Eleventh U.S. National Conference on Earthquake Engineering
Integrating Science, Engineering & Policy
June 25-29, 2018
Los Angeles, California

UNCERTAINTIES IN SMALL-DISPLACEMENT-BASED ALGORITHMS FOR SIMULATING STRUCTURAL COLLAPSE

K.K.F. Wong[1]

ABSTRACT

Can small-displacement-based dynamic structural analysis software packages be used to accurately simulate large displacement responses and structural collapse due to earthquake excitations? This question has been raised by many researchers and practitioners as to whether different small-displacement-based solution algorithms can simulate the same collapse patterns. In this research, an investigation is made by comparing the structural dynamic responses using three small-displacement-based solution algorithms with those obtained using a large-displacement-based finite element solution algorithm. Under the assumption that a large-displacement-based solution algorithm gives the most precise responses for a given structural model, the comparison shows that consistencies are obtained among solution algorithms using different "dynamic" solvers, but inconsistencies occur due to implementation of different "nonlinear" solvers which do not necessarily occur only at large displacement responses. These nonlinear solvers make different assumptions for coupling geometric nonlinearity and material nonlinearity into their respective solution algorithms. For this reason, the quantification of uncertainties in the output responses is performed to identify the solution algorithm that is more appropriate for simulating structural responses at and near collapses.

[1]Research Engineer, Nat. Institute of Standards & Technology, Gaithersburg, MD 20899 (email: kfwong@nist.gov)

Wong KKF. Uncertainties in small-displacement-based algorithms for simulating structural collapse. *Proceedings of the 11th National Conference in Earthquake Engineering*, Earthquake Engineering Research Institute, Los Angeles, CA. 2018.

Eleventh U.S. National Conference on Earthquake Engineering
Integrating Science, Engineering & Policy
June 25-29, 2018
Los Angeles, California

Uncertainties in Small-Displacement-Based Algorithms for Simulating Structural Collapse

K.K.F. Wong[1]

ABSTRACT

Can small-displacement-based dynamic structural analysis software packages be used to accurately simulate large displacement responses and structural collapse due to earthquake excitations? This question has been raised by many researchers and practitioners as to whether different small-displacement-based solution algorithms can simulate the same collapse patterns. In this research, an investigation is made by comparing the structural dynamic responses using three small-displacement-based solution algorithms with those obtained using a large-displacement-based finite element solution algorithm. Under the assumption that a large-displacement-based solution algorithm gives the most precise responses for a given structural model, the comparison shows that consistencies are obtained among solution algorithms using different "dynamic" solvers, but inconsistencies occur due to implementation of different "nonlinear" solvers which do not necessarily occur only at large displacement responses. These nonlinear solvers make different assumptions for coupling geometric nonlinearity and material nonlinearity into their respective solution algorithms. For this reason, the quantification of uncertainties in the output responses is performed to identify the solution algorithm that is more appropriate for simulating structural responses at and near collapses.

Introduction

Performance-based seismic engineering is a useful approach for designing new structures and improving the seismic performance of existing structures. Modelling and simulation of structural response for determining the seismic demands is an essential part of the process. While the seismic demands are compared with the corresponding seismic capacity in the design, the analysis used to determine the demands may be sensitive to the software package used to conduct the analysis. Many of the seismic analysis software packages today use small-displacement theory, but claim to solve a wide-variety of nonlinear structural dynamic problems, up to and including structural collapse. References [1-4] present examples of using small-displacement-based software packages to solve collapse problems over the past decade.

---

[1]Research Engineer, Nat. Institute of Standards & Technology, Gaithersburg, MD 20899 (email: kfwong@nist.gov)

Wong KKF. Uncertainties in small-displacement-based algorithms for simulating structural collapse. *Proceedings of the 11th National Conference in Earthquake Engineering*, Earthquake Engineering Research Institute, Los Angeles, CA. 2018.

Being able to capture structural collapse generally requires software packages to handle significant coupling between geometric and material nonlinearities. The small-displacement-based software packages that are available today, either commercial or research-oriented, have typically been developed to give reasonable results when analyzing models with material nonlinearities and separately when analyzing models with geometric nonlinearities. However, the complexity of nonlinear analysis can lead to inconsistent results when the analysis requires significant coupling between geometric and material nonlinearities. This coupling between yielding components for material nonlinearity and member stability for geometric nonlinearity makes the problem difficult to solve, and being able to address this nonlinear coupling in small-displacement-based solution algorithms remains a challenge.

This paper presents recent research concerning the evaluation of several small-displacement-based software packages based on various geometric nonlinearity and material nonlinearity formulations. Doing so can facilitate the understanding of how each embedded solution algorithm handles the coupling issue to capture large-displacement structural dynamic responses up to and including collapse. Numerical simulations are performed to simulate the nonlinear structural dynamic responses of an eight-story steel moment-resisting frame based on three different geometric nonlinearity formulations of small-displacement-based solution algorithms. The simulated responses are compared with each other and with those obtained using a large-displacement-based finite element analysis software package. Through this study, the applicability and limitations of using small-displacement-based software packages in simulating large displacement responses and structural collapse are examined with uncertainties quantified.

Structural Model

To investigate the coupling effects of geometric nonlinearity and material nonlinearity on the structural dynamic responses among solution algorithms, consider an eight-story, three-bay steel moment-resisting frame as shown in Fig. 1(a). The structural model of this frame consists of 8 lateral degrees of freedom (DOFs) and 112 plastic hinge locations (PHLs) as shown in the figure. Let the mass be 74 075 kg on each floor. No leaning column is used in the model, and therefore gravity loads acting on the frame as shown in Fig. 1(b) have been slightly magnified to reflect the additional gravity load that would have otherwise been acting on the leaning column. The initial periods (without consideration of geometric nonlinearity) and elastic periods (consideration of geometric nonlinearity due to gravity loads) for the eight modes of vibration based on flexural stiffness only, labeled as T1 to T8, are summarized in Table 1. Assume all 112 plastic hinges exhibit elastic-perfectly-plastic behavior to eliminate the differences in simulated responses caused by different implementation of hardening and strength loss rules in the solution algorithms. All plastic hinges are assumed to have 152 mm offset from the center of the beam-column connection, and panel zones are not modeled to simplified the analysis. Let the elastic modulus be 200 GPa and yield stress of steel be 345 MPa for all members. A more detailed explanation of the structural model for the implementation of each solution algorithm is discussed in the following subsections.

Table 1. Periods of vibration using different stiffness representations of the eight-story frame.

Stiffness	T1	T2	T3	T4	T5	T6	T7	T8
Initial	1.70 s	0.62 s	0.34 s	0.22 s	0.16 s	0.12 s	0.10 s	0.08 s
Elastic	1.81 s	0.65 s	0.36 s	0.23 s	0.16 s	0.12 s	0.10 s	0.08 s

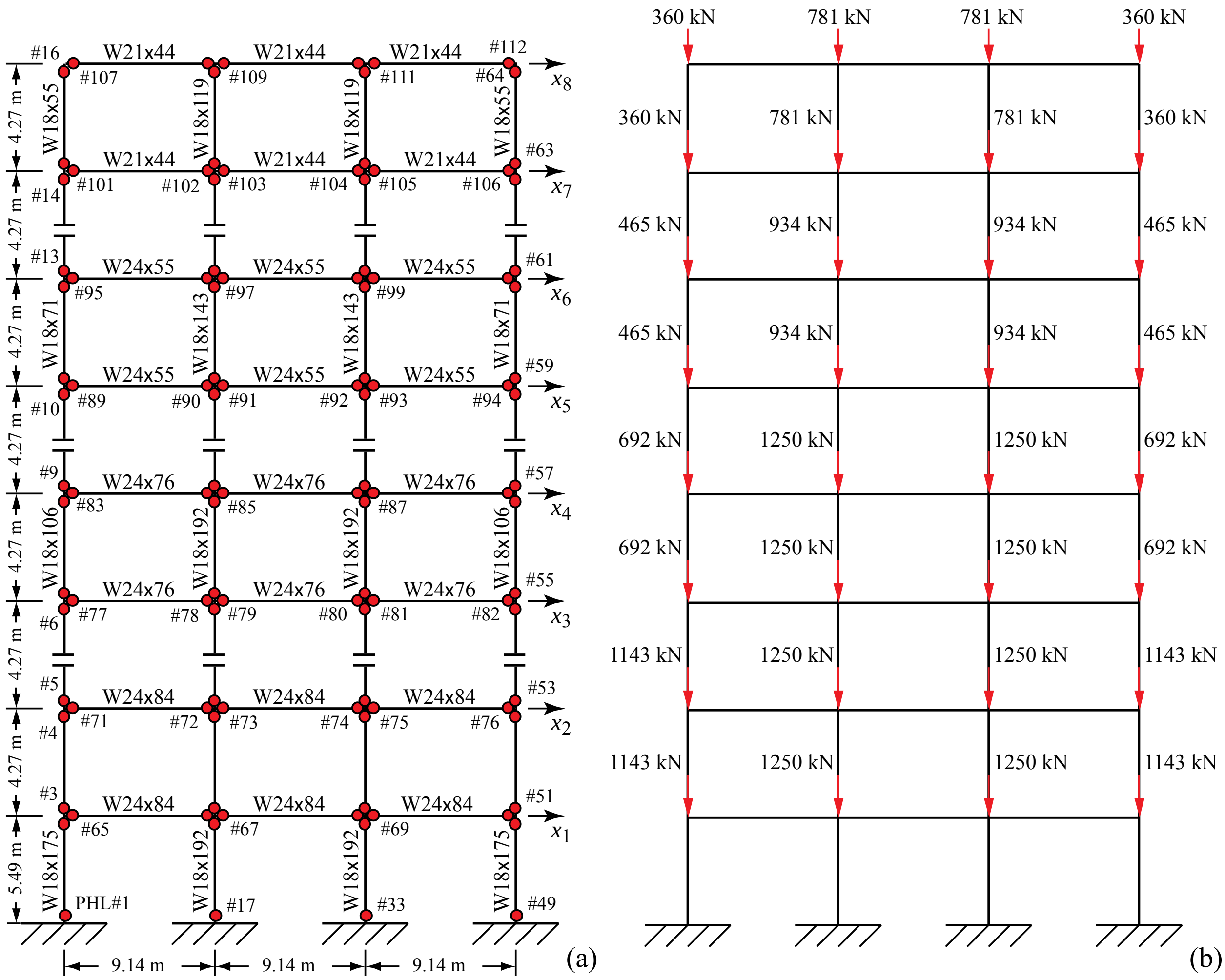

Figure 1. Eight-story three-bay steel moment frame with gravity loads.

A Small-Displacement-Based Algorithm Using P-Delta Stiffness Formulation (PD)

The investigated PD solution algorithm is embedded in a commercial nonlinear structural analysis software package that is commonly used in current practice to perform 3-D nonlinear response history analysis. For this reason, various features have been automatically included in the model development to simplify the input process, such as elastic shear deformation and yielding of column plastic hinges due to axial force and moment interactions. Including shear deformations in the structural model reduces the initial stiffness that is based on flexure only, and therefore the mass on each floor of the frame is reduced to 69 487 kg (a 6.2 % reduction) to give consistent periods of vibration of the frame as summarized in Table 1. In addition, this PD solution algorithm uses P-Delta stiffness formulation [5] that considers only large P-Δ effects while ignoring small P-δ effects. The target for damping is to achieve a Rayleigh damping having a mass proportional constant of 0.25 and a stiffness proportional constant of 0.0. However, because the PD solution algorithm automatically adds numerical damping to the analysis when geometric nonlinearity is considered, the damping constants are calibrated and scaled down in this case such that Rayleigh damping with mass proportional constant of 0.2132 and a stiffness proportional constant of 0.0 is used in the model.

A Small-Displacement-Based Algorithm Based on Corotation Formulation (CR)

The investigated CR solution algorithm is embedded in a research-oriented open-source nonlinear structural analysis software package for performing advanced nonlinear response history analysis. This algorithm provides the option to choose between the P-Delta stiffness formulation that considers only large P-Δ effects or the corotation formulation [6-7] that considers large rigid-body displacement with small strains. The corotation formulation has been selected for this study, yet it is still considered here as small-displacement-based because of its use of a small strain formulation. This CR solution algorithm takes Rayleigh damping as inputs, and therefore Rayleigh damping having a mass proportional constant of 0.25 and a stiffness proportional constant of 0.0 is used. Lumped plasticity is selected to model the yielding of plastic hinges, but the CR solution algorithm allows only the yield moment as input without consideration of axial force and moment interaction. In order to match the yielding characteristics of CR with those defined of PD, the yield moments of the plastic hinges are therefore computed based on the column axial forces from gravity loads.

A Small-Displacement-Based Algorithm Based on Stability Functions Formulation (SF)

The investigated SF solution algorithm is based on the nonlinear structural dynamic analysis theory to address material nonlinearity in a dynamic context [8] and uses the stability functions formulation [9] with both large P-Δ and small P-δ effects included for the geometric nonlinearity formulation. It requires damping inputs be in the form of modal damping, where the periods of vibration are based on the elastic stiffness of the frame as shown in Table 1. To achieve the target of Rayleigh damping having a mass proportional constant of 0.25 and a stiffness proportional constant of 0.0, the damping ratios are calculated by performing eigenvalue and eigenvector analyses and found to be 3.41 %, 1.22 %, 0.67 %, 0.43 %, 0.31 %, 0.23 %, 0.18 %, and 0.15 % among the eight modes of vibration. The material nonlinearity options of the SF solution algorithm are the least sophisticated among all the algorithms used in this study. Only a bilinear backbone curve with kinematic hardening is available for the model with a pre-defined yield moment. Therefore, in order for yielding characteristics of the model to be compatible with those defined in PD, the yield moments of the plastic hinges are computed based on the column axial forces due to the gravity loads, similar to the computation performed for the CR solution algorithm.

Large-Displacement-Based Algorithm Using Large Displacement Formulation (LD)

The investigated LD solution algorithm is embedded in a commercial finite element analysis software package that is based on a large displacement formulation with explicit time integration [10-11]. This type of formulation is often used in finite element analysis capable of capturing significant inelastic deformation, thereby capturing both geometric nonlinearity and material nonlinearity in every element. The drawback is that each member must be subdivided into many finite elements to capture the displacement profile. In the structural model, 10 elements are used to model each column member and 18 elements used to model each beam member, resulting in a significant increase in computational efforts. This LD solution algorithm takes Rayleigh damping as inputs, and therefore Rayleigh damping having a mass proportional constant of 0.25 and a stiffness proportional constant of 0.0 is used. For the yielding of plastic hinges, the interaction between the axial force and moments is considered at the integration points of the cross-section of each element, and therefore the yield stress for the members must be calibrated to match the yielding characteristics used in PD, CR, and SF. The yield stress for the beams are calibrated to 354 MPa for having no interaction with axial force, while the yield stresses for the columns are calibrated at every two floors. These yield stresses are calculated as 365 MPa, 379 MPa, 386 MPa, and 400 MPa, respectively, from the bottom two floor columns up to the top two floor columns.

Earthquake Ground Motions

A total of 7 earthquake ground motions is used in this study, and these ground motion time histories are presented in Fig. 2. Various scaling factors are used to intensify each earthquake ground motion to cause at and near collapses among LD, PD, CR, and SF solution algorithms.

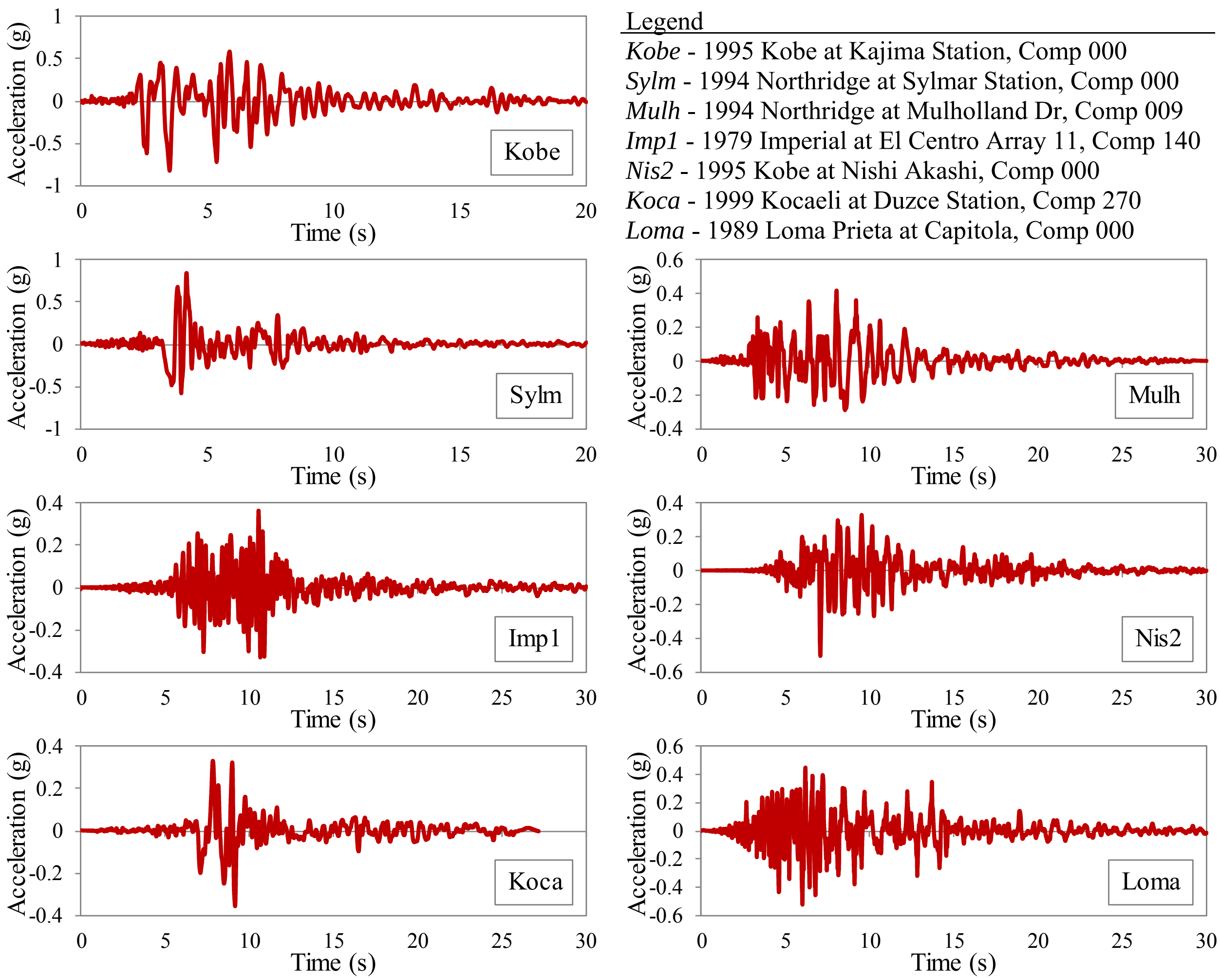

Figure 2. Investigated earthquake ground motions.

Nonlinear Response History Analyses

The eight-story frame shown in Fig. 1(a) is subjected to the seven earthquake ground motions shown in Fig. 2 with various scaling factors. Fig. 3 shows the roof displacement responses in two charts due to different scaling factors of the *Kobe* earthquake. The right chart shows that at least one solution algorithm indicates that the frame collapses at a scaling factor of 2.2. Based on this scaling factor, the left chart shows the roof displacement responses due to a slightly reduced scaling factor of 2.0 to cause the frame to reach the point of having large displacement but remains standing, which is defined here as near-collapse. Similar analyses are performed for the remaining six earthquake ground motions and the results are shown in Figs. 4 to 9. It can be seen from these figures that the responses during the first few seconds among the four solution algorithms are nearly identical, indicating that the structural models are consistent.

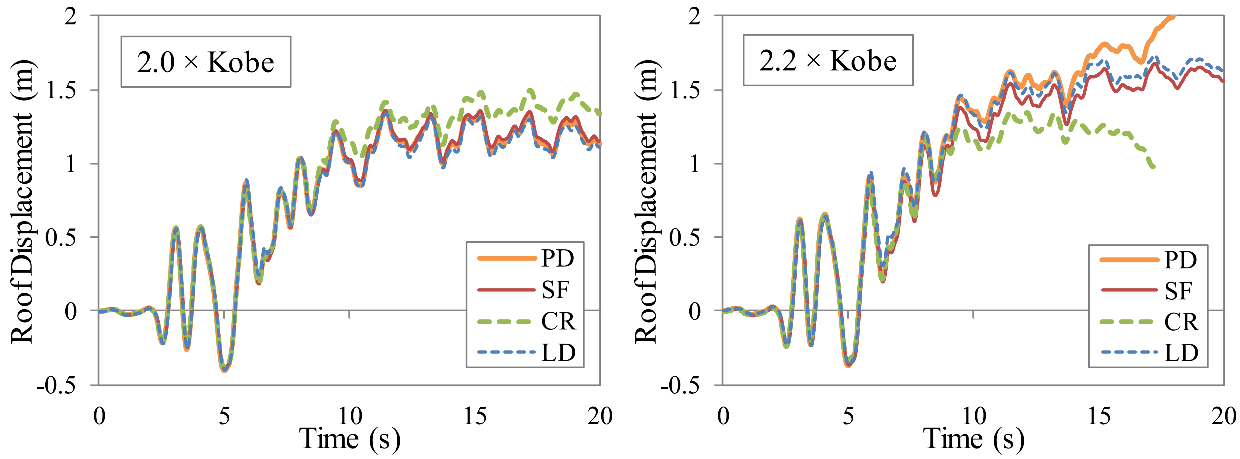

Figure 3. Roof responses of the eight-story frame at and near-collapse due to *Kobe*.

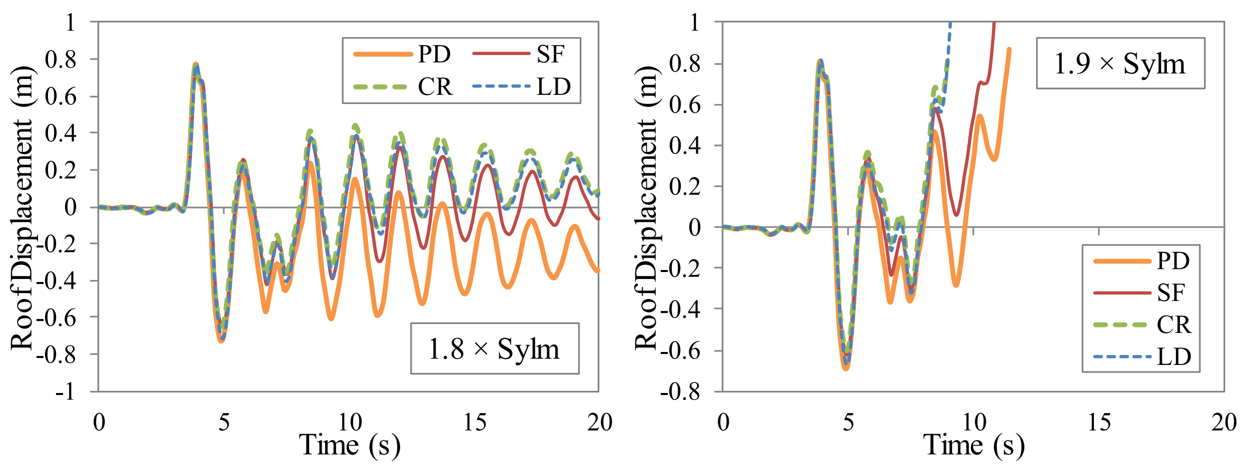

Figure 4. Roof responses of the eight-story frame at and near-collapse due to *Sylm*.

As shown in Figs. 3 to 9, the amplitudes and frequencies of oscillations of the post-yield responses among the four solution algorithms are about the same, except that the centers of oscillation have been shifted due to residual drifts caused by yielding of plastic hinges. This is particularly evident in Fig. 7 where significant differences for the center-of-oscillation on the left chart is observed among the solution algorithms toward the end of the 30s analysis. This suggests that the amplitudes and frequencies of oscillations have been captured consistently by the "dynamic" solvers among the four solution algorithms, but the "nonlinear" solvers among each algorithm are unable to capture consistent residual drifts that result in different offsets among the simulations. This observation highlights the differences in how the coupling between geometric nonlinearity and material nonlinearity in the nonlinear solver among each algorithm affects the simulated responses. Based on the observations of residual drifts in the figures, SF matches LD very well for *Kobe* (Fig. 3), *Imp1* (Fig. 6), and *Nis2* (Fig. 7), while CR matches LD very well for *Sylm* (Fig. 4). PD consistently produces different residual drifts from LD, but it matches LD the best for *Mulh* (Fig. 5) and *Loma* (Fig. 9). Finally, none of the three algorithms matches LD for *Koca* (Fig. 8). This suggests that SF is most suitable for capturing large displacement responses among the solution algorithms in terms of residual drifts.

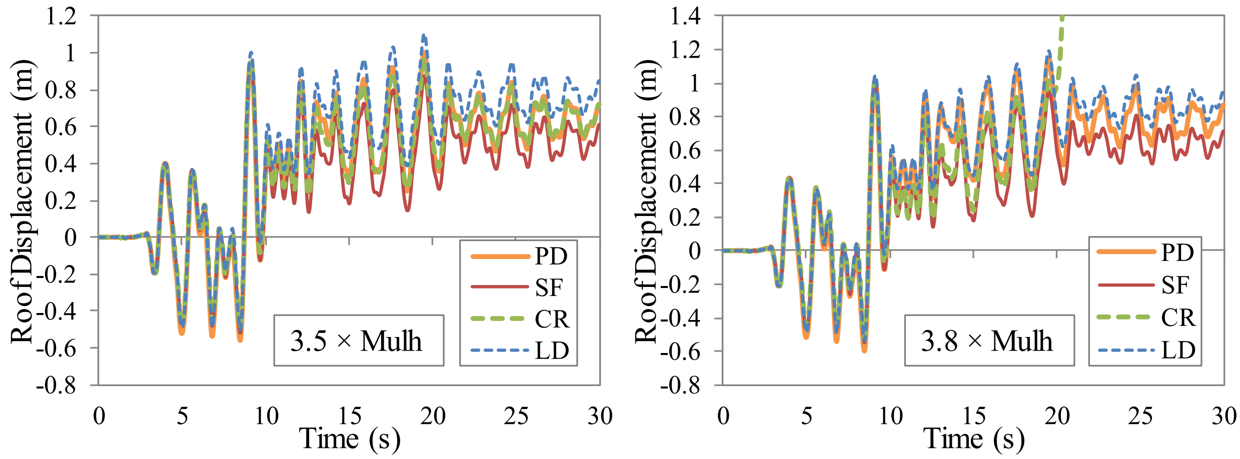

Figure 5. Roof responses of the eight-story frame at and near-collapse due to *Mulh*.

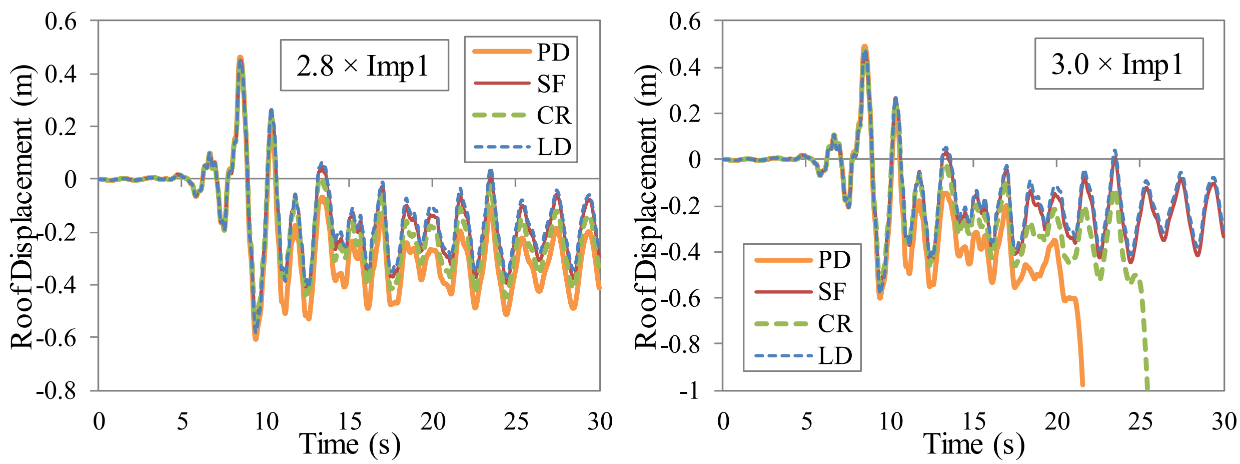

Figure 6. Roof responses of the eight-story frame at and near-collapse due to *Imp1*.

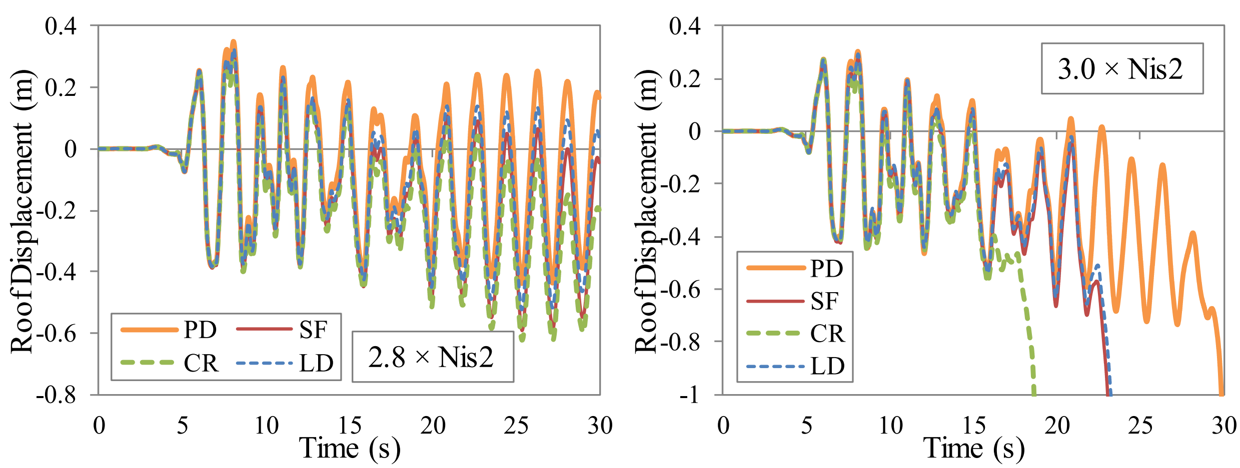

Figure 7. Roof responses of the eight-story frame at and near-collapse due to *Nis2*.

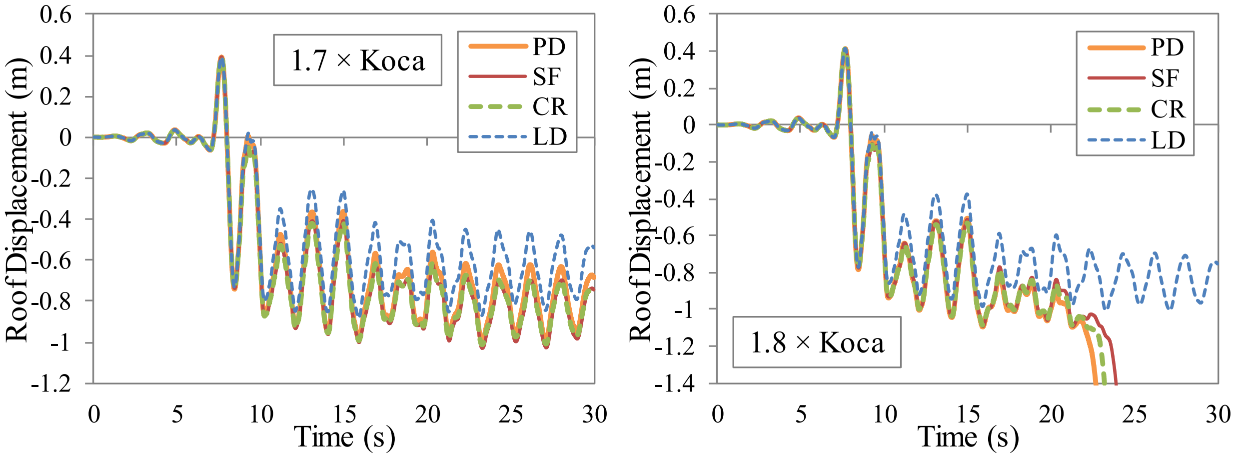

Figure 8. Roof responses of the eight-story frame at and near-collapse due to *Koca*.

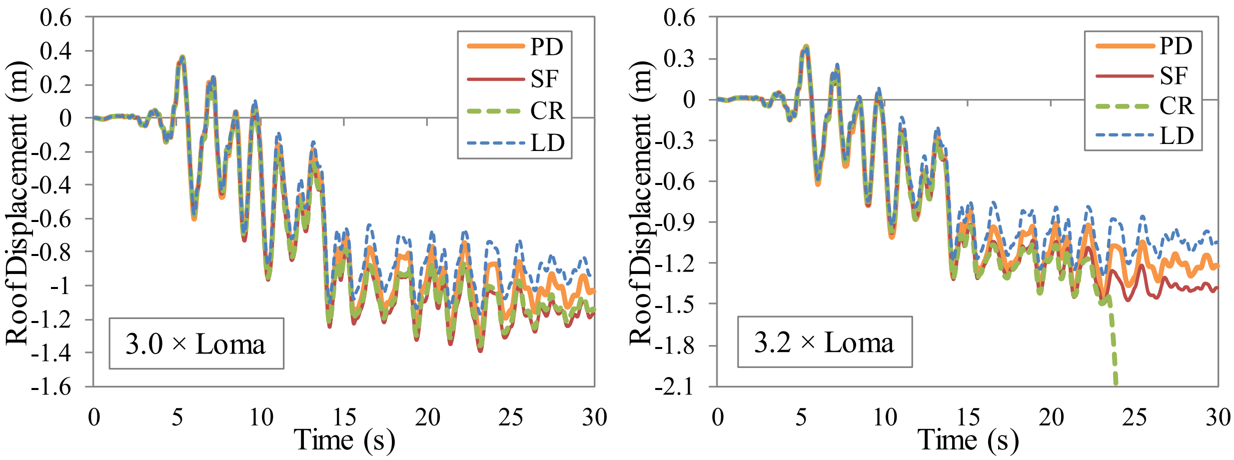

Figure 9. Roof responses of the eight-story frame at and near-collapse due to *Loma*.

Table 2 summarizes the maximum roof displacement responses at near-collapse (i.e., left charts of Figs. 3 to 9) of the 8-story frame. Here, the LD results are used as the comparison standard. The percentage differences between the maximum responses simulated using LD and those simulated using PD, CR, and SF are summarized in the table. Similarly, Table 3 summarizes the maximum roof displacement responses at collapse (i.e., right charts of Figs. 3 to 9) and the associated errors in the prediction (i.e., the situation where LD predicting collapse while PD, CR, or SF predicting the frame remains standing, or vice versa). The percentage differences highlighted in red with an '×' symbol indicate that the prediction of collapse is an error. As shown in Table 3, SF has only one prediction error, while PD has three and CR has five.

The percentage differences highlighted in green in Tables 2 and 3 indicate that those values are minimum across each row or those indicate that collapse is predicted correctly with a '√' symbol, demonstrating which solution algorithm has the smallest percentage difference. It is observed that in some cases one solution algorithm predicts a closer maximum roof displacement response than another solution algorithm, and vice versa for others. In particular, PD shows a better prediction of maximum roof displacement responses when the frame is at near-collapse, while SF shows a better prediction when the frame collapses. Therefore, there is no consistent trend on which small-displacement-based solution algorithm can produce results that match closer to those simulated using large-displacement-based solution algorithm, except that CR consistently produces poor results.

Table 2. Maximum roof displacements of 8-story frame at near-collapse.

EQ	LD	PD		CR		SF	
	Disp (m)	Disp (m)	% Diff	Disp (m)	% Diff	Disp (m)	% Diff
2.0 × *Kobe*	1.340	1.344	0.3 %	1.494	11.5 %	1.357	1.3 %
1.8 × *Sylm*	0.771	0.771	0.0 %	0.768	0.4 %	0.771	0.0 %
3.5 × *Mulh*	1.104	1.002	9.2 %	0.972	11.9 %	0.943	14.6 %
2.8 × *Imp1*	0.582	0.607	4.4 %	0.545	6.2 %	0.551	5.3 %
2.8 × *Nis2*	0.524	0.439	16.3 %	0.623	19.0 %	0.592	12.9 %
1.7 × *Koca*	1.104	1.002	9.2 %	0.972	11.9 %	0.943	14.6 %
3.0 × *Loma*	1.170	1.312	12.2 %	1.358	16.2 %	1.393	19.1 %

Table 3. Maximum roof displacements of 8-story frame at collapse.

EQ	LD	PD		CR		SF	
	Disp (m)	Disp (m)	% Diff	Disp (m)	% Diff	Disp (m)	% Diff
2.2 × *Kobe*	1.728	∞	×	∞	×	1.683	2.6 %
1.9 × *Sylm*	∞	∞	√	∞	√	∞	√
3.8 × *Mulh*	1.192	1.115	3.4 %	∞	×	0.993	16.7 %
3.0 × *Imp1*	0.582	∞	×	∞	×	0.549	5.7 %
3.0 × *Nis2*	∞	∞	√	∞	√	∞	√
1.8 × *Koca*	1.006	∞	×	∞	×	∞	×
3.2 × *Loma*	1.294	1.448	11.9 %	∞	×	1.497	15.6 %

A more objective way of comparing Table 2 is through uncertainties as shown in Table 4, where the means and standard deviations of the percentage differences are evaluated at near-collapse of the frame. Table 4 shows the PD solution algorithm simulates responses that have only 7.4 % differences from the LD responses with a standard deviation of 5.6 %, indicating that the PD solution algorithm simulates the maximum responses with less uncertainty. This is followed by the SF solution algorithm with a mean of 9.7 % and finally by CR with a mean of 11.0 %.

Table 4. Uncertainties in percentage differences of maximum roof displacements.

	PD	CR	SF
Mean	7.4 %	11.0 %	9.7 %
Standard Deviation	5.6 %	5.7 %	6.9 %

Conclusions

Different small displacement formulation makes different assumptions in their solution algorithms to produce nonlinear structural dynamic responses. The assumption of geometric nonlinearity has been investigated by selecting three small-displacement-based software packages using P-Delta stiffness formulation (PD), corotation formulation (CR), and stability functions formulation (SF). Coupling these differences in geometric nonlinearity with each inherent material nonlinearity assumptions result in nonlinear solvers that are quite different among each small-displacement-based solution algorithm. This results in output responses that are quite difficult to track. In this study, a consistent 8-story steel moment-resisting frame model has been developed using PD, CR, and SF, and the output roof displacement responses due to seven earthquake ground motions are compared with those from LD, a large-displacement-based finite element analysis software package that is assumed to produce the most precise responses. Comparison of responses shows that consistency is obtained in the amplitudes and frequencies of oscillations, suggesting that the dynamic solvers among each solution algorithm are reasonably consistent. The inconsistency occurs in the residual drifts predicted by each solution algorithm, suggesting that the nonlinear solvers for handling coupled material and geometric nonlinearities are the source of inconsistency.

Uncertainties are quantified due to the inconsistency among the three small-displacement-based solution algorithms. Based on comparisons with LD responses, results show that SF has better performance based on residual drifts, while PD has better performance based on maximum displacements. At the same time, SF has fewer number of errors in predicting collapses. This suggests that further research on a more objective method of quantifying uncertainties in response history analyses is needed.

References

1. Lignos DG, Krawinkler H, Whittaker AS. Prediction and validation of sidesway collapse of two scale models of a 4-story steel moment frame. *Earthquake Engineering and Structural Dynamics* 2011; **40** (7): 807-825.
2. Grigorian M, Grigorian CE. Lateral displacements of moment frames at incipient collapse. *Engineering Structures* 2012; **44**: 174-185.
3. Eads L, Miranda E, Krawinkler H, Lignos DG. An efficient method for estimating the collapse risk of structures in seismic regions. *Earthquake Engineering and Structural Dynamics*, 2013; **42** (1): 25-41.
4. Domizio M, Ambrosini D, Curadelli O. Experimental and numerical analysis to collapse of a framed structure subjected to seismic loading. *Engineering Structures* 2015; **82**: 22-32.
5. Powell GH. *Modeling for Structural Analysis: Behavior and Basics*, Computers and Structures: California USA, 2010.
6. Belytschko T, Hsieh BJ. Nonlinear transient finite element analysis with convected coordinates. *International Journal for Numerical Methods in Engineering* 1973; **7** (3): 255-271.
7. Sivaselvan MV, Reinhorn AM. Collapse analysis: Large inelastic deformations analysis of planar frames. *Journal of Structural Engineering ASCE* 2002; **128** (12): 1575-1583.
8. Li G, Wong KKF. *Theory of Nonlinear Structural Analysis: The Force Analogy Method for Earthquake Engineering*. John Wiley & Sons: Singapore, 2014.
9. Wong KKF, Speicher MS. Improved Method for the Calculation of Plastic Rotation of Moment-Resisting Framed Structures for Nonlinear Static and Dynamic Analysis. *Computational Methods in Earthquake Engineering Vol. 3*. Springer: Switzerland, 2017; 129-175.
10. Belytschko T. Liu WK. *Nonlinear Finite Elements for Continua and Structures, 2nd Edition*. John Wiley & Sons: Chichester UK, 2014.
11. Reddy JN. *An Introduction to Nonlinear Finite Element Analysis, 2nd Edition*. Oxford University Press: Oxford UK, 2015.