# True value and uncertainty in the GUM



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# True value and uncertainty in the GUM

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**Abstract.** The signal contribution of the Guide to the Expression of Uncertainty in Measurement (GUM) is the operational view of the uncertainty in measurement as a parameter, associated with a result of a measurement (measured value), that characterizes the dispersion of the values that could reasonably be attributed (assigned) to the measurand. Subsequent documents from the Joint Committee for Guides in Metrology (JCGM) have restored an essentially pre-GUM view of uncertainty and called it a coverage interval. The idea of a coverage interval requires the measurand to have a unique true value; therefore, it does not apply to an ordinary measurand that has a range of true values. Most measurands of interest in metrology are ordinary measurands. Therefore, the JCGM idea of a coverage interval is of little use in metrology. We think that the divergence of the JCGM documents from the GUM is a consequence of misunderstanding (i) the true value and related concepts in the GUM, and (ii) the operational view of uncertainty. This paper is an attempt to correct these misunderstandings.

## 1. Introduction

We follow the GUM definitions of a measured value (result of measurement in the GUM) and uncertainty in measurement [1]. We follow the JCGM 200 (VIM3) definitions of a quantity, measurand, quantity value, true value, measurement, and result of measurement [2]. Measurement is not anymore determination of a true quantity value but an assignment of a result of measurement to describe (characterize) an unknown quantity. In this paper, definitions and phrases displayed in the *italic* font are direct quotes from a cited reference; sometimes additional words are inserted in parentheses to clarify the intended meaning. We will introduce some new definitions which are also shown in the *italic* font, but underlined to differentiate them from the cited definitions and phrases. We will use the shorter term 'uncertainty' for 'uncertainty in measurement'.

A measurand-affecting quantity is a *quantity which affects the magnitude of measurand*. Measurands can be divided into two classes: (i) special measurands and (ii) ordinary measurands. A special measurand is *a measurand that is defined to have a unique true value*. Fundamental physical constants of nature are special measurands. An ordinary measurand is *a measurand that is specified by its description, including specification of significant measurand-affecting quantities*. Every ordinary measurand has an interval (range) of true values [1], [2]. Quantity values can be divided into two classes: (i) theoretical quantity values, and (ii) assigned quantity values. A theoretical quantity value is *a conceptual value that is assumed to exist, but is unknowable even in principle*. A true value of a

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quantity is a theoretical value. Error in a measured value relative to a true value is conceptual and unknowable. An assigned quantity value is *a value assigned to a quantity to characterize it, based on measurement*. The reference value of a material measure is an assigned value [2]. Error in a measured value relative to an assigned value is known.

Everybody wants a measured value to be sufficiently close to true value; that is, accurate for the purpose. Accuracy is a qualitative concept. A system of legal metrology and measurement laboratories help assure the accuracy of measured values by maintaining the measuring systems in a calibrated state. Suppose the observed error relative to a reference value is sufficiently small, and the calibrated measuring system is subsequently used to measure an unknown quantity. In that case, it would be reasonable to expect that the error in a measured value relative to a true value of the unknown quantity would also be small, albeit unknowable. The reference values of measurement standards used for calibration of measuring systems are assigned values. Thus, theoretical true values have no operational role in metrology. Nonetheless, measured values of any required qualitative accuracy can be obtained.

The essential GUM is the GUM excluding the Annex G and its linkages with the rest. The essential GUM is a stand-alone, complete, and logically consistent guide to the expression of uncertainty in measurement. The GUM declares Type A evaluations (based on formulas from conventional descriptive statistics) as parameters of a probability distribution expressing the state of knowledge [1]. Thus, the essential GUM has only one interpretation of a probability distribution and is logically consistent. Therefore, any future revision of the GUM must be based on the essential GUM. We refer only to the essential GUM.

The description of every ordinary measurand is necessarily incomplete. Knowingly or unknowingly, a quantity which affects the magnitude of measurand is always left unspecified. When a quantity which affects the magnitude of measurand is left unspecified, a range of quantity values satisfy the definition of true value. Also, the magnitude of an ordinary measurand can change with time. Thus, every ordinary measurand has a range of true values and positive definitional uncertainty. Generally definitional uncertainty is neglected; however, it can be included in the combined uncertainty (when it is suspected to be non-negligible). Most measurands of interest in metrology are ordinary measurands. Therefore, a useful concept of uncertainty must apply to an ordinary measurand.

## 2. True value and related concepts in the GUM

The GUM states that "*This Guide is primarily concerned with the expression of uncertainty in the measurement of a well-defined physical quantity - the measurand - that can be characterized by an essentially unique value*" [1], [3]. This is the 'essentially unique value requirement' of the GUM. The only requirements on the measurand for the applicability of the GUM are that the measurand is (i) sufficiently well-defined in view of the desired qualitative accuracy of the measurement and that (ii) it can be adequately characterized (represented) by a single scalar measured value with standard uncertainty. Therefore, the essentially unique value requirement of the GUM can only mean that the range of true values of an ordinary measurand is sufficiently small for it to be adequately characterized by a single scalar measured value and its associated standard uncertainty. It does not mean that the measurand is required to have a unique true value. In reference [4], the chairman of JCGM/WG2 connects (i) the essentially unique value requirement, (ii) the neglect of definitional uncertainty, and (iii) the GUM interpretation of a probability distribution. He writes, "*it is necessary that … definitional uncertainty be small compared to the measurement uncertainty, or otherwise the probabilistic interpretation of the GUM becomes very complicated, if not meaningless. …. if the definitional uncertainty is significant … then there is no essentially unique true value of the measurand, … and so the meaning of the PDF (probability density function) curve becomes somewhat obscure*" [4]. The reference [4] is wrong. In the GUM, there is no connection between the essentially unique value requirement and the neglect of definitional uncertainty. Further, in the GUM, the interpretation of a probability distribution is unaffected by the neglect or inclusion of definitional uncertainty.

## 3. The operational view of uncertainty in measurement as established by the GUM

Uncertainty in measurement is an operational concept that does not refer to the theoretical concepts of true value and error. The following additional clarifications appear in the GUM [1].

*The focus of this Guide is on the measurement result and its evaluated uncertainty rather than on the unknowable quantities "true" value and error (see Annex D). By taking the operational views that the result of a measurement (best assigned value) is simply the value attributed to the measurand and that the uncertainty of that result is a measure of the dispersion of the values that could reasonably be attributed to the measurand, this Guide in effect uncouples the often-confusing connection between uncertainty and the unknowable quantities "true" value and error.*

The first paragraph of the Annex D in the GUM states that *Two figures are presented to illustrate why the concept of uncertainty adopted in this Guide is based on the measurement result and its evaluated uncertainty rather than on the unknowable quantities "true" value and error.*

*The result of a measurement (after correction) can unknowably be very close to the value (a true value) of the measurand (and hence have a negligible error) even though it may have a large uncertainty.*

*But, even if the evaluated uncertainties are small, there is still no guarantee that the error in the measurement result is small.*

In reference [4], the chairman of JCGM/WG2 quotes the GUM definition of uncertainty and writes, "*Here (in the GUM definition of uncertainty) the term 'the values' is used, but again what is meant is 'the true values'. Without the concept of true value, this definition also does not make sense"* [4]. The reference [4] is wrong. A value attributed to a measurand is a measured value. Therefore, in the definition of uncertainty the term 'the values' refers to 'measured values'. Thus, the uncertainty in measurement characterizes the dispersion of measured values. A reasonable inference is that to the chairman of JCGM/WG2, the operational view of the uncertainty does not make sense. Furthermore, the chairman of JCGM/WG2 writes, "*3.6.1. Possible new definition of measurement uncertainty. Looking ahead a little to the next editions of the VIM and GUM, a definition of measurement uncertainty might be considered more along the lines of the definition in the first edition of the VIM shown earlier, with the important addition of the concept of degree of belief, expressed as a probability: [VIM4 ?] measurement uncertainty: parameter (or parameters) characterizing how well the (essentially unique) true value of the measurand is believed to be known"* [4]. Thus, the chairman of JCGM/WG2 would like to supplant the operational view of uncertainty in measurement with an essentially pre-GUM view from the VIM1 [5]. The confidence in a result of measurement assigned to a measurand comes from scientific and technical understanding of the measuring technique, authenticity of the current observations and previous data, adequacy of the model of measurement procedure, and reasonableness of the probabilistic assumptions made.

## 4. Coverage interval is a pre-GUM concept of uncertainty that does not apply to an ordinary measurand

The JCGM 101 has introduced a concept of uncertainty in measurement called a coverage interval with a stated coverage probability. The JCGM 101 defines a coverage interval as an *interval containing the (true) value of a quantity (the measurand) with a stated probability, based on the information available* [3]. The corresponding coverage probability is the *probability that the (true) value of a quantity (the measurand) is contained within a specified coverage interval* [3]. The 1984 edition of the International Vocabulary of Metrology (VIM1) defines uncertainty in measurement as an estimate characterizing the range of values within which the true value of a measurand lies [5]. Thus, the JCGM 101 has restored an essentially pre-GUM view of uncertainty that is concerned with stating the probability that the true value is captured in a computed interval. The JCGM 101 concept of a coverage interval requires the measurand to have a unique theoretical true value [6]. So, the JCGM 101 concept of a coverage interval does not apply to an ordinary measurand that has an interval (range) of true values.

## 5. Summary and concluding remarks

Uncertainty in measurement is an operational concept that refers only with the quantity values assigned to a measurand based on the available information, the model of measurement procedure, and the probabilistic assumptions used. Uncertainty does not refer to the theoretical concept of true value. So, it applies equally well to an ordinary measurand (which has a range of true values and positive definitional uncertainty) and a special measurand (which is defined to have a unique true value). The JCGM documents have restored an essentially pre-GUM view of the uncertainty and called it a coverage interval with a stated coverage probability. The idea of coverage interval requires the measurand to have a unique true value. Therefore, the idea of coverage interval does not apply to an ordinary measurand that has a range of true values, however small that range may be. In metrology, most measurands of interest are ordinary measurands. So, the idea of coverage interval is of little use in metrology.

## 6. References

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