# Equivalent Isotropic Response as a Surrogate for Incident Field Strength

Daniel G. Kuester, Duncan A. McGillivray, John Ladbury, Adam Wunderlich,
Ari Feldman, William F. Young, and Sheryl Genco

Communications Technology Laboratory
National Institute of Standards and Technology
Boulder, CO, USA
daniel.kuester@nist.gov, duncan.a.mcgillivray@nist.gov, john.ladbury@nist.gov, adam.wunderlich@nist.gov,
ari.feldman@nist.gov, william.young@nist.gov, sheryl.genco@nist.gov

***Abstract*—The strength of an electromagnetic plane wave incident in the free field can be characterized in terms of power output by an idealized isotropic antenna probe. We refer to the parameter as equivalent isotropic incident power (EIIP), though it lacks an accepted name. This parameter has begun to enter use in various industry standards, technical reports, and peer-reviewed papers. To our knowledge, however, it has not been defined or studied in detail by prior work. We start to address this gap here with a proposed a definition, physical interpretation, and comparison to field strength.**

## I. Introduction

Stakeholders in various spectrum sharing scenarios are increasingly asked to specify and test impacts of new systems upon incumbent spectrum users. In these problems, plane waves from multiple radiators impinge upon each receive system with different angles, frequencies, and waveforms. The strength of the wave incident from each radiator upon each receiver must be understood clearly in order to enable direct comparison or combination among simulations, tests, and analytical models. The ideal parameter fits into 1) established terminology, 2) radiated "black box" testing of receiver systems with integrated antennas, and 3) simple, direct application to link budgeting.

A parameter that is an alternative to incident field strength has quietly entered use for this purpose [1]–[7]. The idea is to characterize plane wave strength in terms of the output power response of a hypothetical isotropic probe antenna. It is the complement to EIRP on the receive side of the Friis equation.

We summarize here this "equivalent isotropic" receive parameter, which we call EIIP. We propose an explicit physical and mathematical definition, offer some interpretation of the parameter, and discuss its relationship with the standardized antenna terminology.

## II. Definitions

Terminology standardized in [8] includes the well-known effective isotropic radiated power (EIRP) as

$$\mathrm{EIRP} = P_t G_t. \tag{1}$$



Here, $G_t$ is the transmit antenna absolute gain (polarization losses are not included – there is no standardized "partial EIRP"). An interpretation of EIRP is: "the power absorbed by a lossless isotropic antenna that excites far-field plane waves equivalent to the transmit system along a free space path."

The parameter finds use in regulation, system models, and tests for which internal "subsystem" parameters are not known. Like gain, an EIRP value could be specified as a pattern plot, a value at some specified transmit antenna orientation like boresight, or an implied maximum value.

On the other side of the link, the receiving antenna is impinged by an incident electric field with magnitude $|E_r|$, polarized with the transmit antenna. The receive antenna outputs available power $P_r$, depending on the partial gain of its antenna $G_r e_p$, where $G_r$ is receive antenna absolute gain and $e_p$ is the link polarization efficiency. Consider the following definition of "equivalent isotropic *incident* power" to relate these parameters:

$$\mathrm{EIIP} = \frac{P_r}{G_r e_{pr}} = \frac{|E_r|^2}{\eta_0} \frac{\lambda_0^2}{4\pi}, \tag{2}$$

where wavelength $\lambda_0$ and $\eta_0 = \sqrt{\mu_0/\epsilon_0} \approx 377\,\Omega$. The name is meant to emphasize the nature of the parameter — a surrogate for incident field strength and complement to EIRP. A physical interpretation of EIIP is: "the output power available from an isotropic antenna impinged upon by a plane wave with field strength $|E_r|$ and $e_{pr} = 1$." The EIIP does not vary with receive antenna orientation because the reference antenna is defined as isotropic — the factor $1/(e_{pr} G_r)$ cancels the orientation dependence in $P_r$.

## III. EIIP in Link Analysis and Testing

*a) Reference Polarization:* We decompose the complete Friis link polarization as follows:

$$e_p = |\hat{\rho}_r \cdot \hat{\rho}_t^*|^2 = |\hat{\rho}_r \cdot \hat{\rho}_{\mathrm{ref}}^*|^2 |\hat{\rho}_{\mathrm{ref}} \cdot \hat{\rho}_t^*|^2 = e_{pr} e_{pt}, \tag{3}$$

since dot products commute and $|\hat{\rho}_{\mathrm{ref}} \cdot \hat{\rho}_{\mathrm{ref}}^*| = 1$.

The reference polarization efficiencies for the transmitter and receiver ($e_{pt}$ and $e_{pr}$) are determined by the corresponding antenna polarization vectors ($\hat{\rho}_t$ and $\hat{\rho}_r$) and some specified

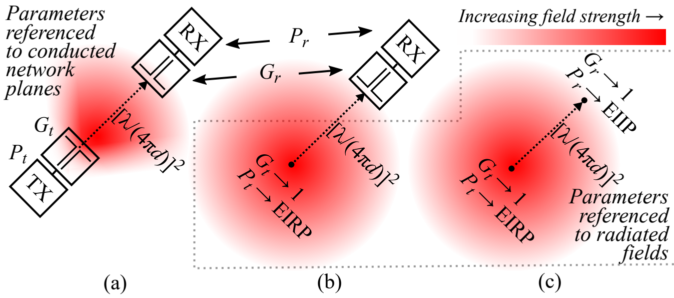


Fig. 1. An actual transmit system (a) and the "equivalent" radiator (b)-(c) excite plane waves with strength that is equal only along the dotted lines. The available output power from the actual receive antenna (a)-(b) is $P_r$; the output power response of an isotropic antenna probe (c) given an equivalent incident plane wave is EIIP.

*reference* polarization vector ($\hat{\rho}_{\text{ref}}$). The $\hat{\rho}_{\text{ref}}$ can be chosen arbitrarily to suit an application, but needs to be specified (as with partial gain parameters).

*b) "Equivalent Isotropic" Friis Transmission Equation:* The EIRP and EIIP definitions in (1) and (2) can substitute directly into the Friis transmission equation, as illustrated in Fig. 1. The relationship between equivalent isotropic parameters in free space is

$$\text{EIIP} = \text{EIRP}\left(\frac{\lambda_0}{4\pi d}\right)^2 e_{pt}, \tag{4}$$

where $d$ is the separation distance between antennas. Equation (4) quantifies radiated field strength excited by the transmit antenna only along the path between the antennas. Like incident field strength, it does not depend on parameters defined inside the receive system (such as $P_r$, $e_{pr}$, and $G_r$), making it a "black box" characterization.

*c) Units:* Labeling EIRP values with power units is standard practice. The same can apply to EIIP. A potential source of confusion, however, is that these parameters do not correspond with any measurable conducted power. One approach to emphasize this distinction could be to borrow the "i" from the "dBi" of antenna gain: "dBWi" or "dBmi" for EIRP, or "dBW/i" or "dBm/i" for EIIP.

*d) Receiving System Response to EIIP:* If a characterized receiving system is excited at some known EIIP level, then

$$P_r\,(\text{dBm}) = \text{EIIP}\,(\text{dBm/i}) + G_r\,(\text{dBi}) + e_{pr}\,(\text{dB}) + e_m\,(\text{dB}). \tag{5}$$

This equation is a means to determine received power in wireless link budgets from 1) internal receive parameters $G_r$, $e_{pr}$, and matching efficiency $e_m$, and 2) the incident plane wave strength via EIIP. The $P_r$ result is subject to the usual far-field link estimation constraints and has the expected orientation dependence via $G_r$ and $e_{pr}$.

*e) Modulated Field Approximation:* If the incident field is modulated, its power spectral density is distributed across a range of frequencies, not the single tone implied by (2). An approximate relationship in terms of RMS power is

$$\text{EIIP} \approx \frac{\text{E}\left[|E_r(t)|^2\right]}{\eta_0}\frac{\lambda_c^2}{4\pi}, \tag{6}$$

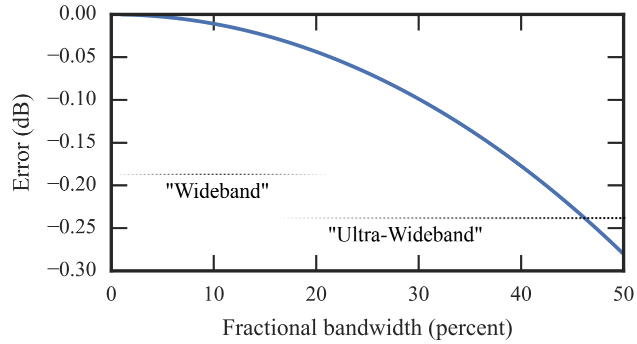


Fig. 2. Error in converting RMS EIIP to mean-squared field strength with the approximation (6) for band-limited white Gaussian noise signals.

by substitution into (2). Now $\lambda_c$ is the wavelength at the modulation center frequency, and $\overline{|E_r|^2}$ is the expected value of $|E_r(t)|^2$ ("mean-squared" field strength). The approximation error in (6) depends on the power spectral density function of the field modulation. For the special case of band-limited white Guassian noise, this error is shown in Fig. 2.

## IV. Conclusion

The EIIP parameter has the properties we sought in the introduction. Further, the definition of (2) means that an EIIP can be computed from measurements of either antenna gain and RF power or field strength, producing a derived metrology quantity. Errors caused by under-defined probe response to modulation present issues of definitional uncertainty.

Still, there are caveats to use of EIIP. Analytical conversion between field strength and EIIP involving ultra-wideband modulated waveforms need to be treated carefully. Few commercial field strength probes are specified for use with modern communications waveforms, so the most appropriate class of test equipment to measure these quantities is not clear. The implications of the parameter's use in realistic propagation conditions are also unclear. Application of EIIP in these problem areas could benefit from more research in the future.

## References

[1] "User Equipment (UE) Conformance Specification for UE Positioning; Part 1: Conformance Test Specification," 3GPP, Tech. Rep. TS 37.571-1, 2016.

[2] "Measurements of radiated performance for MIMO and multi-antenna reception for HSPA and LTE terminals," 3GPP, Tech. Rep. TR 37.976 v11.0.0, 2012. [Online]. Available: http://www.3gpp.org/DynaReport/37976.htm

[3] P. V. Nikitin and K. V. S. Rao, "Antennas and Propagation in UHF RFID Systems," in *2008 IEEE Int. Conf. RFID*, 2008, pp. 277–288.

[4] S. Saunders and A. Aragón-Zavala, *Antennas and Propagation for Wireless Communication Systems*, 2nd ed. John Wiley & Sons, 2007.

[5] "High Rate 60 GHz PHY, MAC and HDMI PAL," Ecma International, Geneva, CH, Tech. Rep. ECMA Std. 387, dec 2008.

[6] A. Saakian, *Radio Wave Propagation Fundamentals*. Artech House, 2011.

[7] "Standard for Air Interface for Broadband Wireless Access Systems," IEEE, Tech. Rep. Std. 802.16n-2013, 2013.

[8] "IEEE Standard for Definitions of Terms for Antennas," IEEE Standards Association, Tech. Rep. Std. 145-2013, 2013.