimensioned, the only cause of missed advertisements is collisions [6]. Whether SCIs are missed due to collisions or the half-duplex effect, UEs that miss advertisements will not be able to receive the corresponding data that is sent during the subsequent occurrence of the PSSCH.

The PSSCH consists of a set of periodically repeating PRBs that occur after the PSCCH in the time domain. The band of PRBs spanned by the PSSCH in the frequency domain is divided into N_{sb} sub-bands, while the set of subframes spanned by the PSSCH in the time domain is divided into multiple Time Resource Patterns (TRPs); each TRP spans N_{TRP} subframes. An OOC UE with data to send chooses a sub-band at random and also randomly chooses a set of k_{TRP} out of N_{TRP} subframes to use to transmit data in each TRP; the UE uses the same set of subframes in each TRP. The chosen pattern of subframes is called the UE's TRP mask.

[1]If the Signal to Interference and Noise Ratio (SINR) at the receiver is high enough, it may be possible to decode one of the interfering messages.

UEs choose resources in the PSSCH randomly, so there is a risk that they can interfere with each other. For example, if two UEs choose the same sub-band and also choose TRP masks that partially overlap, then some of their transmissions will collide, causing interference. In addition, a UE that transmits in a given subframe will be unable to receive transmissions from other UEs in the same subframe if UEs cannot transmit and receive simultaneously, due to the half-duplex effect.

ProSe uses the Hybrid Automatic Repeat Request (HARQ) mechanism to mitigate the impact of collisions. UEs do not provide feedback over the sidelink for each HARQ transmission [4, Clause 14.1.1]. A transmitting UE sends four Redundant Versions (RVs) of data over the PSSCH; each RV is composed of information and error correction bits [5]. For this paper, we assume that each HARQ transmission takes up all the available PRBs in a subframe; i.e., it fills the chosen sub-band. For example, a set of four HARQ transmissions with $k_{\mathrm{TRP}} = 1$ would be sent over the course of four TRPs, as shown in the top row of Fig. 1. If $k_{\mathrm{TRP}} = 2$, then a UE can send two messages, with the first occupying the first two TRPs and the next occupying the last two TRPs; if $k_{\mathrm{TRP}} = 4$, 4 RVs can be sent during every TRP.

B. Purpose of this work

In this paper, we develop a performance metric for the PSSCH. The analytical model underlying this metric incorporates the major features of the PSSCH: random sub-band selection, random TRP mask selection, the half duplex effect, and HARQ. We define our metric with respect to a single UE of interest in a group of N_u UEs, which we call UE_0. We define $\mathcal{R}^C_\rho$ to be the event, "ρ UEs decode UE_0's SCI," and $\mathcal{R}^S_\delta$ to be the event, "δ UEs decode UE_0's data on the PSSCH." Our performance metric is the conditional probability distribution of the number of UEs that decode UE_0's data: $\mathsf{Pr}\{\mathcal{R}^S_\delta \,|\, \mathcal{R}^C_\rho\}$. We can remove the condition on the number of UEs that decode the SCI by using the distribution of this number that we derived previously [6], giving

$$\mathsf{Pr}\{\mathcal{R}^S_\delta\} = \sum_{\rho=0}^{N_u-1} \mathsf{Pr}\{\mathcal{R}^S_\delta \,|\, \mathcal{R}^C_\rho\}\mathsf{Pr}\{\mathcal{R}^C_\rho\}. \qquad (1)$$

This metric will allow a network operator to characterize the performance of an OOC group of UEs, and thus dimension the PSSCH to optimize the Quality of Experience (QoE) for public safety users. The metric can also be used to determine what input parameters (number of UEs, PSSCH pool dimensions, etc.) have the greatest impact on performance.

The rest of this paper is organized as follows. We briefly survey related work in Section II. In Section III, we develop the conditional distribution of the number of UEs that decode a transmitted message given that ρ of them decoded the corresponding SCI message. We validate our model in Section IV, and also examine the sensitivity of the metric to various input parameters. We summarize our discussion in Section V.

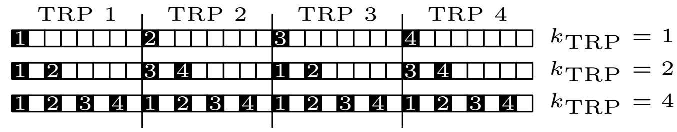

Fig. 1. Examples of transmission occurences of the four HARQ repetitions during a PSSCH period consisting of four TRPs, for various values of k_{TRP}.

II. Survey of Other Work

In [6], we developed a closed-form expression for the distribution of UEs that receive an SCI over the PSCCH, and we showed that the half-duplex effect can be eliminated by properly sizing the control resource pool. To the best of our knowledge, ours is the first work that models the PSSCH in this fashion, although some other works have considered D2D communications. Yoon, Park, and Choi developed a feedback scheme for the sidelink that uses a feedback pool that follows the PSSCH in time and has the same "shape" as the PSCCH [7]. Park et al. developed a resource selection scheme for the PSCCH that aims to improve performance by using measured interference to inform the selection process, rather than using only random selection [8]. Shih et al. developed an autonomous resource selection algorithm for out of coverage UEs [9]. Their approach partitions the control channel resource pool so that UEs sense the energy in the first PRB that they choose and then pick a different resource if they sense a collision. They use a collision analysis for the SCI and the transmitted data, but their performance analysis does not account for the half duplex effect in the PSCCH or the PSSCH, and does not account for the effect of HARQ in the PSSCH. The analysis also does not include the 3GPP mapping from resource index to PSCCH PRBs.

III. Theoretical Analysis

A. Model description

For the following model description, we provide a list of variable definitions in Table I. Consider a group of OOC UEs and let N_u be the number of UEs in the group. We assume that all UEs have data to send. We focus on a single transmitting UE of interest, which we call UE_0. All UEs contend for PSCCH resources to send their SCI messages. In this analysis, we assume that a subset of ρ UEs successfully decoded UE_0's SCI message.

To model the HARQ function, we define ψ_i to be the probability that a UE successfully decodes UE_0's message given that it received i HARQ transmissions, where $i \in \{0, 1, 2, 3, 4\}$. The number of messages that a UE can send during a period depends on k_{TRP}, as shown in Fig. 1. The rows in Fig. 1 each represent a period consisting of four TRPs, and correspond to the cases $k_{\mathrm{TRP}} = 1, 2, 4$. Example subframe masks are shown in black in each row.

From the example masks in Fig. 1, we can determine which decoding probabilities ψ_i to use for a given value of k_{TRP}. Since a message requires four HARQ transmissions, and since each UE uses a single mask for all TRPs in a

TABLE I
LIST OF VARIABLES

Symbol	Definition
N_u	UE group size
UE_0	Randomly chosen UE of interest
$\mathcal{R}_\rho^C$	Event where $\rho \leq (N_u - 1)$ UEs decode UE_0's SCI
$\mathcal{R}_\delta^S$	Event where $\delta \leq \rho$ UEs decode UE_0's data on the PSSCH
N_{TRP}	Number of subframes per TRP
k_{TRP}	Number of subframes per TRP used by UEs to send data
Y	Number of subframes per TRP used by UE_0 not affected by collisions with other UEs operating in the same sub-band
N_{sb}	Number of sub-bands partitioning the PSSCH in the frequency domain
ρ	Number of UEs that decode UE_0's SCI message
ι	Number of UEs that do not decode UE_0's SCI message
s	Number of UEs that transmit in UE_0's sub-band
d	Number of UEs that transmit in sub-bands different than UE_0's sub-band
s''	Number of same-sub-band (SSB) UEs that decode UE_0's SCI
d''	Number of other-sub-band (OSB) UEs that decode UE_0's SCI
ψ_i	Probability that a UE decodes UE_0's SCI given that it received i transmissions
S_ρ	Number of receiver UEs that transmit in UE_0's sub-band
S_ι	Number of interferer UEs that transmit in UE_0's sub-band
S	Total number of UEs that transmit in UE_0's sub-band
D_ρ	Number of receiver UEs that transmit in different sub-bands
D_ι	Number of interferer UEs that transmit in different sub-bands
s'	Value taken by S_ρ
σ	Value taken by S_ι
n	Value taken by Y
ω_n	$\Pr\{\text{SSB receiver UE decodes UE}_0\text{'s SCI} \mid Y = n\}$
ϕ_n	$\Pr\{\text{OSB receiver UE decodes UE}_0\text{'s SCI} \mid Y = n\}$
N_{run}	Number of Monte Carlo runs per validation simulation
N_{trials}	Number of trials per validation run
T	Number of UEs decoding UE_0's message per trial

TABLE II
VALUES OF ψ_n VERSUS n FOR VARIOUS VALUES OF k_{TRP}

n	k_{TRP}		
	1	2	4
0	ψ_0	ψ_0	ψ_0
1	ψ_4	ψ_2	ψ_1
2	–	ψ_4	ψ_2
3	–	–	ψ_3
4	–	–	ψ_4

given PSSCH period, the decoding probability depends on the number of UE_0's subframes that are not subject to interference from collisions and that other UEs can receive because they are not transmitting during those subframes. If $k_{\text{TRP}} = 1$, UE_0's message requires four TRPs to transmit, and a collision will impact all four transmissions, and all receivers decode UE_0's message with probability ψ_0; conversely, if there is no collision, then all four of UE_0's transmissions can be received (provided that a receiver in another sub-band is not using UE_0's mask) with probability ψ_4. By similar reasoning, we can construct the table of message decoding probabilities shown in Table II for other values of k_{TRP}. We will use these values in the table in the model development in Section III-C.

B. Constructing the model

By examining the selection of resources by different UEs in a particular sequence, which does not affect the fidelity of the model, we can determine subframe choices of UEs that operate in UE_0's sub-band affect outcomes for UEs that are not in UE_0's sub-band.

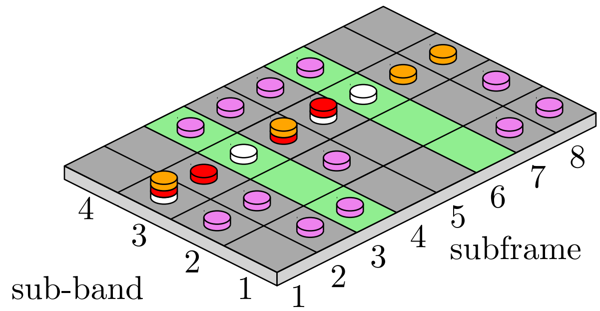

Fig. 2. An example of sub-band and TRP mask choices by UE_0 (white disks), UEs that choose UE_0's sub-band (red and orange disks), and UEs that choose other sub-bands (purple disks).

In the example shown in Fig. 2, $N_{\text{TRP}} = 8$ subframes, $k_{\text{TRP}} = 4$ subframes, and $N_{\text{sb}} = 4$ sub-bands. The figure shows an example outcome of UE_0's choice of a subframe mask, and a random sub-band. We show UE_0's choice with white tokens placed in spaces corresponding to subframes $1, 3, 5, 6$ in sub-band 3. Fig. 2 also shows transmissions by two UEs that have chosen UE_0's sub-band. We show these two UEs' choices using red and orange tokens placed in spaces corresponding respectively to subframes $1, 2, 4, 5$ and $1, 4, 7, 8$ in sub-band 3. Because the two other UEs' transmissions overlap some of those of UE_0, UE_0 has two collided and two non-collided transmissions. Thus the two UEs that transmit in UE_0's subframe decode UE_0's message with probability ψ_2.

Finally, Fig. 2 shows the effect of transmissions by UEs that have chosen sub-bands other than UE_0's sub-band. For clarity, only one UE is shown per sub-band; we show the other-sub-band (OSB) UEs' choices with purple tokens placed in spaces corresponding to the masks that they have chosen. The UE that chose sub-band 1 chose a mask that overlaps one of UE_0's two uncollided transmissions in subframe 3, so this UE decodes UE_0's message with probability ψ_1. The UE that chose sub-band 2 has a mask that overlaps neither of UE_0's uncollided transmissions; this UE decodes UE_0's message with probability ψ_2. Finally, the mask chosen by the UE that chose sub-band 4 overlaps both of UE_0's uncollided transmissions, so this UE decodes UE_0's message with probability ψ_0.

C. Development of $\Pr\{\mathcal{R}_\delta^S \mid \mathcal{R}_\rho^C\}$

Our model uses Jordan's formula [11, Eqs. (3,4)], which we briefly describe here. Given a probability space Ω with equally likely outcomes $\{\omega \in \Omega\}$, define n events $A_1, \ldots, A_n$, each of which corresponds to a subset of Ω; the probability of an event A_i is $\Pr\{A_i\} = \mathcal{N}(A_i)/\mathcal{N}(\Omega)$, where $\mathcal{N}(A)$ is the number of elements in the set A. Let the random variable ν be the number of events that occur due to an outcome ω. The probability that exactly k out of n events occur (i.e., $\nu = k$)

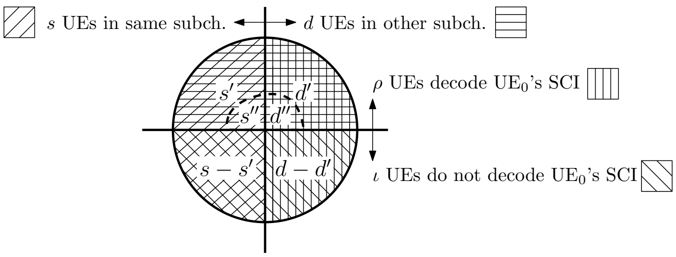

Fig. 3. Partitioning of the set of (N_u-1) UEs with respect to the UE of interest, UE$_0$.

is

$$\mathsf{Pr}\{k \text{ events occur}\} = \sum_{r=k}^{n} (-1)^{r-k} \binom{r}{k} \mathsf{E}\left\{\binom{\nu}{r}\right\} \tag{2}$$

where

$$\mathsf{E}\left\{\binom{\nu}{r}\right\} = \sum_{1\le i_1<i_2<\cdots<i_r\le n} \mathsf{Pr}\{\cap_{j=1}^{r} A_{i_j}\} \tag{3}$$

is the rth binomial moment of ν.

To derive $\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C\}$, we partition the set of (N_u-1) UEs in UE$_0$'s group based on whether they received UE$_0$'s SCI and whether they randomly select the sub-band chosen by UE$_0$. We show this partitioning in Fig. 3. We call the ρ UEs that received UE$_0$'s SCI, "receivers;" and the remaining $\iota = (N_u-1-\rho)$ UEs, "interferers." We define the random variables S_ρ and S_ι to be, respectively, the number of receivers and interferers that use UE$_0$'s sub-band. Let $S = S_\rho + S_\iota$ be the number of UEs in UE$_0$'s sub-band. We define s' and σ to be values taken by S_ρ and S_ι, respectively, and s to be the value taken by S, so that $s = \sigma + s'$. In addition, we define D_ρ and D_ι to be random variables that are the number of receivers and interferers that use sub-bands other than UE$_0$'s; d' is the specific value taken by D_ρ.

Next, we condition on the number of receivers and interferers that choose UE$_0$'s sub-band, which gives us

$$\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C\} = \sum_{s'=0}^{\rho}\sum_{\sigma=0}^{\iota} \mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C,\, S_\rho = s',\, S_\iota = \sigma\} \times \mathsf{Pr}\{S_\rho = s',\, S_\iota = \sigma\}. \tag{4}$$

The probability that a given UE picks UE$_0$'s sub-band is $1/N_{\text{sb}}$. Thus the probability that s' out of ρ receivers and $\sigma = s - s'$ out of ι interferers pick UE$_0$'s sub-band, which is $\mathsf{Pr}\{S_\rho = s',\, S_\iota = \sigma\}$ in Eq. (4), is

$$\begin{aligned}\mathsf{Pr}\{S_\rho = s',\, S_\iota = \sigma\} &= \tbinom{\rho}{s'}\left(\tfrac{1}{N_{\text{sb}}}\right)^{s'}\left(1-\tfrac{1}{N_{\text{sb}}}\right)^{\rho-s'}\tbinom{\iota}{\sigma}\left(\tfrac{1}{N_{\text{sb}}}\right)^{\sigma}\left(1-\tfrac{1}{N_{\text{sb}}}\right)^{\iota-\sigma}\\ &= \tbinom{\rho}{s'}\tbinom{\iota}{\sigma}\left(\tfrac{1}{N_{\text{sb}}}\right)^{s}\left(1-\tfrac{1}{N_{\text{sb}}}\right)^{N_u-1-s}.\end{aligned} \tag{5}$$

Let the random variable Y be the number of UE$_0$'s transmitted subframes in a TRP that do not experience interference from other UEs in UE$_0$'s sub-band. By conditioning on the value of Y, we can expand $\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C,\, S_\rho = s',\, S_\iota = \sigma\}$ from Eq. (4) as follows:

$$\begin{aligned}&\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C,\, S_\rho = s',\, S_\iota = \sigma\}\\ &\quad= \sum_{n=0}^{k_{\text{TRP}}} \mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C,\, S_\rho = s',\, S_\iota = \sigma,\, Y = n\} \times \mathsf{Pr}\{Y = n \,|\, S = s' + \sigma\}.\end{aligned} \tag{6}$$

The probability that Y takes the value n is

$$\mathsf{Pr}\{Y = n \,|\, S = s\} = \sum_{\ell=n}^{k_{\text{TRP}}} (-1)^{\ell-n}\binom{\ell}{n}\mathsf{E}\left\{\binom{Y}{\ell}\,\middle|\, S = s\right\} \tag{7}$$

where

$$\mathsf{E}\left\{\binom{Y}{\ell}\,\middle|\, S = s\right\} = \sum_{1\le i_1<\cdots<i_\ell\le k_{\text{TRP}}} \mathsf{Pr}\{\cap_{j=1}^{\ell} A_{i_j} \,|\, S = s\}, \tag{8}$$

and we define the set of events $\{A_i\}_{i=1}^{N_{\text{TRP}}}$ as follows. Event A_i occurs when UE$_0$ chooses subframe i and no other UE transmitting in UE$_0$'s sub-band chooses that subframe. Thus $\mathsf{Pr}\{\cap_{j=1}^{\ell} A_{i_j} \,|\, S = s\}$ is the probability that the ℓ subframes chosen by UE$_0$ whose indices are $i_1, i_2, \ldots, i_\ell$ are not used by s other UEs. We compute this conditional probability by taking the ratio of the number of ways that a single UE can pick a mask such that it does not use subframes $i_1, i_2, \ldots, i_\ell$, to the total number of ways that a UE can pick a mask. To avoid picking subframes $i_1, i_2, \ldots, i_\ell$, a UE has $(N_{\text{TRP}} - \ell)$ subframes from which to choose k_{TRP} subframes for its mask, and there are $\binom{N_{\text{TRP}}-\ell}{k_{\text{TRP}}}$ ways to do this. The total number of masks that any UE can pick is $\binom{N_{\text{TRP}}}{k_{\text{TRP}}}$, so the probability that a UE chooses a mask so that it does not overlap subframes $i_1, i_2, \ldots, i_\ell$ is $\binom{N_{\text{TRP}}-\ell}{k_{\text{TRP}}}/\binom{N_{\text{TRP}}}{k_{\text{TRP}}}$ [2]. UEs choose masks independently, so if the number of other UEs that choose UE$_0$'s sub-band is $S = s$, then

$$\mathsf{Pr}\{\cap_{j=1}^{\ell} A_{i_j} \,|\, S = s\} = \left[\frac{\binom{N_{\text{TRP}}-\ell}{k_{\text{TRP}}}}{\binom{N_{\text{TRP}}}{k_{\text{TRP}}}}\right]^{s},\; s = 0, 1, \ldots, N_u - 1. \tag{9}$$

All of the $\mathsf{Pr}\{\cap_{j=1}^{\ell} A_{i_j} \,|\, S = s\}$ terms in the sum in Eq. (8) are identical and are given by Eq. (9). There are $\binom{k_{\text{TRP}}}{\ell}$ terms in the sum in Eq. (8), because this is the number of ways to pick ℓ subframes from the set of k_{TRP} subframes that make up UE$_0$'s mask. Thus, if $S = s = s' + \sigma$,

$$\begin{aligned}&\mathsf{Pr}\{Y = n \,|\, S = s' + \sigma\}\\ &\quad= \sum_{\ell=n}^{k_{\text{TRP}}} (-1)^{\ell-n}\binom{\ell}{n}\binom{k_{\text{TRP}}}{\ell}\left[\frac{\binom{N_{\text{TRP}}-\ell}{k_{\text{TRP}}}}{\binom{N_{\text{TRP}}}{k_{\text{TRP}}}}\right]^{s'+\sigma}.\end{aligned} \tag{10}$$

To get $\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C,\, S_\rho = s',\, S_\iota = \sigma,\, Y = n\}$ in Eq. (6), we define s'' to be number of same-sub-band (SSB) UEs that decode UE$_0$'s message, and d'' to be the number of OSB UEs that decode UE$_0$'s message. It follows that $s'' \le s'$ and $d'' \le$

[2] Note that $\binom{N_{\text{TRP}}-\ell}{k_{\text{TRP}}}/\binom{N_{\text{TRP}}}{k_{\text{TRP}}} = 0$ if $N_{\text{TRP}} - \ell < k_{\text{TRP}}$.

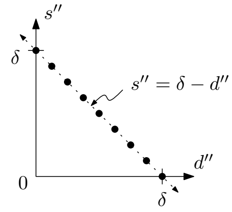

Fig. 4. Illustration of the relationship between d'' and s''.

d', as shown in Fig. 3. If δ is the total number of UEs that decode UE_0's message, then $s''+d''=\delta$, as shown in Fig. 4, and we let s'' vary from 0 to δ and then set $d''=\delta-s''$.

Conditioning on the number of UEs in UE_0's sub-band that decoded the transmitted message gives

$$\begin{aligned}&\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C,\, S_\rho=s',\, S_\iota=\sigma,\, Y=n\}\\ &\quad=\sum_{s''=0}^{\delta}\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C,\, S_\rho=s',\, S_\iota=\sigma,\, Y=n,\, S=s''\}\\ &\qquad\times\mathsf{Pr}\{S=s''\}. \end{aligned}\tag{11}$$

Given $Y=n$, the probability that s'' receivers out of the s' receivers in UE_0's sub-band decode the message is

$$\mathsf{Pr}\{S=s''\}=\binom{s'}{s''}\omega_n^{s''}(1-\omega_n)^{s'-s''}.\tag{12}$$

where $\omega_n=\psi_{4n/k_{\text{TRP}}}$ is the probability that a SSB UE decodes UE_0's message given that it received n transmissions, as given in Table II.

Let ϕ_n be the probability that an OSB receiver decodes UE_0's message given $Y=n$. We condition on the value of m, the number of unblocked subframes that the OSB receiver can access, given n unblocked subframes, and use the same approach that we used to develop Eq. (9) and Eq. (10) (with $s=1$ in this case):

$$\begin{aligned}\phi_n&=\sum_{m=0}^{n}\psi_{4m/k_{\text{TRP}}}\mathsf{Pr}\{\text{receive } m \text{ of } n \text{ unblocked subframes}\}\\ &=\sum_{m=0}^{n}\psi_{4m/k_{\text{TRP}}}\left[\sum_{\ell=m}^{n}(-1)^{\ell-m}\binom{\ell}{m}\binom{n}{\ell}\frac{\binom{N_{\text{TRP}}-\ell}{k_{\text{TRP}}}}{\binom{N_{\text{TRP}}}{k_{\text{TRP}}}}\right].\end{aligned}\tag{13}$$

Then the probability that d'' out of d' OSB receivers decode UE_0's message is

$$\begin{aligned}&\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C,\, S_\rho=s',\, S_\iota=\sigma,\, Y=n,\, S=s''\}\\ &\quad=\binom{d'}{d''}\phi^{d''}(1-\phi)^{d'-d''}\\ &\quad=\binom{\rho-s'}{\delta-s''}\phi^{\delta-s''}(1-\phi)^{(\rho-s')-(\delta-s'')}.\end{aligned}\tag{14}$$

Note that we have to have $s''\le s'$ for Eq. (12) to be non-zero and $\delta-s''\le\rho-s'$ for Eq. (14) to be non-zero; thus the series in Eq. (11) runs from $s''=\max(0,s'-\rho+\delta)$ to $s''=\min(\delta,s'')$.

To obtain the final form for the conditional distribution, $\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C\}$, we substitute Eq. (12) and Eq. (14) into Eq. (11), then combine the resulting expression with Eq. (10) in Eq. (6), and insert this result into Eq. (4) along with Eq. (5). The result is the expression in Eq. (15).

IV. Numerical Results

A. Monte Carlo simulations

To validate the theoretical model, we implemented a set of Monte Carlo simulations in Matlab whose output is an empirical conditional probability mass function of the number of UEs that decode transmitted data from a randomly chosen peer. For each set of input parameters, we performed $N_{\text{runs}}=5$ runs, with N_{trials} 10 000 trials per run.

In each trial, we use the following procedure. We initialize T, the tally of UEs that decoded UE_0's message, to zero, and we randomly assign a sub-band and mask to UE_0. Next, we assign sub-bands and masks to the other (N_u-1) UEs in the group, and we determine n, the number of UE_0's subframes that are not blocked by transmissions from other UEs in UE_0's sub-band. For each receiver UE in UE_0's sub-band, we generate U, a $U[0,1]$ random variate; if $U\le\psi_n$, we increment T. For each receiver in other sub-bands, we determine k, the number of UE_0's un-collided subframes that do not overlap with subframes chosen by the OSB receiver UE. Then we generate U for that UE and if $U\le\psi_k$, we increment T. After processing the last OSB UE, we record the value of T for the trial, and move on to the next trial in the run.

Once all the runs were complete, we computed an empirical probability mass function (PMF) for each run. For the rth run, for $0\le m\le N_u-1$, we computed $\widehat{p}_r(m)$, which is the number of times that $T=m$ divided by N_{trials}. Then we generated $\widehat{p}(m)=\sum_{r=1}^{N_{\text{runs}}}\widehat{p}_r(m)/N_{\text{runs}}$. We also estimated the standard deviation of $\widehat{p}_r(m)$, $\varsigma(m)$. The approximate 95 % confidence interval for the PMF is $\widehat{p}(m)\pm1.96\,\varsigma(m)/\sqrt{N_{\text{runs}}}$.

B. NS3 simulations

We simulated a group of N_u UEs using the ns3 simulation tool as an additional check of the model. In order to fix the number of UEs that successfully receive a given UE's SCI message to have a given value of ρ during each simulated period, we set the SINR threshold for SCI messages so that all transmitted SCI messages will be received by all other UEs, even if collisions occur in the PSCCH. We performed 5 simulation runs for each set of input parameters; for each run, we simulated 400 s of activity. Each instance of the PSSCH contained 4 TRPs whose duration was 8 subframes each, and the PSCCH's duration was 8 subframes. This results in a period duration of 40 ms, so we simulated 10 000 periods per run. We picked $(\rho+1)$ UEs to send SCIs that were received by all UEs in the group. We randomly picked one of the $(\rho+1)$ SCI-transmitting UEs to be UE_0 and all N_u UEs in the group then sent data over the PSSCH. We counted the number of UEs that were able to decode UE_0's data and we saved this tally as the result for the run. We combined the results from the

$$\mathsf{Pr}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C\} = \sum_{s'=0}^{\rho}\sum_{\sigma=0}^{\iota}\binom{\rho}{s'}\binom{\iota}{\sigma}\left(\frac{1}{N_{\text{sb}}}\right)^{s'+\sigma}\left(1-\frac{1}{N_{\text{sb}}}\right)^{(N_u-1)-(s'+\sigma)}$$
$$\times\left(\sum_{n=0}^{k_{\text{TRP}}}\left[\sum_{\ell=n}^{k_{\text{TRP}}}(-1)^{\ell-n}\binom{\ell}{n}\binom{k_{\text{TRP}}}{\ell}\left[\frac{\binom{N_{\text{TRP}}-\ell}{k_{\text{TRP}}}}{\binom{N_{\text{TRP}}}{k_{\text{TRP}}}}\right]^{s'+\sigma}\right]\right.$$
$$\left.\times\left[\sum_{s''=\max(0,s'-\rho+\delta)}^{\min(\delta,s')}\binom{s'}{s''}\omega_n^{s''}(1-\omega_n)^{s'-s''}\binom{\rho-s'}{\delta-s''}\phi_n^{\delta-s''}(1-\phi_n)^{(\rho-s')-(\delta-s'')}\right]\right) \quad (15)$$

set of runs in the same manner as in Section IV-A to produce an empirical PMF with 95 % confidence intervals.

C. Discussion of results

To generate our results, shown in Fig. 5, we used $N_u = 11$ UEs, and the PSSCH bandwidth was 50 PRBs, or 10 MHz. We examined two partitions of the PSSCH bandwidth: $N_{\text{sb}} = 8$ sub-bands and $N_{\text{sb}} = 16$ sub-bands, and we considered two mask types: $k_{\text{TRP}} = 2$ and $k_{\text{TRP}} = 4$. We also investigated two values for the number of UEs that received UE$_0$'s SCI: $\rho = 3$ UEs and $\rho = 9$ UEs. We validated our model for other values of N_u and ρ, and for $k_{\text{TRP}} = 1$, but we do not show these results due to space limitations.

In all four sub-figures, we observe excellent agreement between the theoretical results and the empirical results from both sets of simulations, which is strong evidence for the accuracy of our model. All four sub-figures also show that increasing the number of sub-bands shifts the mass of the conditional distribution to the right, i.e., we see a reduction in the probability that no receiver UEs decode UE$_0$'s while we also see an increase in decoding probabilities for the greatest number of UEs. However, we note that the price of increasing N_{sb} is a reduction in the number of PRBs per subframe that a UE is able to use to send data; the trade-off that results in the maximum throughput is a topic for further study. In addition, we see that while increasing k_{TRP} increases the number of possible masks (70 when $k_{\text{TRP}} = 4$ vs. 28 when $k_{\text{TRP}} = 2$), this does not produce higher decoding probabilities; masks that cover more subframes create greater chances for interference or loss of transmissions due to the half duplex effect. The extreme example of this effect occurs when $k_{\text{TRP}} = 8$; all UEs will block or miss each others' transmissions with probability one, since they transmit in every subframe. However, reducing k_{TRP} also reduces the number of PRBs available for a UE to use, and so there is a second trade-off with respect to throughput.

To further illustrate the trends shown in Fig. 5, we show the theoretical values of the conditional means and variances, $\mathsf{E}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C\}$ and $\mathsf{Var}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C\}$, as ordered pairs in Table III for the cases plotted in the figure. We note from the table that if the number of UEs that decoded UE$_0$'s SCI message is small, then varying other parameters such as ρ or k_{TRP} or N_{sb} does not significantly affect the conditional statistics associated with the number of these UEs that decode UE$_0$'s message on the PSSCH. If the PSCCH is configured to allow most UEs to decode the SCI message, then the design of the PSSCH has more impact, with the greater impact coming from the mask design (i.e., the value of k_{TRP}).

TABLE III
($\mathsf{E}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C\}$, $\mathsf{Var}\{\mathcal{R}_\delta^S \,|\, \mathcal{R}_\rho^C\}$) FOR PARAMETERS IN FIG. 5

	$N_{\text{sb}} = 8$		$N_{\text{sb}} = 16$	
	$k_{\text{TRP}} = 2$	$k_{\text{TRP}} = 4$	$k_{\text{TRP}} = 2$	$k_{\text{TRP}} = 4$
$\rho = 3$	(1.69 , 1.11)	(0.84 , 0.78)	(1.95 , 0.91)	(1.13 , 0.85)
$\rho = 9$	(5.07 , 6.72)	(2.52 , 3.96)	(5.86 , 4.83)	(3.38 , 3.83)

V. Summary and Conclusions

In this paper, we developed a mathematical model to characterize the performance of the PSSCH for out-of-coverage D2D communications. The model produces the distribution of the number of devices that receive a UE's data transmission given the number of UEs that received the corresponding SCI message. This result can be combined with existing models of the PSCCH. We validated our model using two sets of simulations: a simple Monte Carlo model and a full simulation in ns3 of an OOC group of UEs using D2D communications. The results that we obtained show that increasing the number of sub-bands in the PSSCH improves the likelihood of a UE's decoding a transmitted message, although this is at the expense of throughput, which we intend to quantify in future work. We also showed that the value of k_{TRP} has a significant impact on performance, with lower values resulting in a greater likelihood that a receiver UE decodes the message on the PSSCH, but that this also reduces throughput. In future work, we will discuss how to use this model to maximize the throughput on the sidelink.

Acknowledgment

The authors would like to thank the NIST Summer Undergraduate Research Fellowship (SURF) program, which made it possible for Aneta Galazka to work on this project.

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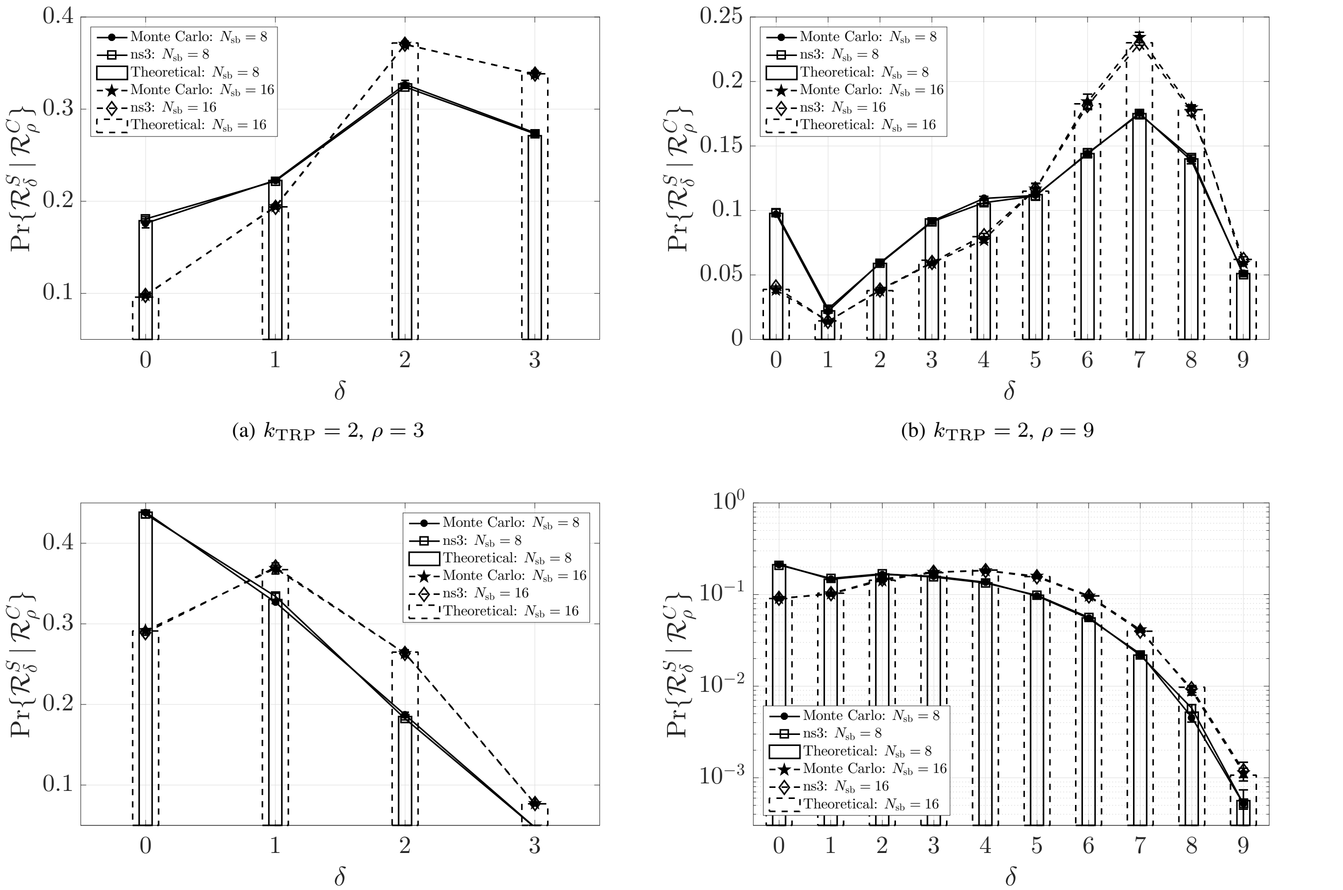

(a) $k_{\mathrm{TRP}} = 2$, $\rho = 3$

(b) $k_{\mathrm{TRP}} = 2$, $\rho = 9$

(c) $k_{\mathrm{TRP}} = 4$, $\rho = 3$

(d) $k_{\mathrm{TRP}} = 4$, $\rho = 9$

Fig. 5. Bar graphs of theoretical PMF values from Eq. (15) with scatter plots of empirical PMF values from Monte Carlo simulations using Matlab and ns3, with 95 % confidence intervals shown for all simulation results.

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