Design of a Table-Top Watt Balance

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Abstract:

Measurements of the Planck constant with watt balances using 1 kg masses have achieved relative standard uncertainties below 2×10^{-8}. Having established a metrological link between the kilogram and this fundamental constant of nature, a redefinition of the International System of units (SI) is planned and likely to occur in 2018. In the revised SI, watt balances can be used to realize the unit of mass at any value, not just at the cardinal point of 1 kilogram. In this article, we discuss two models of table top watt balances that we have recently built at the National Institute of Standards and Technology. We aim to have a capacity of 10 g with a relative standard uncertainty of few parts in 10^6 with these table top watt balances.

1. Introduction

The watt balance is an electromechanical balance that is self-calibrating. The idea was first published in 1976 by Bryan Kibble [1]. The measurement is basically performed in two modes. In the first mode, the weight of a mass is compensated by an electromagnetic force,

$$mg = IBl, \tag{1}$$

where I is the current in a coil of wire with length l that is immersed in a magnetic field with flux density B. The current can be easily measured to high precision by passing it through a known resistor R and measuring the voltage across it.

In the second mode, the geometric factor Bl is obtained by moving the coil vertically through the magnetic field with a velocity v resulting in an induced voltage V. The symmetry in Maxwell's equations is such that the quotient of induced voltage to velocity is exactly the geometric factor

$$Bl = \frac{V}{v}. \tag{2}$$

Substituting Bl from equation (2) into equation (1) yields the watt balance equation,

$$mgv = VI. \tag{3}$$

It is named after the unit of power, the watt. The left side of equation (3) is the mechanical power necessary to move a mass vertically in the gravitational field with a velocity v and the right side of the equation is the electrical power. In the measurement described above, the power comparison is only virtual, i.e., the two factors on each side of the equation are measured in different modes and not at the same time. For example, in the weighing mode, the current I is measured, while in the velocity mode the voltage V is measured.

A detail left out so far, is the fact that the velocity and the force are vectors, whereas the voltage and the currents are scalars. Hence, it is important that the velocity and the force be parallel to each other so we can use the dot product on the left side, i.e.,

$$m\vec{g} \cdot \vec{v} = VI. \tag{4}$$

The electrical power, right side of equation (4) can be expressed as a product of two frequencies and the Planck constant, see e.g., [2] for details.

Currently, the watt balance is used to measure a precise value of the Planck constant. It seems likely that the International System of Units (SI) will be redefined in the next few years. In the new SI, seven fundamental constants will have a fixed numerical value, with no uncertainty. These seven constants can be used to realize the units. The unit of mass, the kilogram, will be realized by a fixed number of the Planck constant, the speed of light and the hyperfine splitting frequency of Caesium-133.

Watt balances that operate on masses at the kilogram level are costly and complex, because they require relative measurement uncertainties of order 2 parts in 10^8 to be competitive with the current realization of the kilogram. At lower nominal masses values, larger relative uncertainties occur in the current dissemination chain. For example according to the recommendation of the Organisation Internationale de Métrologie Légale (OIML) [3] for a 10 g mass of class E_2 a one-sigma-standard uncertainty of 10 µg is permissible. This corresponds to a relative uncertainty of 1 part in 10^6. Hence, we formulate our goal to build a table top watt balance capable of measuring 10 g with a standard uncertainty of 10 µg.

A direct realization of the mass at the 10 g level to the Planck constant can have far reaching consequences for weighing applications at the factory floor. Such a balance would eliminate the need of calibration weights.

Kibble and Robinson [4] showed recently that for a mechanical system where the motion can be completely described by one variable, i.e., a system with one degree of freedom the watt balance equation holds. This insight greatly reduces the complexity of the watt balance and allows to build inexpensive table top watt balances.

2. Designs for two table top watt balances

As a first step toward measuring a 10 g mass with a relative uncertainty of 10^{-6}, we have built two table top watt balances to study the principles and their limitations. The two balances are different in design. One design was built around a seismometer suspension, the other is a classic beam balance using a jewel bearing to support the beam.

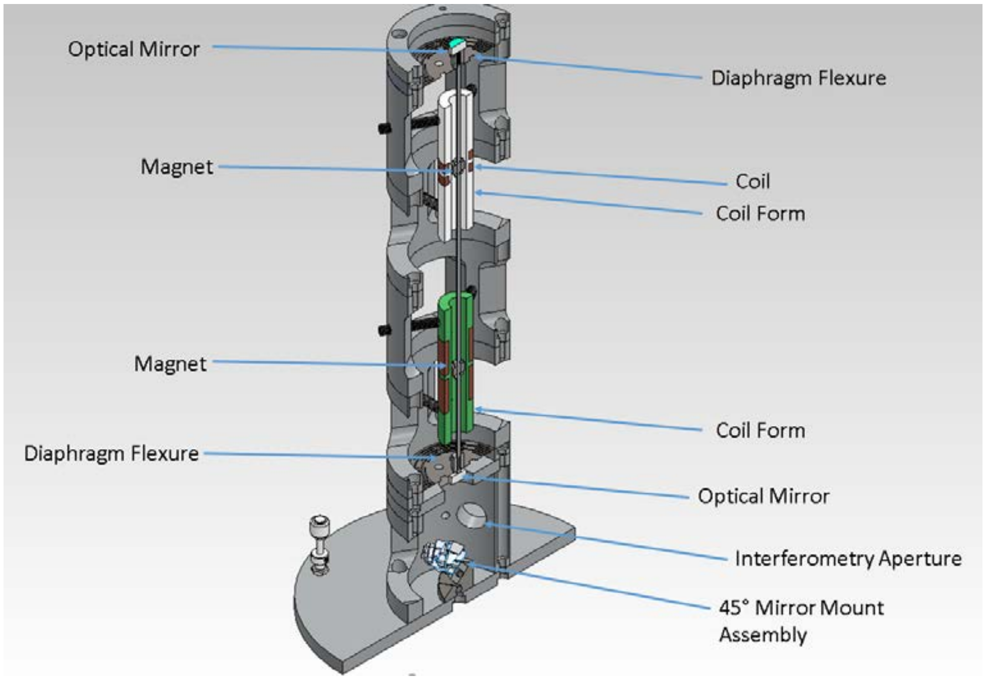

Figure 1 Cut away drawing of the seismometer watt balance.

2.1 The seismometer watt balance

Figure 1 shows a conceptual drawing of the seismometer watt balance. A thin rod holding two magnets is guided by two diaphragm flexures clamped at the top and bottom of an aluminum tube tower. The tower has two magnet coil systems. One can be used as a motor and the other as a generator during velocity mode. Unlike in a conventional watt balance, where the coil moves and the magnet is stationary, here the magnet moves and the coil is stationary. The rationale for this is to keep the moving part as light as possible for reasons that are explained in the following paragraph. The small ring magnet can be made much lighter than the copper windings. As an added bonus, no electrical connections need to be made to the moving part. Two stationary coils, wound in opposite directions and connected in series surround the permanent magnet. This coil geometry is only sensitive to the magnet moving within the coil system and insensitive to stray magnetic fields, which would induce the same but opposite electromotive force in both coils.

In contrast to a beam balance, where the central pivot supports the dead weight of the mass pans and the beam, but only a mass imbalance creates a torque around the pivot, the flexures in the seismometer balance have to support the dead weight and measure along the same degree of freedom, the vertical axis. The sensing degree of freedom is the same as the load bearing degree of freedom. This coincidence requires an important design compromise between having soft flexures to improve sensitivity and stiff flexures to minimize sag of the moving part if none of the coils is energized. Consequently, the moving part should be designed to be as light as possible to increase the sensitivity to the signal. In our case the total mass of the moving part is

12 g and the flexures were designed to have 1 mm sag for this load.

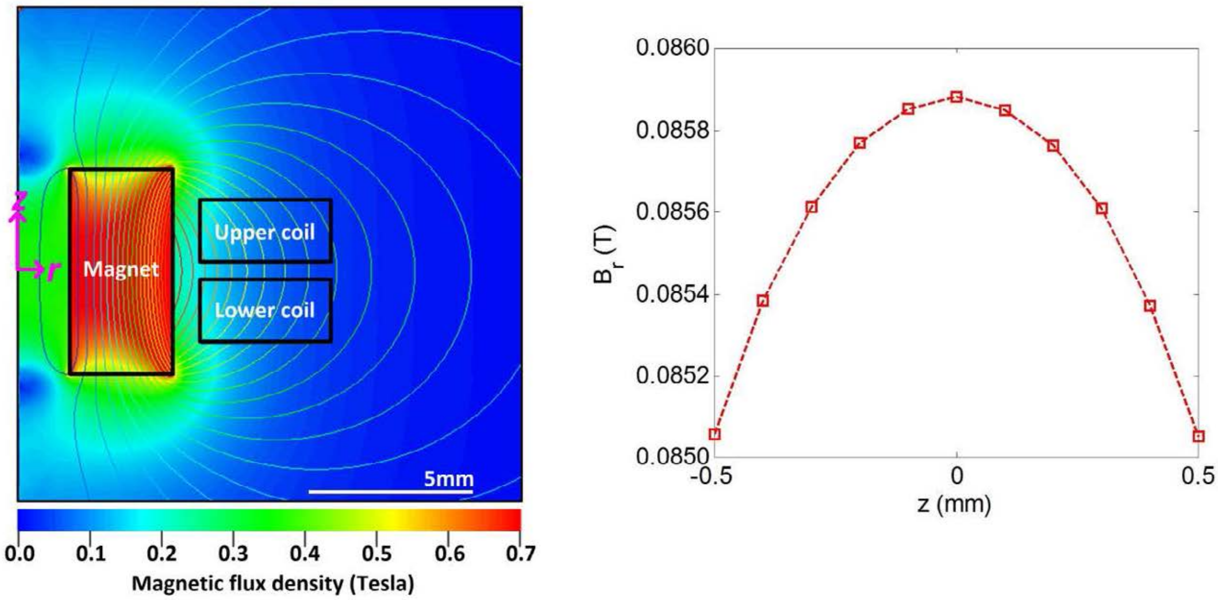

Figure 2 Design of the new field coils for the seismometer balance. On the left: Finite element calculation of the magnetic flux density as a function of radius and vertical position. On the right:The radial component of the magnetic flux density as a function of position.

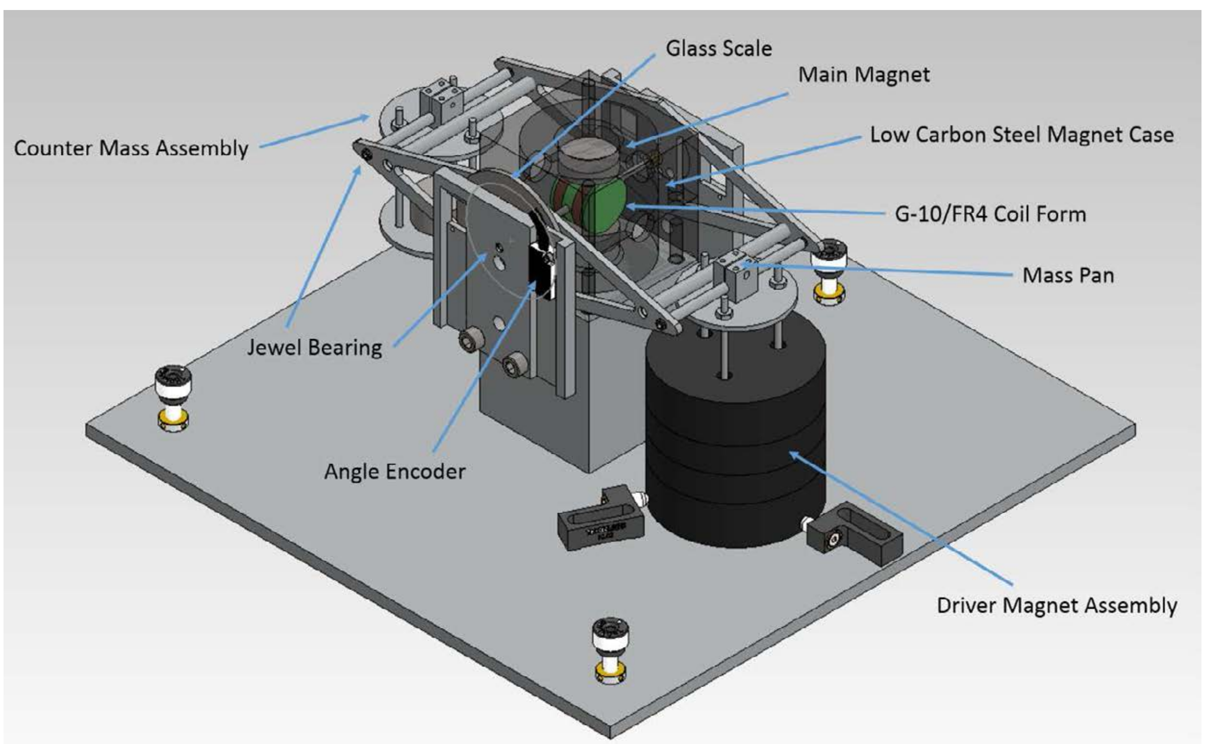

Figure 3 Drawing of the beam balance. Currently an angle encoder is used to operate the balance, but we are planning on using a n interferometer to read the position and velocity of the coil.

In its current design, the upper coil has a geometric factor of $Bl = 4.4\ \mathrm{T\ m}$. The radial distance between the coil and the permanent magnet is too large. This distance is given by the clearance of the magnet and the inner diameter of the coil form and the thickness of the coil form. Both dimensions can be reduced by a factor of two and we hope to increase our geometric factor by four. The new design of the magnet system is shown in Figure 2.

Laser interferometry is used to measure the position and velocity of the rod. The measurement beam enters an aperture at the base of the instrument and is reflected off a mirror that is mounted on the bottom of the lower diaphragm spring.

One concern of this design is cross-talk between the upper and lower coil system. By driving the oscillator at its resonance frequency this coupling can be made small. In resonance, the quotient of the motion amplitude to the current amplitude is enlarged by the quality factor of the system when compared to the same quotient at very low frequencies. In the system described here the resonance frequency is 9.2 Hz and the quality factor is 150.

It is technically possible to build a system that has only one coil magnet system. The measurement would then be performed by driving the system at resonance. After a certain amplitude is reached, a relay switches the coil from the current source to a voltmeter and the geometric factor is measured as the oscillation decays. We have tried this mode of operation, but the data quality with a constantly driven system is better.

We have built the system described above and are currently working on the data acquisition and analysis. First measurements in velocity mode have been made. We are currently working on improving the timing of the data acquisition, in particular the synchronization between the

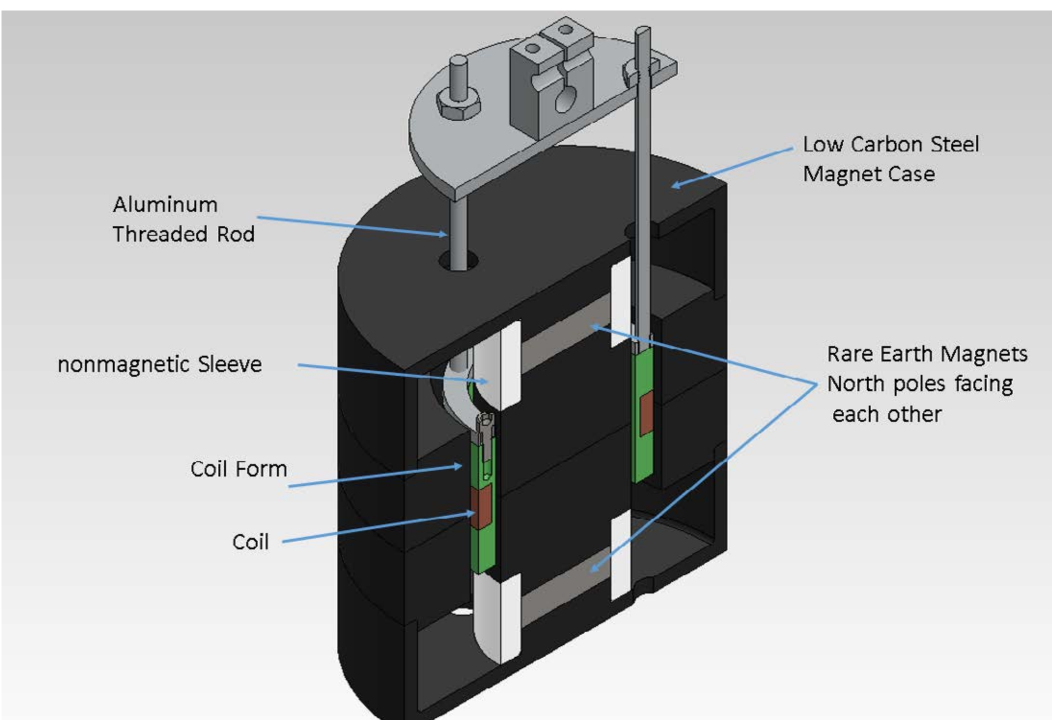

Figure 4 Cut away view of the permanent magnet system used for the beam balance.

voltage and velocity measurements.

2.2 The beam balance

Figure 3 shows a three dimensional model of the beam balance. Jewel bearings are used for the three pivots (the center of the beam and on each end of the beam). Two magnet systems are implemented. Again, one is used as motor and the other as a generator during velocity mode. One magnet system is located around the central pivot axis, the other at the end of the beam. The system at the center of the beam is similar to a direct current (DC) motor, a planar coil on a shaft inside a homogeneous magnetic field. The system at the end of the beam is modeled after the magnet system that has been designed for the watt balance built by the International Bureau of weights and Measures (BIPM) [5]. A cut away view of the magnet system is shown in Figure 4. The position readout is performed using a commercial angle encoder.

The magnet systems of the beam balance are much more massive and larger geometric factors are achieved. For the magnet system at the end of the beam we measure $Bl = 42$ T m.

The beam balance has been built and we have been focusing on the force mode measurement. We use the magnet system at the end of the beam to balance the weight of a mass.

3. Discussion and Summary

We have built two table top watt balances that are capable of weighing masses on the order of 10 g. We are currently finishing our data acquisition and analysis routines. For each balance, we are close to getting one mode to work. At the symposium, we hope to present the results of both modes combined for both balances.

3.1 Advantages and Disadvantages of the seismometer balance

The biggest disadvantage of the seismometer balance at the moment is its relatively small geometric factor. The magnet has to be small and light. The flux is not concentrated with an iron yoke causing an inefficient use of the magnetic energy. The next largest disadvantage is the fact that the weight of the moving part has to be carried by the spring, reducing the effective sensitivity. The third disadvantage of this system is that it is a seismometer. Vibrations of the ground and the table couple into the measurement. The rod is inertial for frequencies higher than the resonance frequencies, but the surrounding coils are moving. Another issue to investigate is the frequency dependence between the two modes, because the weighing is performed at DC, but the velocity is performed at 9.2 Hz. The first advantage of the seismometer balance is ease of construction, because of its small part count. The second advantage is the fact that the flexures don’t suffer from static friction. A small mass will cause a small deflection. The third advantage is that no electrical connection needs to be made to the moving part.

3.2 Advantages and Disadvantages of the beam balance

The friction in the pivots is the largest concern with the beam balance. Will the balance be sensitive enough? If not, the balance needs to be redesigned with flexures which might cause problems for the largest motions. Another disadvantage is that the beam balance requires routing of a total of four wires to the two coils on the moving parts of the balance. Special care must be taken routing these wires to avoid torques and especially time varying torques. The torque produced by these wires can change with temperature of the wires which depends on the amount of current these wires are carrying. Since the current in the wires is correlated with the weighing this can cause a systematic bias. The big advantage of the beam balance is that it allows the design of a powerful magnet system. The weights of the coil can be compensated by weights on the other side of the beam. For comparison, we achieved a geometric factor of 42 T m, about 8 times more than in the seismometer balance. Also the beam balance allows for slightly larger motions. In the seismometer balance the linear range of the spring motion is about ±1 mm, whereas the beam allows for ±3 mm.

3.3 Summary

While both balances have their advantages and disadvantages, it is unclear which of the two concepts is more promising regarding our goal to achieve a relative uncertainty of 10^{-6}. In the next few weeks, the software and the alignment of the balances will be finished. Then, a detailed assessment of the uncertainties in both systems will be performed. Based on the uncertainty budgets of the two systems, a decision which to develop further can be made.

References

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